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ORDINARY MEETING.

15 December, 1936.

Sir ALEXANDER GIBB, G.B.E., C.B., F.R.S., President, in the Chair.

The Council reported that they had recently transferred to the class of

Members.

Cyril Bernard Harvey Clark, B.Sc. (Eng.) (*Lond.*).
Jack Duvivier, B.Sc. (Eng.) (*Lond.*).
Samuel Thomas Farnsworth, B.Sc. (Eng.) (*Lond.*).
Walter Fidkin.
John Alfred Hardie.
John Lamb Paterson, B.Sc. (*Edin.*).
Donald Hugh Baillie Reynolds, M.A. (*Cantab.*).

And had admitted as

Students.

Rex Archibald Addison, B.Sc. (Eng.) (*Lond.*).
Guy Dunn Allison, B.Sc. (*Durham*).
Robert Lawrence Armstrong, B.A. (*Cantab.*).
Ivan Dussard Arscott.
Lewis Donald Bannister.
Eric Arthur Joseph Barnett.
Arthur John Batchelor.
John Walter Baxter, B.Sc. (Eng.) (*Lond.*).
Hubert Leslie Binning.
Reginald Walter Bishop, B.Sc. (Eng.) (*Lond.*).
Donald Robinson Blackburn.
Henry Kenneth Wykeham Blood.
John William Boyle.
Arthur Thomas Boyson.
Stewart Harman Brown.
Dennis George Cartwright.
Henry Goudoever Cochrane, B.Sc. (Eng.) (*Lond.*).
Wilfrid Willis Cotton.
David Thomas Wayne Davies.
Harold Joseph Davies.
William Oliver Davies.
Paul Davison.
Peter Dendy.
Ferdinand Shaw Eiloart.
William Evans.
William Arthur French.
Eric Harpur Fryer.
Ronald George Geering.
Martin Oatley Glew.

Frederick Dale Goodwin, B.A. (*Cantab.*).
Gerald Christopher Goulder.
Arthur George Chapman Hannaford.
Robert Joseph Harding.
William John Hempsall.
Ernest Hockley, B.Sc. (Eng.) (*Lond.*).
Homi Nusservanji Kanga, B.Sc. (*Edin.*), B.Sc. (*Bombay*).
Geoffrey Morris Kenworthy.
Henry William Richard Kingston.
Michael Landy.
Edward Lewis.
Eldred Philip Linton.
Charles Alexander Macdonald.
Terence Harrison McGrath, B.Eng. (*Liverpool*).
Keith Haig McGregor.
Colin Archibald MacNicol, B.Sc. (*Durham*).
Charles Bertram Marshall, B.Sc. (*Manchester*).
Maurice Milne, B.Sc. (*Aberdeen*).
Denis Edmund Nash.
Richard Gordon Nelson.
Howel Griffith Nicholas, B.Sc. (*Wales*).
Rhys John Nicholas.
John Walton Norris.
Philip James Pye.
Royston Wilfred Querèe.
William Thomas Noel Reeve, B.A. (*Cantab.*).
Alan Peter Rose.
Ernest Percy Scott.
George Herbert Scott, B.Sc. (*St. Andrews*).
James Simpson.
Ian Small.
Kenneth Hinchliffe Soulsby.
Frank Lionel Stark.
Anthony George Temple, B.A. (*Cantab.*).
George Ingham Temple.
Philip Thomson Thomson-Walker, B.A. (*Cantab.*).
David Sydney Tilzey.
Eric Tranter, B.Sc. (*Bristol*).
Frederick Stanley Warwick.
Idris George White.
Dermot Francis Wilkin, B.Sc. (*Belfast*).
Roy Harry Willmington.
John Stuart Woosley, B.Sc. (Eng.) (*Lond.*).

The following Paper was submitted for discussion, and, on the motion of the President, the thanks of The Institution were accorded to the Author.

Paper No. 5089.

"The Second-Stage Development of the Lochaber Water-Power Scheme."[1]

By Arthur Holden Naylor, M.Sc., B.Sc. (Eng.), M. Inst. C.E.

TABLE OF CONTENTS.

Introduction.

The Lochaber water-power scheme of the British Aluminium Company, Ltd., has been described by Mr. W. T. Halcrow in a Paper[2] read before The Institution. That Paper described the complete scheme in general terms and gave a detailed account of the first stage of development. In the present Paper it is proposed, therefore, merely to sketch an outline of the whole scheme in sufficient detail for its proper understanding before proceeding to the consideration of the second-stage development or Laggan–Treig works.

Physical Features.

A general idea of the physical features is essential in order to appreciate the evolution of the design of the works. At the end of the last glacial epoch the fringe of a great ice cap was retreating towards the west coast of Scotland. Great glaciers flowed therefrom in a general easterly direction, gouging out in their progress the valleys of the Treig and Spean and ultimately feeding the Spey. At a later stage Glen Roy, the Spean valley, loch Treig and loch Laggan

[1] Correspondence on this Paper can be accepted until the 15th June, 1937. —Sec. Inst. C.E.

[2] Minutes of Proceedings Inst. C.E., vol. 231 (1930–31, Part I), p. 31.

formed one huge lake, the geological loch Spean, hemmed in on the west by the ice barrier and overflowing eastward into the Spey at the point where now the river Pattack turns sharply to the west. Fine sand deposited around the shores of this lake now forms an extensive elevated beach at a level of + 860 feet O.D. The glacial action explains the great depths of lochs Treig and Laggan and the general absence of soil on the slopes of the valleys, a thin covering of peat masking hard rock planed smooth and scratched with glacial markings. The rock surface thus exposed consists of mica schist in the Treig valley, of granite at Laggan dam and of metamorphic schist higher up the Spean valley.

The sides of the valley ascend steeply to high mountains. Consequently not only is the rainfall high but the water losses, which are entirely due to evaporation, are small and the run-off is rapid. It was necessary, therefore, to make provision for high flood-discharges in the design.

The Complete Scheme.

In the complete scheme the upper Spey is diverted westward (Figs. 1 and 2, Plate 1) * into the river Pattack which flows into loch Laggan, so that whereas originally the river Spean flowed into the river Spey to discharge into the North Sea, the Spey will flow into the Spean and will discharge into the Atlantic Ocean. A dam 900 feet long and 30 feet high is to be constructed across the Spey 2 miles above Laggan bridge, and an aqueduct of 1,600 cusecs capacity will convey the water across the watershed into the river Pattack. These works will form the third and final stage of development.

The second stage comprises essentially the collection of the discharge from the loch Laggan and loch Ossian catchment areas in a reservoir formed by the construction of Laggan dam, its diversion through a tunnel to loch Treig, and the provision of further storage capacity by the construction of Treig dam.

From loch Treig a 15-mile tunnel conveys the water under the Ben Nevis group of mountains and the flow is augmented on the way by the absorption of numerous mountain streams. Emerging on the slopes of Ben Nevis overlooking Fort William at a level of about + 600 O.D., steel pipe-lines convey the water to the power-station, where the turbine nozzles are at a level of + 19 O.D.; a tail-race ¾ mile long discharges into the river Lochy near its mouth. These works comprised the first stage of development and were completed in 1929. Water from a total catchment area of 303 square

* Reproduced from Mr. Halcrow's Paper.

miles will be led to this power-station which, with a final capacity of 120,000 HP., is the largest water-power unit possible in Great Britain, apart from the tidal power-station of the Severn barrage scheme. The completion of the Laggan–Treig works renders available an amount of water equal to $\frac{5}{6}$ths of the ultimate supply.

The Second-Stage Development: General Considerations of Design.

The determination of the requisite storage capacity, position of dams, and size of tunnel involved consideration of many factors. In a water-supply scheme the continuity of a uniform supply is essential and calculations of storage are therefore based upon the average flow during the 3 driest consecutive years. In a water-power scheme overall economy is the ruling consideration. The amount of storage required will depend upon the method of regulation. The power obtainable from every cusec of flow is proportional to the pressure head, and therefore where, as at loch Treig, the reservoir is connected to the power-house by a pressure tunnel, the maintenance of a high water-level in the reservoir is as important as the prevention of wastage of water by overflow. By reducing the consumption of water by n cubic feet when the reservoir is low, n more cubic feet are available for power at a higher water-level, and so the total power produced with a given storage capacity is increased. From this reasoning it would appear to be advisable to vary the flow of water between wide limits so as to concentrate it as far as possible at high water-levels, especially as there would be little difficulty in storing until required any excess of aluminium produced. On the other hand, a consideration of the amount of plant, size of factory, diameter of tunnels, etc., indicates the desirability of maintaining a constant output. Economy lies between these extremes. When the Lochaber works are fully developed there will be required sufficient plant to permit the load to be increased by about 30 per cent. when the reservoir is nearly full and the water-level is likely to rise, in order to reduce the loss of energy through overflow. In any case a certain amount of additional plant would be required to serve as a standby during repairs.

Loch Treig is the main storage reservoir. By the construction of a dam to raise the overflow level from its original height of + 784 O.D. to + 819 O.D., a capacity of 7,838 million cubic feet is obtained above the lowest level to which it can be drawn down, namely + 695 O.D. By arranging that loch Laggan reservoir should be capable of being drawn down to + 804 O.D. and should spill at + 820 O.D., a further storage capacity of 1,480 million cubic feet is rendered available. This additional storage, although relatively

small, is very important, as about three-quarters of the catchment drains into the Laggan reservoir and local storage is necessary to prevent excessive flood-loss due to the limited capacity of the Laggan–Treig tunnel. There is the further advantage that by creating a large surface area for the combined reservoirs at high water-levels, it enables the Treig reservoir to be maintained at a higher average level, with consequent gain in power.

Records of the flow of the river Spean and of the overflow of loch Treig had been kept since the commencement of the first-stage works; from these it was possible to calculate by a step-by-step process the flood-losses with various schemes, and to arrive at a design embodying the most economical combination of factors.

Actually, the possible economic storage was closely defined by physical conditions. The overflow level of + 820 O.D. for Laggan dam could not be exceeded on account of the danger of flooding Ardverikie Lodge on the shore of the loch and portions of the main road from Fort William to Newtonmore. In order that, in general, overflow should commence at about the same time at Laggan and Treig dams, the crest level of the latter was set 1 foot lower, namely + 819 O.D., to allow for the hydraulic gradient in the tunnel. In order to be able to draw-down the water-level in loch Laggan it was necessary to dredge a channel through the sand-bar at its lower end. The lower limit of + 804 O.D. in the Laggan reservoir was determined by the high cost of dredging this channel to a lower level. This would have entailed much rock excavation in addition to a large increase in the soft excavation.

The fluctuation in the water-level at Laggan dam, combined with the high overflow-level at Treig dam, put beyond consideration the use of an open aqueduct connecting the two reservoirs. In order that the capacity of such an aqueduct should be adequate at low-water level it would have had to have a full cross-sectional area below + 804 O.D., and at the same time its sides would have had to extend up to about + 820 O.D.; a tunnel was therefore adopted. The good quality of the rock was a further argument in favour of this course.

The level of the tunnel invert at the entrance was fixed at + 789 O.D. so as to give the full calculated cross-sectional area below + 804 O.D., the lowest level of Laggan reservoir. There is a fall of 15 feet in its length of $2\frac{3}{4}$ miles. An increase of gradient would be of no advantage as the capacity of the tunnel and the flood-loss at Laggan are determined by the difference in the water-levels of the lochs. The above gradient allowed the tunnel to be taken under the London and North Eastern Railway, which passes down the Treig valley skirting the eastern shore of the loch, and to dis-

charge at the level of the valley floor. In calculating the size of the tunnel consideration had to be given to flood-losses and the level of loch Treig. A small diameter would result in an increase of flood-loss at Laggan dam, some reduction of flood-loss at Treig dam, and a generally lower water-level in loch Treig. With a sufficiently large diameter of tunnel the reservoirs would behave together as one. At the economic size the capitalized value of the increase of power with increase of diameter equals the increase of cost. In this way a diameter of rather more than 14 feet was indicated. In the calculations of discharge a value of 142 for C in Chezy's formula was assumed, as in the calculations for the Ben Nevis tunnel.

Position of Laggan Dam.

The lower end of loch Laggan is partially blocked by a part of the raised beach mentioned in the early part of this Paper. Below this the river Spean ran at a flat slope through sand and gravel for 3½ miles, after which it fell over a sill of rock, and below that it tumbled down at a steep gradient. It was clear that the length of tunnel must be reduced by the dredging of a channel from loch Laggan. At first sight this sill would appear to be the obvious position for a dam; a height of dam above river-level of only 40 feet would be required, with a crest length of about 550 feet. The alternative site was the position chosen at a constriction in the valley 1¼ mile lower down, where the dam is 130 feet high above river-level and 700 feet long. It was estimated that there would be a saving in cost by adopting the higher dam and shorter tunnel. The diameter of the shorter tunnel would be somewhat less for the same capacity. In addition to the saving in cost, there was the advantage of increased storage capacity, and the need for an aqueduct to divert the waters of the Roughburn into the reservoir disappeared. A further possible advantage was that a large volume of dead storage below the level of the tunnel invert is available for the accumulation of sand and silt should this be necessary.

Detailed Considerations of Design.

Dredged Channel for River Spean.

The bed of the river Spean at the outlet from loch Laggan was at a level of + 816·5 O.D. It was necessary to lower the outlet by dredging in order that the storage capacity of loch Laggan might be rendered available. The dimensions and gradient of the dredged channel were regulated by the need to be able to draw down the loch-level to + 804 O.D. A channel with a bed-level of + 802 O.D. at loch Laggan and a base-width of 35 feet as far as the confluence with the river Ghuilbinn (the width thereafter being increased to

52 feet), was adopted ; the bed was formed to a gradient of 1 in 7,000. The side slopes were formed at $1\frac{1}{2}$ to 1 except where fine sand or clay rendered a flatter slope desirable. In the event of the channel tending to scour the presence of a rock-bar in the bed will maintain the requisite minimum water-level in loch Laggan. The new channel cut across unnecessary bends in the river, and in general the course was straightened and the curves were " sweetened."

Laggan Dam.

A very marked constriction of the valley at the Laggan dam and numerous exposures of hard granite on the north bank and high up on the south bank suggested a very suitable dam site. Prickings gave a good indication of the position of the rock surfaces over the remainder of the south bank. The river-bed was filled with boulders, gravel, and sand, so that to determine the depth of bed rock would have entailed the sinking of trial pits in mid-stream. This was not considered necessary as a rough estimate of the depth could be made from consideration of the side slopes. The fact that the sides of the gorge at the dam site had withstood the tremendous pressures, shearing forces, and erosive action of the glacial period with less wear than the adjacent reaches of the valley was felt to be a guarantee of good-quality rock.

This fact, and the fairly regular steep side-slopes, pointed to the possible economy of constructing an arch dam. Designs for arch and gravity dams were prepared, but it was found that there would be no saving in cost with an arch dam. The length of Laggan dam, 700 feet, appeared to be about the limit beyond which an arched dam ceased to show an advantage. In recent years this length has been exceeded in an arch dam, but the wisdom of such a design is open to question in view of the lack of knowledge of the stresses set up by temperature and shrinkage. A gravity dam, slightly curved in plan (2,000 feet radius) was adopted (Figs. 3 and 4, Plate 1). This curvature did not involve any increase in volume. It could not, in view of the inevitable contraction of concrete in setting and cooling, appreciably affect the stresses in the dam. The advantage lies in the fact that in the extremely unlikely event of failure of a sector or block of the dam it would immediately become wedged between the adjacent blocks, and a disastrous failure would be impossible. A further advantage is that a curved dam was considered to harmonize better with the natural surroundings.

In view of the sound nature of the rock it was considered that hydrostatic uplift pressures would be negligible. Nevertheless the design was based on the assumption of 50 per cent. of uplift being present. On this basis the maximum vertical compressive stress

at the upstream face with the reservoir empty will be about 11 tons per square foot, and at the downstream toe with the reservoir full from 9 to 10 tons per square foot. This corresponds to a maximum principal stress at the downstream toe of under 15 tons per square foot. As this is carried well down into sound rock, and as the curved toe is of considerable thickness, there is no likelihood of tensile stress being set up in the concrete of the downstream toe. To minimize uplift pressures the excavation at the upstream face was carried down in a cut-off trench to close-jointed rock. Behind this a rubble drain was formed connecting to copper relief pipes discharging into an inspection gallery in the heart of the dam. The concrete was of 7-to-1 mix with 2-inch aggregate, with the exception of the facing, which was of 4-to-1 concrete with 1-inch aggregate.

The dam was constructed in blocks separated by radial contraction joints at about 45-foot intervals; watertightness at the joints was ensured by a joggled copper strip embedded in the adjacent blocks. The joggle is surrounded by a 6-inch diameter cylindrical space filled with a bituminous mixture consisting of equal parts of natural Trinidad asphalt and coal-tar pitch with 15 per cent. of refined coal tar. These proportions were determined as the result of extrusion tests designed to indicate the tendency for the mixture to escape into an open contraction joint. A roadway was carried across the top of the dam on a series of arches to provide access to the estate on the south side of the river.

Laggan Dam Spillway.—The estimated maximum flood-discharge at Laggan dam is 14,000 cusecs. This includes a contribution of 1,600 cusecs from the Spey conduit and is based on a run-off of 3·17 inches per day from the catchment area of the Laggan reservoir. The upper limit of water-level as stipulated in the Lochaber Water Power Act of 1921 was not given simply as a maximum permissible flood-level. It was laid down that not only should the water-level in the Laggan reservoir not exceed + 822 O.D. for more than 24 hours, nor + 820·5 O.D. for more than 72 hours, but also that the spillway-level should not be higher than + 820 O.D. The adoption of an all-siphon spillway or an all-tilting-gate spillway would have permitted the observance of a maximum water-level of + 822 O.D. with a spillway-crest level of about + 821·5 O.D., and this would have given a valuable increase in storage capacity at little additional expense. Such a scheme was, however, not permissible. With a plain overflow spillway extending the full length of the dam the crest would have had to be set below + 820 O.D. in order that the specified flood-levels should be complied with. It was therefore necessary to adopt an intermediate scheme, and a plain

overflow spillway was combined with a battery of six siphons of 3,600 cusecs capacity, both with crest levels of + 820 O.D.; the siphons are, however, set to operate as such at higher levels. Some additional increase in the discharge can be obtained by a scour culvert of 5-foot outlet diameter. Stop-logs have also been provided in the two bays at the south end of the dam, whereby the crest-level may be lowered by 3 feet. In a reservoir 12 miles long, such as Laggan, the effect of wind and of flood run-offs of high intensity from the catchment area makes it difficult to forecast the variation in water-level along the reservoir itself. Should some adjustment be found necessary the stop-logs provide a ready means for effecting this.

The scour culvert set at a level of + 712 O.D. is required to operate under a head of about 110 feet. A Glenfield free-roller sluice gate was fitted at the entrance to the culvert (Fig. 4, Plate 1). The latter, lined with mild steel through the dam, is 6 feet in diameter. At the downstream face a Glenfield combined needle-valve and disperser of 5 feet outlet diameter was fitted. Both roller-gate and needle-valve are operated from a chamber formed at the crest of the dam, the needle-valve being hand-operated through shafting carried down the downstream face of the dam, and the roller-gate being suspended by steel ropes from a petrol-driven winch.

The downstream face of the dam terminates in a curved toe protected by granite pitching set in cement mortar, in order that the overflow should leave the dam horizontally without impact on the foundations. On the north bank little erosion of the valley slopes could occur as the rock was exposed at the surface. A considerable depth of soil and fir plantation extending down to the river on the south bank made it desirable to confine the overflow to the width of the toe of the dam by a training wall. The design of this wall so that it should not be overtopped by the turbulent overflow was determined as a result of experiments with a small-scale model $\frac{1}{96}$th full size. An overhang coping which would throw the surging water back on to itself was found to be an advantage.

Siphons.—As Laggan dam represents the first instance in Great Britain of the embodiment of a large siphon spillway in the design, it may be of interest to describe the siphons in some detail.[1] After consideration of the uncertainty of the precise depth of overflow at which priming of a siphon will take place if permitted, it was decided to attempt to arrange for the priming of two siphons with

[1] A fuller account will be found in the Author's book, "Siphon Spillways," London, 1935.

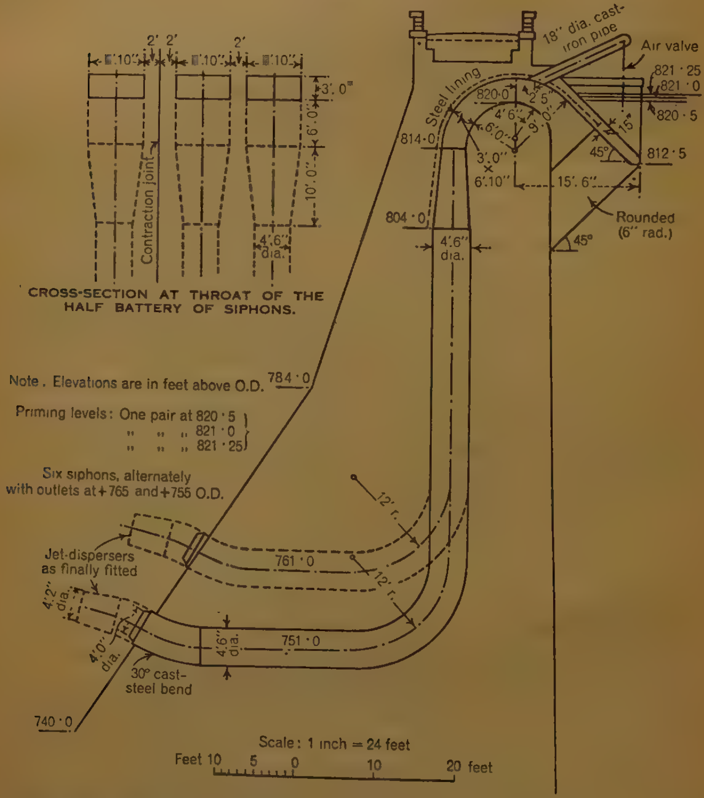
6 inches of overflow, two siphons with 1 foot, and two siphons with 1 foot 3 inches of head above crest-level. Experience abroad indicated that priming could be arranged to take place with a head above the crest of $d/3$, where d denotes the depth of the throat of the siphon, and that, if permitted, it would probably take place earlier. This was confirmed by experiments carried out on a scale model $\frac{1}{8}$th full size by Professor S. M. Dixon, M.A., B.A.I., M. Inst. C.E., at the City and Guilds College; these indicated that with a throat depth of 3 feet priming at 6 inches head would be a possibility, and at 1 foot a practical certainty. A depth of 3 feet was accordingly chosen.

The distribution of velocity in the upper bend of a siphon approximates to that of a free vortex, the velocity at the crest being greater than that at the crown and the pressure at the crest being correspondingly less than at the crown. Where, as at Laggan dam, the operating head may be made as high as is desired, the velocity at the crest may be made as high as about 40 feet per second, which would correspond to a vacuum pressure of about 24 feet of water. Experience indicates that it is not advisable to exceed such a vacuum. The velocity at the crown will depend upon the radius of the upper bend, and so for maximum discharge the radius must be as large as possible. By forming the mouth of the siphons outside the section of the dam a radius equivalent to 6 feet at the crest is obtained and an intensity of discharge of 96 cusecs per foot of crest is possible. Considerations of strength and the spacing of contraction joints in the dam influenced the choice of the width of the siphons at the throat; the width was finally made 6 feet 10 inches. Such a throat design is suitable for a discharge up to 650 cusecs. It was decided to instal six siphon units. The section of the siphon is shown in *Figs. 5*, p. 12. An alternative arrangement with the siphon carried externally down the dam was considered, but it has several disadvantages: greater cost, greater liability to vibration and the concentration of flow on the toe of the dam.

With a total fall to the river bed of about 130 feet, a "high head" type of siphon (that is, one in which the passages have to be tapered to prevent the formation of an excessively high vacuum at the throat) is indicated. The outlets were made 4 feet diameter and set alternately at 55 feet and 65 feet below the crest in order to reduce the stresses set up in the dam. By inclining the outlets at 30 degrees above the horizontal, the issuing jet should strike the river-bed at about a maximum distance from the toe of the dam, and should thus reduce to a minimum any danger of the dam being affected by vibration or undermining of the toe. The siphon passages were made circular and of 4 feet 6 inches diameter up to within 16 feet of

crest-level. In the interest of economy in construction the section was kept constant over as long a length as possible. Above a level of 16 feet below the crest it was necessary to reduce the velocity in order to prevent the pressure falling unduly low, and a transition length was introduced in which the cross-section was gradually changed, attaining full throat section at the horizontal diameter of the crest.

Figs. 5.



The good priming qualities of a siphon of this type, with the lower limb vertical and no special priming device, such as an auxiliary siphon or a priming step or projection in the lower limit, depend in the first place upon the overflowing water springing clear from the siphon wall and being broken up so as to carry down with it the air in the vertical limb. Theory indicated that water flowing over a circular weir should spring clear at a point about 60 degrees from

the vertical radius, and this was verified on a full-scale model, the water being found to spring clear when the head above crest-level exceeded 2 inches.

The inlet cowl forming the mouth of the siphons is bell-mouthed and the lip is set well below crest-level, being immersed over 8 feet when the siphons are in operation. In this way the loss of head at the entrance is reduced, the tendency for the water surface at the siphons to be drawn down becomes negligible, and the formation of air vortices is discouraged. Intermittent making and breaking of the siphons due to the lip becoming uncovered by wave action is also prevented. Another advantage of a low inlet is that in the extremely unlikely event of the mouth being sealed by a sheet of ice it would most certainly be burst inward by the upward pressure of water before the loch rose to crest-level. Gratings of parallel flat bars across the siphon mouths prevent the entry of large debris. They are set at an angle of 45 degrees to the horizontal in order to prevent the possibility of a large floating object such as a boat being held down and submerged as the water-level rose. By interposing a resistance to rotational motion the gratings also tend to discourage the formation of vortices. Nevertheless, in spite of all these precautions it is interesting to note that small vortices have sometimes formed when the siphons have been in operation. The siphons as thus designed give calculated discharges of 640 and 590 cusecs according to whether the outlet is set at + 755 or + 765 O.D. respectively. The $\frac{1}{8}$th-scale model indicated a discharge for the siphon with the higher head of 613 cusecs, which, considering the greater effect of viscosity in the model, constitutes good agreement.

The carrying down of the inlet lip below crest-level makes necessary the provision of some means of breaking the siphons, as otherwise once they had primed they would continue to operate until the reservoir had been drawn down to lip level. The usual method is to provide an air vent or a pipe with its mouth set at the desired level, communicating with the interior of the siphon hood. It is desirable that this air passage should not be too large, as its mouth, being only just submerged when the siphons are in operation, would be intermittently unsealed if the flow of water through it was sufficient to produce a local draw-down of the water surface. As it is not possible to calculate the requisite size of vent recourse must be had to experience elsewhere. The ratio of area of air vent to area of siphon throat has varied from 1/4·3 to 1/28 in successful installations. A model-test with a siphon of $\frac{1}{8}$th scale in cross-section but with full suction head of water indicated that a ratio 1/22 would be sufficient. To allow an ample margin 18-inch diameter air pipes were provided, giving a ratio of areas of 1/11·5.

It was decided to adopt simple air-inlet lips to be sealed by the rising water-level for the central pair of siphons, and in the case of the other siphons to utilize automatic air valves. The simple inlets each consist of a straight lip formed by a bent steel plate 6 feet 4 inches long, enclosing a considerable inlet area so as to reduce velocity and prevent local draw-down of the water surface when the siphons are in operation. These inlets are shielded from the effect of wave-action by a curtain wall carried down 8 feet below the water surface. The amplitude of wave-motion diminishes rapidly with depth below the surface, so that although the curtain wall is quite open at the bottom the wave motion inside has been scarcely perceptible.

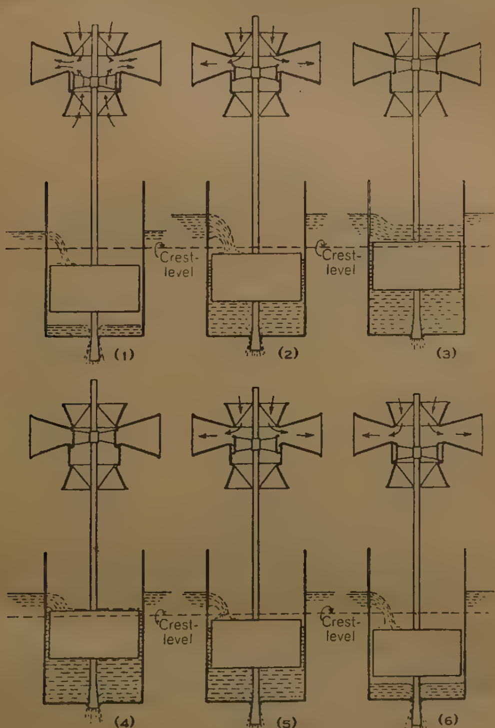
On each of the remaining two pairs of siphons, an air-valve designed by Mr. H. D. Morgan, M.Sc., Assoc. M. Inst. C.E., of the staff of Messrs. C. S. Meik and Halcrow, was installed. Diagrams illustrating the principle of action are given in *Figs. 6*. The air-valve consists of a balanced cylindrical valve operated by a float to which it is rigidly connected. This float lies in a tank into which water flows over a notch as the reservoir-level rises; the water then passes out through a drain led through the dam and discharging on its downstream face. At a certain level the rate of inflow exceeds the capacity of the drain and the tank fills, causing the float to rise and so close the air valve, and the siphon primes. The reverse process takes place with falling water-level and the valve opens, breaking the siphon positively and quickly when the water-level is only 3 inches below priming level.

Tests on the siphons since the completion of the works show that priming with air valves closed or lips sealed will take place with a head above crest-level of only 5 inches, or $d/7$; this is very satisfactory. With the plain air lips there was no draw-down of the water surface and no tendency for air vortices to form. Priming took place as soon as the water surface sealed the lip. Breaking took place at a rather lower level than was anticipated. As the water surface fell below the level of the air lips and air commenced to be drawn in, a to-and-fro oscillation was set up behind the wave curtain, which was greatly augmented by the placing of the air lips for the adjacent siphons in opposition to each other. This checked the inflow of air and delayed the breaking of the siphons. The prevention of this surging by a partition wall and a slight modification to the air lips should ensure earlier cessation of siphonic action. The performance of the air valves was excellent.

A modification to the siphon outlets was decided upon during the course of construction. When exposed, the rock in the river-bed, although sound granite, was so intersected by vertical joints that it

was felt that the impact of 600 cusecs in a concentrated jet of high velocity might result in the blowing-out of blocks of granite. Accordingly the Glenfield-Kennedy jet-disperser was fitted, which, by giving a high velocity of rotation to the issuing jet causes it to diverge in the form of a cone, the water breaking up into a fine spray which falls harmlessly on to the river-bed. For the same discharge the outlet diameter at the disperser had now to be increased from the

Figs. 6.



4 feet required for a plain outlet to 4 feet 2 inches, and the angle of tilt was reduced to 15 degrees to allow for the effect of the now large air resistance on the trajectory of the jet.

The siphon passages, where they were of circular section, were lined with 4-to-1 concrete and were reinforced both circumferentially and longitudinally. For the transition length and the upper bend where the highest vacua would be experienced, a lining of welded mild-steel sheeting was provided. At the downstream face the 30-degree bend is of cast steel as it may be subjected to high

stresses due to shrinkage of the concrete or to thermal expansion of the steel, as well as the reaction of the jet when the siphon is in operation. The jet-disperser units are of cast iron.

Laggan–Treig Tunnel.

The Laggan–Treig tunnel passes through hard rock throughout its $2\frac{3}{4}$-mile length, with the exception of 250 feet at the outfall which passes through the gravel and sand of a raised beach. The first 11,000 feet from Laggan dam is in granite, the remainder being in mica schist.

A lined tunnel was adopted; an unlined tunnel of the same capacity would have been costlier. The cross-section shown in

Fig. 7.

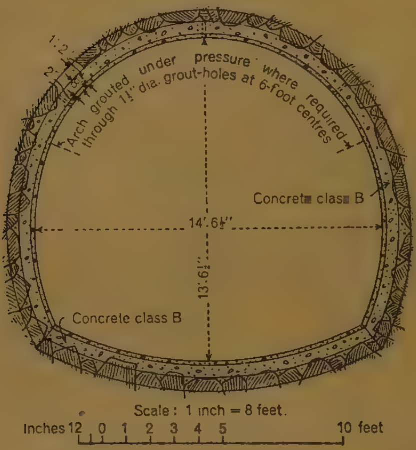


Fig. 7 was adopted for the tunnel in rock. The flattened invert permits the use of a double track during construction and facilitates the concreting of the invert. Holes were drilled through the arch in three rows at 6-foot centres to equalize the water-pressure on the two sides of the lining. The thickness of the lining could thus be reduced to the minimum requisite for continuity of surface. The lining was formed of 4-to-1 Portland-cement concrete with aggregate of 1-inch maximum size. The section through the raised beach was made circular and was formed in cast-iron segments; it was then lined with concrete. A control gate capable of withstanding pressure in either direction was installed at the tunnel intake for use in emergency. Grooves for stop-logs were provided at both the intake and the tunnel outfall.

Side-Stream Intakes.

Three streams passing over the tunnel are diverted by small dams into shafts discharging into the tunnel. As in the Ben Nevis tunnel, air shafts were provided to allow the escape of air entrapped by the falling water; such air might otherwise collect along the tunnel arch and, travelling against the flow, blow-out at the Laggan intake with possible disturbance to the control gate. The design of two of the shaft junctions with the tunnel is similar to that used and found satisfactory in the Ben Nevis tunnel and is described and illustrated in the earlier Paper.[1] As the remaining shaft was utilized as a working shaft during construction it was formed directly above the tunnel and the design of the water shaft and junction was modified so as to avoid additional excavation.

Treig Dam.

Treig dam is situated at a constriction in the valley at the end of Idir loch, a small loch at the northern extremity of loch Treig. The bed rock of mica schist is exposed on the west side but on the east side it passes under a raised beach of sand and gravel. The floor of this valley is filled with sand and gravel running back at a flat gradient from a rock-bar at Fersit about a mile to the north. The rock level in the centre of the valley on the line of the dam is at a general level of + 750 O.D., with a deep depression on both sides. During the period of construction loch Treig was drawn down to a considerable extent, and at about this same level a layer about 1 inch thick consisting of millions of hazel nuts was uncovered. This correspondence with the general level of the rock at Treig dam indicates that at some time subsequent to the glacial period, and before the accumulation of detritus on the floor of the valley, the loch had overflowed at this level.

The extent and depth of the raised beach of fine sand and gravel at the east end of Treig dam made it imperative to adopt a design of dam embodying an impervious core-wall which would form a watertight seal up to the crest-level + 819 O.D. The design is illustrated in Figs. 8 and 9, Plate 2. The exposed dam has a length of 380 feet and a height of 40 feet, whereas the length of the core-wall at crest-level is 675 feet and the maximum depth is 122 feet. The absence, save in small variable pockets, of clay suitable for a puddle core and impervious filling, and the availability of a large quantity of rubble from the tunnel excavation, were responsible for the choice of a rock-fill type of dam with a concrete core-wall keyed into

[1] W. T. Halcrow, "The Lochaber Water-Power Scheme." Minutes of Proceedings Inst. C.E., vol. 231 (1930–31, Part 1), p. 31.

the bed-rock. On the downstream side the core-wall is supported by rubble filling formed to a 3-to-1 slope with an inverted wedge of hand-packed rubble immediately adjacent to the wall. Soft sandy filling with impervious material against the core-wall is formed on the upstream side to a 3-to-1 slope, and is protected by 18 inches of granite rubble pitching bedded on an 18-inch layer of gravel and quarry ridd. The concrete core-wall is of 5-to-1 concrete below ground, except where it is encased in metal sheeting, in which case 7-to-1 concrete was permitted, the aggregate being of 2 inches maximum size. Above ground the mix was 4 to 1. The base of the core-wall is well keyed into the sound bed-rock. It has a minimum width of 6 feet at the junction with the rock and increases to a maximum of 10 feet at the surface. Above this it tapers uniformly to 5 feet at the top of the dam. On the upstream side the wall is reinforced by $1\frac{1}{4}$-inch diameter vertical bars at 12-inch centres.

Above ground-level, where there is a possibility of the core-wall becoming de-saturated, with consequent shrinkage, vertical contraction joints were provided at intervals of about 35 feet. They were rendered watertight by joggled copper strips, and relative deflexion between the adjacent sections of core-wall was prevented by forming a vertical key, tapered and greased to permit shrinkage, at the joints.

Treig Dam Spillway.

The Treig catchment has an area of $43\frac{1}{2}$ square miles. It contains several high mountains with steep rocky slopes running down to the loch, with the result that precipitation is high and the run-off is rapid. For the purpose of calculating the maximum flood-discharge at Treig dam it has been assumed that the maximum rate of run-off is 5 inches per day; this, with some additional flow from the Laggan–Treig tunnel and the back-flow from the Ben Nevis tunnel, which would occur in times of exceptional flood, gives a total discharge of 7,600 cusecs.

The design of a spillway capable of dealing with an overflow of such magnitude required careful consideration. Several alternative designs were investigated. The original idea was to pass the overflow over two circular spillways to a large discharge culvert formed at the western end of the dam. The rock was, however, found to fall away so steeply at this place that to have provided suitable foundations and free access to the spillway would have been very expensive. In another tentative design advantage was taken of the presence of rock at a relatively high level in the centre of the valley to found a spillway thereon, in the form of two diverging arms each consisting

of a double spillway, projecting upstream from the centre of the core-wall, and discharging into a culvert passing through it. In addition, a siphon spillway situated between the arms was incorporated in the design. The most economical design, however, which was the one adopted, consists of a plain overflow spillway in which the top of the core-wall over a length of 330 feet forms the spillway crest. This necessitated the protection of the downstream rubble filling. A concrete apron was formed in situ on the rubble filling of slabs of reinforced concrete 14 feet 6 inches by 15 feet by 12 inches thick, with interlocking edges. In the event of any settlement of the downstream filling taking place the apron, being composed of articulated panels, would be able to follow the settlement without cracking. It was terminated against a mass-concrete toe arched between and supported by six piers, four of which were founded on the bed-rock, one on gravel and one on piles. The space under the arches between the piers was sealed by a curtain-wall resting on steel sheet-piling. The upper surface of the toe is curved so as to ensure the formation of a hydraulic jump, and is lined with granite ashlar. This is necessary in order to destroy the high velocity of the overflowing water in order to prevent serious erosion of the bed of the valley below the dam. At the sides of the apron vertical walls 8 feet high with an overhang coping protect the sides of the valley and deflect the overflow towards the river-channel. It is desirable that the overflow should be fairly uniformly distributed over the length of the curved toe in order to give a minimum of scouring action on the river-bed below and to prevent intense local erosion. In order to arrive at the best design a scale model, $\frac{1}{72}$nd full size, was made in wood. As was expected, with the plain apron the flow was concentrated at the two ends of the toe. Rows of inclined baffle walls were then inserted in order to deflect portions of the overflow towards the centre. After many trials it was found that fairly uniform distribution of flow over the toe was obtained with one inclined baffle wall on each side in the positions shown in Figs. 8, Plate 2. The model enabled the necessary height of side-wall and the effectiveness of the overhang in throwing the water back on to the apron to be determined. It showed also that for all flows the hydraulic jump would form on the concrete apron before reaching the curved toe. It was considered, however, that a curved toe was a desirable safeguard, and that it would reduce the tendency to scour the river-bed immediately below. Large rubble was filled in against the toe as an additional deterrent against scouring action.

It may be of interest to include here the results of tests carried out at the City and Guilds College on a model, $\frac{1}{6}$th full size, of the spillway crest, in order to determine the coefficient of discharge

corresponding to various heights of overflow. These tests indicate that for a smooth concrete crest of radius $R = 15$ feet with 3-to-1 tangential slopes both upstream and downstream, the coefficient C in the discharge formula $Q = Cbh^{\frac{3}{2}}$ has the following values, where Q denotes discharge in cusecs, b denotes the length of crest in feet, and h denotes the height of still water upstream above crest-level in feet:—

h :	0·6	1·0	1·5	2·0	3·0	4·0
$\frac{h}{R}$:	0·04	0·07	0·10	0·13	0·20	0·27
C :	3·07	3·09	3·13	3·16	3·20	3·24

In order that the water-level in Treig reservoir may be lowered when required, a culvert 5 feet 6 inches high by 5 feet wide was formed through the dam. An operating shaft was built into the core-wall, and at the bottom of it double hand-operated sluice gates were fitted. The culvert was founded on piles to reduce settlement during consolidation of the filling, and joints, rendered watertight with bitumen, were provided at the junction with the core-wall and along the culvert to allow for unequal settlement.

London and North Eastern Railway Diversion.

The raising of the overflow-level of loch Treig by 35 feet from + 784 O.D. to + 819 O.D. made it necessary to raise the L.N.E. Railway for a distance of nearly a mile above Treig dam. Below the dam the diversion falls at a gradient of 1 in 71 to meet the old line $\frac{2}{3}$ mile to the north, a total length of diversion of $1\frac{1}{2}$ mile. Since it was not possible to interrupt the railway services the new line was formed clear of and to the east of the old line. The diversion afforded some improvement on the original line; the maximum gradient is reduced and curves of larger radii have been introduced. At the 80th mile, where a large rock bluff descends steeply to the loch, and around which the old line wound on a narrow benching, a tunnel 150 yards long has been driven. An open rock cutting could not have been driven at this point without endangering traffic on the line below.

Road Diversions and Access Bridges.

The extension of loch Laggan by the extension of Laggan dam renders necessary the diversion to higher ground of the main road from Fort William to Newtonmore over two short lengths totalling $1\frac{1}{8}$ mile. On occasion this road used to be flooded to a depth of as much as 3 feet, as a consequence of which a detour of 120 miles via Inverness was necessary, so that

here, too, an improvement has been effected. Access for wheeled road traffic to the south of the river Spean above its confluence with the Treig was only possible at one place about a mile below loch Laggan where there was a ford. After the formation of the dredged channel a ford was impossible, and it was replaced by a 104-foot span bow-string girder bridge. The access road was raised above high water-level and a further bridge was constructed over the river Ghuilbinn.

Construction.

Temporary Works.

The temporary works merit a brief mention. Camps accommodating 650 and 350 men were built at Fersit and some little distance above Laggan dam, respectively, each forming a self-contained community. Central stores and workshops with railway sidings alongside were established in an old railway ballast-pit near Fersit.

A 3-foot-gauge railway 23 miles long had been constructed during the first-stage works from Fort William to loch Treig to give permanent access to the intakes and adits of the Ben Nevis tunnel, and this was now temporarily extended from loch Treig to the camp above Laggan dam. Passing through Fersit camp, near which a sandpit supplying the whole works was opened up, the line crossed the Treig valley on a timber viaduct, passed over the L.N.E. Railway, and followed the line of the Laggan–Treig tunnel to Laggan dam, where a steel deck-bridge supported on two steel towers with a central span of 200 feet led the railway across the valley at a height of 90 feet. The steel towers were extended upwards to support 7-ton monotower cranes of 100 feet radius, which could command the whole dam with the exception of the ends and the downstream toe. A granite quarry was opened up in a line with the dam on the north side of the main road. The stone was broken to 3-inch size in a slogger, was passed under the main road on a rubber belt, crushed to 2-inch size and the dust separated, and was then stored in hoppers. Thence it was drawn off into skips and transported by aerial ropeway to concrete batching and mixing plants situated below the bridge on both sides of the valley. The sand for concrete was brought by the railway and side-tipped from the bridge into the batching plants. The stone from the Laggan dam quarry was supplemented by granite from the tunnel excavation.

The temporary hydro-electric power-station constructed at Monessie, near Roy Bridge, and described in the previous Paper [1] supplied

[1] *Loc. cit.*

electric power up to 4,000 kilowatts throughout the extent of the works by a network of three-phase 11,000-volt transmission lines. In extremely dry weather, when the flow of the river Spean was insufficient for the power requirements, supplementary power was supplied from the North British Aluminium Company's power-station at Fort William.

Triangulation.

As was done in the case of the main tunnel, a triangulation survey was carried out to ensure the accurate setting-out of the Laggan–Treig tunnel and Laggan dam. This survey was complete in itself although it was linked up with the triangulation for the first-stage works. A base-line about 2,700 feet long was measured along a straight stretch of the L.N.E. Railway between Tulloch station and Fersit, the usual corrections for temperature and tension of the measuring tape and for variation of level being made. This was extended by a network of triangles to cover the Laggan–Treig works. The instrument used was a Watts' 5-inch micrometer theodolite. The type of instrument station and the method of taking observations were as described in the previous Paper[1] and also in another Paper.[2] In reducing the results the triangles were corrected by the method of equal shifts. In one case where the tunnel-line was transferred underground at a working-shaft the Weisbach triangle method was employed. At the meetings of all headings no lack of alignment could be detected. Theoretically an average error of $\frac{7}{8}$ inch in line and $\frac{1}{4}$ inch in level was committed at the four junctions of headings.

London and North Eastern Railway Diversion.

An early start was made on the diversion of the railway as it would not be possible to complete the core-wall of Treig dam until the diversion had been put into service.

From the north end of the diversion to about $\frac{1}{4}$ mile south of Treig dam the new formation lay either in side cutting through raised beach or on embankment. A 6-ton Ruston-Bucyrus crane-navvy mounted on caterpillars made short work of this section. Further south were small rock cuttings and the tunnel. With these particular care had to be exercised in order not to interfere with the operation of the main line below. The cleavage of the mica schist over the tunnel was such as to create a danger of triangular prisms of rock

[1] *Loc. cit.*

[2] R. D. Duncan and E. M. R. Jones, "Survey Work on the Lochaber Water-Power Scheme." Inst. C.E. Selected Engineering Paper No. 137.

slipping down on the railway, especially as the covering of rock over the greater part of the tunnel consists of a mere shell. A bottom heading on the eastern side of the tunnel was first driven, so that only very light blasting was required to enlarge the tunnel to full section. The side-walls and arch were lined with concrete and brick respectively. Immediately to the south of the tunnel prickings revealed a particularly insecure condition; a surface of smooth glacier-planed rock at a slope of $1\frac{1}{2}$ to 1 or even steeper lay buried some 15 or 20 feet below, and parallel to the ground surface. The stability had not been endangered while the level of loch Treig remained undisturbed, but with the large fluctuations of level to which it is now subject there was danger of a landslip. It was therefore decided to support a length of 200 feet by a concrete retaining wall carried up from the rock. In other places large quantities of rubble were tipped to form protective aprons against erosion and the scour below bridges and culverts.

The diversion to the new line was effected in August, 1932.

Treig Dam.

The area to be covered by the dam was first cleared of surface soil and vegetation. The excavation for the core-wall in the centre of the valley was then commenced. Open excavation to a depth of 10 feet was followed for a further 16 feet in depth by timbered trench 10 feet wide, up to 25 feet further depth in Larssen interlocking steel sheet-piling inset to a width of 7 feet 6 inches, and finally, where required, by a second inset consisting again of steel sheet-piles with a width of 6 feet. Over the deepest portion of the core-wall, where a third inset was required, the timbered trench was made 12 feet wide so as to ensure a width at the bottom of not less than 6 feet. The rock surface was excavated down to close-jointed rock, a key thus being formed of average depth below the rock surface of 6 feet. In addition, the rock at the base of the key was drilled and grouted under presure.

At the end of the trench near the L.N.E. Railway running sand was encountered. Settlement of the end of the trench was taking place and the proximity of the railway made it advisable to postpone further excavation until the diversion of the line had been effected. At the west end excavation was delayed in order to maintain the overflow-channel in view of the probability of loch Treig overflowing during the winter; this occurred in January, 1932. Accordingly, after the concrete had been brought up to ground-level across the centre of the valley attention was devoted to the portion of the core-wall east of the railway. Owing to the presence of the raised beach

on this side of the valley a length of 250 feet of core-wall lies entirely below ground. The contractors elected to construct this in open trench with the exception of the last 60 feet where the small height of the core-wall compared with its great depth below the ground surface made it more economical to complete the core-wall in heading (Figs. 8, Plate 2). The western end was constructed next, and the overflow channel was carried over the trench in the form of a timber flume lined with corrugated sheeting. After the railway had been diverted to its new position the excavation of the remaining length of trench was commenced. The difficulty which had been occasioned by the running sand was very successfully dealt with by sinking a cylinder, built up inside the trench, of cast-iron segments, through the fine sand to gravel and using this as a sump. A deep depression in the bed-rock made the maximum depth of excavation below the surface 100 feet. The trench was divided up by the struts into 7-foot bays. In the deeper portions of the trench where, on account of running sand, very high pressures were anticipated, a space of only 12 inches was left between horizontal frames ; 10-inch by 5-inch walings of Oregon pine and 10-inch by 10-inch struts were used. The concreting of the deepest portion was complicated by an inflow of water from all sides amounting to some 400 gallons per minute. There was not room to form pump-sumps both upstream and downstream of the core-wall, and the method adopted consisted essentially of forming down the centre of the trench a partition of short lengths of interlocking steel sheet-piles sealed against the bottom of the key by bags of concrete, by means of which the upstream and downstream leakages were led to a central box-sump partitioned into two halves by the steel sheeting. The sides of the rock key were draped in canvas hung from the bottom walings and sealed against the bottom with concrete bags, so that the flowing water should be kept away from the concrete. Sufficient rubble was filled in on both sides of the steel sheet-piling to cover in the flow of water, so that all concrete could be placed in the dry. Finally, when the concrete had been brought up above ground water-level, the sumps—a subsidiary sump was formed for a second pump—were concreted, thus forming an effective seal at the top of the steel sheet-piling ; the rubble drains and spaces behind the canvas were then well grouted. Some delay was caused by a second overflow of loch Treig during December, 1932, and January and February, 1933, to such a depth that the core-trench was flooded. Somewhat similar, although lesser, difficulties had been experienced in dealing with the influx of water when concreting at the deepest point of the core-trench on the west side of the valley, where the depression in the rock-surface is of smaller depth.

In sinking the piers for the curved downstream toe, rock was reached at a shallow depth except in the case of the two east piers, where as in the case of the core-wall, the rock fell away steeply. In the first of these a foundation of good gravel was available, and the end pier was founded on timber piling, in addition to which the steel sheet-piling around the pier was concreted in place.

The downstream rubble fill was spread in 12-inch layers sloping down towards the core-wall. The rubble stone was for the most part granite, the remainder being mica schist, and each layer was overspread with smaller stones and consolidated by a 10-ton steam roller. Adjacent to the core wall the inverted wedge of hand-packed rubble was founded on a layer of rubble set in concrete in order to reduce settlement. The downstream fill was brought up well ahead of the upstream filling in order to pre-stress the core-wall so that its stress under full reservoir conditions may be reduced. The upstream filling of soft material was also inclined at a flat slope towards the core-wall. Immediately against the wall an impervious fill of micaceous dust from the Laggan–Treig tunnel invert was used in the absence of clay. The whole was well watered and consolidated by a 2-ton horse roller.

The concrete apron was constructed in alternate squares, chess-board fashion, the intermediate spaces being concreted later. Before concreting these the edges of the adjoining slabs were painted with bitumastic paint. An asphalt joint 1 inch thick was provided along the upper edge of the apron at its junction with the core-wall to allow for expansion and contraction of the slabs or for deflexion of the core-wall.

River Spean Channel.

The dredged channel below loch Laggan involved the excavation of 250,000 cubic yards of material, mostly sand and gravel. The greater part of this was carried out by two Ransome drag-line excavators of 2-cubic-yard capacity, working from each bank in turn. They were electrically driven, a transformer mounted on rails enabling the high-tension line to be tapped at any point. With the exception of a small amount of dredging at the lower end of the channel work proceeded downstream, as otherwise the rapid fall in river-level at the head of the dredged section would have caused severe local scour and silting up of the completed portion downstream.

The extension of the channel in the bed of loch Laggan itself to the new low-water line was effected by means of an electrically-driven sand pump mounted on a pontoon and discharging through a 900-foot pipe-line carried on floats into deep water.

The small amount of rock excavation was carried out by subaqueous blasting, the loose rock being removed by a scraper-loader operating from the bank and discharging into 2-foot gauge skips.

Laggan Dam.

A river-bed, filled to a depth of 35 feet with large and small boulders, sand, and gravel, presented a difficult problem in respect of cofferdams. The difficulties were accentuated by a river-flow varying from less than 100 cusecs to over 10,000 cusecs in high flood.

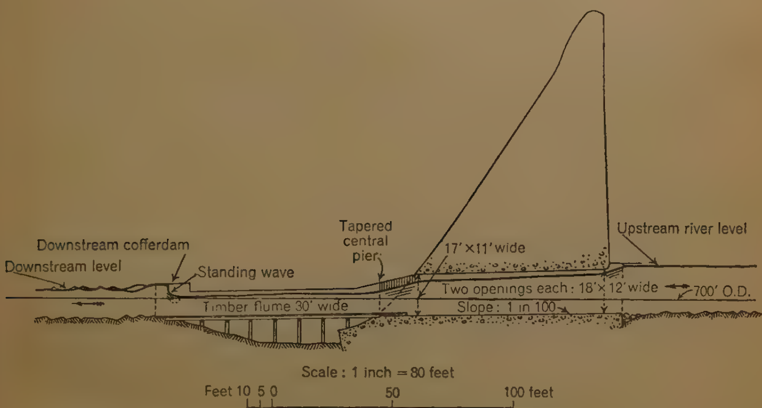
The south bank was the most suitable location for the forming of diversion openings through the dam through which the river could be passed during construction. The dam was to be divided up into blocks by vertical radial contraction joints extending the full height of the dam. The first block to be constructed was that containing the diversion openings. For this purpose a cofferdam was constructed in the river parallel to the bank consisting of close timber sheeting supported by timber frames held down by a platform weighted and backed with rubble. This was made watertight by tipping clay on the river-side and protecting it from scour by rubble gabions. Two diversion openings were formed through the dam, each 18 feet high by 12 feet wide at the upstream and 17 feet high by 11 feet wide at the downstream face. Below these an open timber flume 30 feet wide was constructed to carry the discharge above the toe of the dam to beyond the downstream cofferdam.

In view of the greater depth of the bed-rock surface in the centre of the river, it was decided to construct the main cofferdams across the river of sandbags enclosing a clay hearting and backed with an embankment of sand sufficiently wide to include a hydraulic seepage gradient of 1 in 4. This was stone pitched and provided with a wide berm. The upstream side was further protected by rubble mattresses built into the cofferdam. A mass-concrete wing-wall reinforced by tie-bars grouted into the rock connected the diversion openings with the upstream cofferdam. Even so, it became evident that in the centre of the river the excavation would undercut the upstream cofferdam, and a concrete retaining wall was therefore carried down to the rock in timbered trench and tied down by steel dowels. The cofferdams were very successful and the amount of leakage was small. The foundations were flooded twice during construction. The first flooding occurred during excavation inside the first cofferdam; on this occasion a maximum flood estimated at about 7,500 cusecs occurred, the highest, according to local information, for many years. The later flooding took place after the main cofferdams had been formed across the river, and was due to failure

to carry up the upstream cofferdam to a sufficiently high level. On account of the protection of the inside slopes with stone pitching, it had but little effect on the cofferdams.

The behaviour of the flow through the diversion openings was interesting, and is shown diagrammatically in *Fig. 10.* During floods, although the edges of the upstream openings were rounded off, the openings never ran full at the downstream face of the dam even when the upstream openings were submerged to a depth of several feet. Further, in the open flume below the openings a hydraulic jump occurred except at low flows, its position depending upon the flow; it formed initially at the upstream end and as the flow increased it moved down to the downstream end where the flume joined the rough river-bed. The profile of the flow was

Fig. 10.



checked by theory and by observations of the current. Calculations indicate that the height of opening at the downstream face need not exceed about two-thirds of the height of the upstream water where there is a free outflow or where (as can be calculated from consideration of momentum) a hydraulic jump will occur. Under these conditions the capacity of the openings is not affected by the level of the river downstream. For approximate computations the formula for a broad-crested weir may in fact be applied and an allowance made for friction. Treatment as a submerged orifice may give quite erroneous results.

The flooding of the main cofferdams referred to above had an unexpected and untoward consequence. As the hydraulic jump formed at the downstream end of the flume this contained a relatively shallow depth of fast-flowing water while it was surrounded by water

at a much higher level. The result was the bursting upwards of the timber flooring of the flume. After reconstruction a recurrence of this was guarded against by arranging for the sides of the flume above a certain level to fall in under an excess of external water pressure.

The rock foundation is characterized by vertical joint-planes parallel to the valley. As a result the foundation was in the form of large steps with smooth vertical faces and irregular top surfaces. The rock higher up the sides of the valley where the slope was less severe was of good-quality granite. Lower down it was more jointed and discoloured. The good-quality granite was, however, found to be underlain by clay seams, in some cases an inch thick, running roughly parallel with the surface slopes of the valley. This phenomenon was observed on both sides of the valley. A possible explanation may be that the intense shearing forces exerted by glacial action on the sides of the valley at the constriction ruptured the rock steps along their weakest plane, namely from root to root of the steps. In the river-bed a main fault-plane was discovered perfectly smooth, straight and vertical. No doubt the location of the Spean valley in its present position was determined by the weakness of the rock resulting from this faulting.

The foundation was carried down some 5 feet or more into sound rock at the downstream toe. It was sloped down towards the upstream face at a gradient of about 1 in 12, being stepped-up where sound rock indicated the permissibility of so doing. At the upstream face the excavation was carried down a further 5 to 10 feet in a narrow trench in an attempt to secure not only sound but impervious rock. The resulting trench was in places as much as 20 feet below the level of the rock-surface and 50 feet below river-bed level. In general the jointing of the rock at the bottom of the trench was close and tight, but as a precaution grout-holes of various depths up to 15 feet were drilled to cross the joints, and were grouted at a pressure exceeding the final water pressure at that place. Smooth vertical rock faces were well tied to the concrete by $1\frac{1}{4}$-inch diameter steel dowels. The rock, after being hosed with high-pressure water and brushed clean, was coated with neat-cement grout followed by 2 inches of 2-to-1 cement mortar immediately before concrete was placed. The cut-off trench and the upstream face of the dam to a depth from the face varying from 7 feet at the lowest point to 2 feet at the crest was formed in 4-to-1 concrete with 1-inch aggregate. The remainder was of 7-to-1 concrete with 2-inch aggregate with the exception of a layer of 4-to-1 concrete 1 foot in thickness at the downstream toe. Granite of good quality was available for the coarse aggregate, and the sand was excavated from the raised beach

at Fersit. The grading of the aggregate was analysed and compared with Fuller's maximum-density grading,[1] but no attempt was made to separate out and proportion each grain-size of the material, the stone, crusher dust and sand being porportioned so as to give an amount of fine aggregate (under ¼-inch size) somewhat in excess of theoretical requirements, thus ensuring sufficient workability. In order to assess the relative merit of sand and the fine material from the crusher run for use as the fine aggregate, three sets of 6-inch test-cubes were made to the maximum density grading, using for the fine aggregate crushed granite only, crushed granite and sand in equal quantities, and sand only. Their respective average crushing strengths at 28 days were 209, 255, and 256 tons per square foot. This seemed to indicate that the sand might be replaced up to 50 per cent. without deleterious effect. As the proportion from ¼ inch down to dust was excessive in the 2-inch crusher-run granite, this was separated and then recombined with the stone in order to give a proportion not exceeding 20 per cent. of the latter. For the 1-inch aggregate the stone, after being broken to 2-inch size in a jaw crusher, was then reduced to 1 inch by passing between crusher rolls. As the sand as excavated contained a certain amount of gravel of all sizes it was passed through a ⅜-inch screen before use. Normal proportions of the above stone : sand : cement were 5·6 : 2·1 : 1 parts by volume for the 7-to-1 concrete and 2·9 : 1·5 : 1 for the 4-to-1 concrete. The added water was kept to a minimum quantity consistent with good workability, an average water/cement ratio for the 7-to-1 concrete being about 0·55 by weight, and for the 4-to-1 concrete about 0·4. 6-inch test-tubes were made at frequent intervals of the concrete as placed. At the same time samples of the materials used were taken and the grading analysed. A check was thus kept on any variations. The cubes showed a consistently high density, the average for both the 7-to-1 concrete and the 4-to-1 concrete being 152 lbs. per cubic foot. The average crushing strengths at 28 days were 174 tons per square foot for the 7-to-1 concrete and 301 tons per square foot for the 4-to-1 concrete. The concrete was laid in 3-foot 6-inch lifts sloping downwards at a slope of 1 in 12 towards the upstream face, the continuity of joints being broken by 9-inch steps. Between lifts neat-cement grout, well brushed in, followed by ½ inch of 2-to-1 mortar ensured a good bond. The use of displacers was encouraged and a proportion varying up to about 15 per cent. was attained.

As a consequence of vertical cracks appearing on the radial

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[1] F. W. Taylor and S. E. Thompson, "Treatise on Concrete, Plain and Reinforced." P. 192. Third edition, New York and London, 1917.

faces of the blocks into which the dam is divided, the junctions of lifts of concrete in later blocks were arranged so as to predispose any cracking to take place parallel to the downstream face rather than vertically, so that, in the event of further such cracking, it would have no effect on the stability of the dam. To guard against seepage through the cracks on the upstream face a layer of reinforced gunite, applied in two coats to give a final thickness of $1\frac{1}{2}$ inch, was carried up to + 804 O.D., the new low-water level. The gunite mixture consisted of 3 parts of dried sand (under $\frac{1}{4}$-inch size) to 1 part of cement, although, owing to the rebound of the larger grains, the resulting layer is considerably richer. For a distance of a foot on each side of cracks adhesion of the gunite was destroyed by painting the concrete with bitumastic paint so that the reinforced gunite might stretch elastically in lieu of cracking. The base of the upstream face from its junction with the rock to river-bed level was waterproofed with " Sika " waterproofing compound, and the crevice against the upstream face was then filled in to ground-level with crusher dust.

The closure of the diversion openings without mishap due to possible sticking of the gates was ensured by the temporary damming of the river Spean by tipping an earth bank about 3 miles below loch Laggan. With the steep gradient of the river at Laggan dam the water would have risen in a few minutes to above the level of the gates. The procedure adopted allowed ample time for effecting the closure before the river topped the temporary dam. The openings were then concreted solid, and the junction between concrete plugs and dam made tight by pressure grouting.

Laggan–Treig Tunnel.

The excavation of the Laggan–Treig tunnel was carried out from eight working faces, access being made from each end, by two adits and by a working shaft which was subsequently converted into the down-shaft of a side-stream intake. The tunnel was thus divided into four sections varying from 4,600 feet to 1,800 feet in length.

Three compressor houses, and at each access a battery-charging sub-station, smithy, workshops, and stores, were installed. The track laid in the tunnel was of 2-foot gauge, and electric-battery locomotives were employed.

At the working face the upper half of the tunnel was kept some 10 to 15 feet ahead so that a platform about 5 feet high was formed from which to drill. The normal procedure was to drill twenty-two shot-holes in the upper heading, namely, six cut-holes, four easers, eight trimmers, and four lifters, as well as seven holes in the bench.

A progress of about 6 feet was made with each blast, and a typical charge consisted of about 130 lbs. of gelignite. To avoid overbreak the section as initially blasted was made smaller than the final section, to which it was trimmed by subsequent light blasting. For excavation to final section a total of about 4 to $4\frac{1}{2}$ lb. of gelignite were required per cubic yard of rock. For the most part Holman–Sullivan pneumatic scraper-loaders were used to load the stone and debris into skips. The experience on the Laggan–Treig works proved the efficiency of scraper-loaders. One blast represented the work of each 8 hours shift, and with three shifts per 24 hours a maximum progress for one heading of 118 feet per week was attained. With a specified minimum thickness of concrete lining of 2 inches, it is perhaps of interest to note that the average thickness of lining worked out at $11\frac{1}{2}$ inches. The concreting was carried out with steel shuttering mounted on carriages in the manner described in the earlier Paper.[1]

The moraine section near the outfall presented particular difficulties owing to the quantity of fine running sand encountered. The method adopted was to run a timbered bottom heading through to the rock in order that work on the rock heading might be proceeding. A small top heading was then pushed forward and widened as cast-iron lining segments were bolted on from the crown downwards.

For some distance from the outfall the depth of cover was small, and cut-and-cover construction was adopted. A cofferdam was provided at the outfall in order that work in the tunnel should not be delayed by a high water-level in loch Treig.

Loch Treig was separated from Idir loch at its north end by a spit of shingle, only a narrow channel connecting the two lochs. This bar appeared to be composed for the most part of shingle, and borings had indicated the presence of a large proportion of gravel. The material, having withstood the flow of water from loch Treig for centuries, was considered capable of maintaining the bar with flow in the reverse direction. A small channel was, however, cut through the bar to permit of draining the Laggan–Treig tunnel should it be necessary. This cut was in gravel. After the completion of the tunnel the side streams below Laggan dam were allowed to discharge into it with loch Treig at a low level. The water passed through the narrow cut, and rapid erosion took place, revealing only a small depth of gravel below the bed of the channel and an almost complete absence of gravel in the heart of the bar, which was found to consist of fine sand. It was evident that steps would have to be

[1] *Loc. cit.*

taken to stabilize the channel, or the erosion might ultimately threaten the railway embankment and Treig dam. Scour was therefore permitted to continue until a channel had formed at a steep and fairly uniform gradient between the tunnel outfall and Loch Treig. This was then trimmed to trapezoidal section and lined with reinforced concrete to give a cross-section 30 feet wide at the bed with side slopes of $1\frac{1}{2}$ to 1.

Study of the Internal Temperature and the Formation of Cracks in Laggan Dam.

Advantage was taken of the opportunity provided by the construction of Laggan dam to study the rise of internal temperature and its relationship to cracking in a concrete dam.

Temperature Rise.

Electrical-resistance thermometers were accordingly inserted near the greatest cross-section of the dam in the positions indicated in Fig. 11, Plate 2. These and a temperature-indicator were supplied by the Cambridge Scientific Instrument Company. It was possible to follow the temperature of the concrete from the moment of placing.

The temperature—time curves are given in Fig. 12, Plate 2. There are, unfortunately, two large gaps in these temperature-records, due in the first instance to difficulty of access to the indicating instrument, and in the second case to the sending away of the instrument for calibration. The records suffice, however, to give fairly complete information concerning the thermal properties of the dam. A study of the curves reveals many points of interest. In general the temperature, after rising rapidly, reached a maximum between 36 and 48 hours after placing, after which it fell until shortly after the placing of the succeeding lift when the heat evolved in the latter checked the rate of loss of heat and the temperature again began to rise. The concrete was by then well blanketed by the superimposed lifts and consequently the rate of dissipation of heat was low, so that generally the rise of temperature continued up to a second maximum which occurred after from $1\frac{1}{2}$ to $2\frac{1}{2}$ months, with the exception that thermometers Nos. 5A and 9A near the faces of the dam recorded the second maximum temperature after $\frac{1}{2}$ month. Thereafter cooling set in rapidly near the surface and more slowly as the centre of the dam was approached. With thermometers Nos. 2, 3, 4, 6, 7, and 8 the second maximum temperature is the higher, whereas in Nos. 5A and 9A the initial rise is the higher on account of the greater rate of loss of heat near the faces of the dam.

Thermometer No. 1, in the centre of the base of the dam, showed no second maximum, and this fact is accounted for by its proximity to the rock foundation. The maximum temperature recorded in the 7-to-1 concrete was 111° F., this being the later maximum temperature in the heart of the dam. Under adiabatic conditions a temperature of about 123° F. would have been attained. In the 4-to-1 concrete at the upstream face a maximum temperature of 118° F. was recorded; this was the earlier maximum.

For the thermometers well embedded in 7-to-1 concrete, namely Nos. 2, 3, 4, 6, 7, and 8, the initial temperature rise was fairly uniform and averaged 33° F. For thermometer No. 5A in the 4-to-1 concrete of the upstream face the initial rise of temperature was 55° F. These temperature rises are in approximately the same ratio as the ratio of the cement contents of the concretes. The fall of temperature after the initial maximum, and the subsequent rise of temperature after the placing of the next lift of concrete, depend upon the interval of time between lifts. The greater this interval the more heat is lost at the exposed face of the lifts before it is effectively blanketed by the next lift, and the lower the value of the second maximum temperature. Thus the second maximum temperature of thermometer No. 2 is relatively low because an interval of 7 days elapsed before the placing of the next lift.

The temperature records for Laggan dam were placed at the disposal of the Building Research Station. By comparing them with the rise of temperature with the same cement and concrete mix under adiabatic conditions, and by supplementing the information with records from other places, the station has been able to enumerate the laws governing the initial and second rises of temperature, so that it is possible to forecast the maximum temperature in any particular case.[1] It is not proposed, therefore, to deal further with the matter here.

It will be noticed that the temperature curves of thermometers Nos. 2, 3, 6, and 7 in the centre of the dam are parallel and that the rate of cooling is slow. So, too, are the curves for thermometers Nos. 4 and 8 stationed half-way towards the centre of the dam, but the rate of cooling is greater. The curves for thermometers Nos. 5A and 9A at 3 feet from the faces of the dam are also parallel, but indicate a very much greater rate of cooling. If certain assumptions are made, as explained in Appendix I, the temperature curves can be derived theoretically. The coefficient of conductivity of the concrete of the dam, which may be very different from the

[1] Building Research Technical Paper No. 18. Published by H.M. Stationery Office.

coefficient of conductivity of the same concrete in a dry state, was not known. This has been derived from the curves of thermometers Nos. 3, 4, 5A, and 9A by finding that value which caused closest agreement between theoretical and actual cooling curves. In each case fair correspondence was obtained with a value for the thermal diffusivity of 0·016 C.G.S. units. Taking the specific gravity of the concrete as 2·4 and the specific heat as 0·20, the coefficient of conductivity becomes $0{\cdot}016 \times 2{\cdot}4 \times 0{\cdot}20 = 0{\cdot}008$ C.G.S. units. This is too high: a normal value of the conductivity is 0·003 and a maximum value for water-saturated concrete is 0·006. The excess water present when the concrete was gauged has no means of escape from the heart of the dam, so that the thermal conductivity probably approached 0·006. The explanation of the apparent conductivity being in excess of the actual value lies in an escape of heat laterally in addition to its flow towards the upstream and downstream faces. This happens to be an important factor in this particular block (No. 4B, Figs. 3, Plate 1) as the adjacent block (No. 5A) was the first to be concreted and had been cooling down for a whole year. Thus it will be noticed that, comparing the thermometers at the centre of the dam, No. 6 within 3 feet of block 5A registered temperatures from 15° to 20° F. below the corresponding temperatures at No. 3 thermometer at the centre, and at No. 7 within 3 feet of block No. 4A.

With the assistance of the theoretical curves many interesting deductions may be made. After 1 year the excess temperature at the centre fell to about one-half its original height and after 2 years to one-quarter the original height. The average annual fluctuation of temperature at the surfaces and at the various thermometers appears to be as follows:—

Thermometer No.	Range.
Surface of concrete	24° F.
Nos. 5A and 9A	19° F.
Nos. 4 and 8	5° F.
Nos. 3, 6, and 7	2° F.

Expressed as a proportion of the surface variation, the annual fluctuation at various depths becomes:—

Depth.	Proportionate range of temperature.
0 feet (at the surface)	1
3 „ $= \frac{d}{29}$	0·80 . . . .
22 „ $= \frac{d}{4}$	0·19 . . . .
43 „ $= \frac{d}{2}$ (i.e. centre of dam)	0·08 . . . .

There will be superimposed on the annual cycle a daily fluctuation of temperature, but the effect of this is confined to the surface layers and falls off very rapidly with penetration into the concrete. Even at the surface this will be less than the daily range of air temperature, as the concrete has insufficient time to attain the changing air temperature. Applying the formulas of Appendix I, it is seen that the daily fluctuation ceases to be appreciable before the first thermometer at a depth of 3 feet is reached. Reverting to the yearly cycle, there will be an increasing time-lag with penetration into the concrete. The figures for depths corresponding to the positions of the thermometers are as follows :—

Depth.	Time-lag.	Month of highest periodic temperature effect.
0 feet	0 months	July—August
3 „ $= \frac{d}{29}$	0·6 „	August
22 „ $= \frac{d}{4}$	3·1 „	October—November
43 „ $= \frac{d}{2}$	6·1 „	January—February

It takes 6 months for the summer heat-wave to reach the centre of the dam.

It will perhaps be of assistance to future investigators and to instrument makers to comment on the behaviour of the electrical-resistance thermometers and indicators. Thermometers Nos. 5 and 9 soon became faulty and had to be abandoned. Fortunately it was possible to place additional thermometers, Nos. 5A and 9A, in analogous positions. After 1½ years thermometer No. 2 also failed. The brass capsules containing the resistance elements relied for moisture-tightness on the close fit of the rubber-sheathed electric cable. Inside the capsule there was an air space around the resistance element. As heat is evolved in the concrete, the enclosed air will tend to expand and will probably succeed ultimately in forcing an escape from the capsule. As the dam cools, a partial vacuum will be formed and moisture will then be drawn into the capsule, for the excess water in the heart of a large mass of concrete must be retained for a very long period, if not indefinitely. This will reduce the resistance of the element and will render it useless. It is not sufficient that a resistance thermometer should be watertight: it should be hermetically sealed so as to remain gas-tight under pressure. With a thermocouple there is no need of such precautions, but it is normally less accurate than a resistance thermometer. Again, the indicating instrument, if placed in a hot moist inspection gallery as at Laggan dam, requires to be specially designed. In addition to the fault

which led to its being sent away for recalibration at the beginning of 1935, the instrument has behaved erratically since June, 1935, as evidenced by similar vagaries in the curves of all the thermometers.

Shrinkage.

The longitudinal contraction and relative vertical movement at contraction joints was measured by a taper gauge reading to 0·001 inch between brass stops set on each side of the radial joints at the upstream face a short distance below crest-level. Measurements were also made at the joints in the inspection gallery. Owing to difficulty of access the measurements at the upstream face were discontinued after a few months. They indicated practically no relative vertical movement within that period. From the horizontal measurements it was possible to correlate movement with variation of atmospheric temperature. The movements at different joints were very unequal; this may be due to the variation in foundation level in a block, as a result of which contraction would not be symmetrical about its centre. Taking the average value of the shrinkage, the results appear to be consistent with a normal value for the coefficient of thermal expansion of concrete, namely 0·000006 per ° F.

The movements measured at the contraction joints in the inspection gallery are of great interest. The opening of these joints varies from $\frac{1}{100}$ inch in joint 4A to $\frac{1}{16}$ inch in joint 3A. In all cases there is an annual cycle of variation, but while the annual fluctuation at the lower levels (joints 3/4, 4A and 4/5) was in each case only about 0·005 inch, at the intermediate level (joint 3A) the fluctuation was 0·025 inch, and at the highest level (joint 2/3) 0·035 inch. The theoretical movement due to temperature at the distance from the upstream face corresponding to the inspection gallery can be calculated from the readings of thermometers Nos. 4 and 5A and this was found to coincide with the movement at joint 2/3 both in respect of steady cooling shrinkage and annual fluctuation. This suggests that joint 2/3 is open down to the level of the gauge-stops, that there is slight contact at the intermediate level corresponding to the gauge-stops in joint 3A, and that at the lowest level (joints 3/4, 4A, and 4/5) there are large zones of contact, but that they do not extend quite to the inspection gallery. The depths below crest-level of the upper, intermediate, and lower levels are 67 feet, 95 feet, and 114 feet respectively. It would appear, therefore that beyond a depth of about 100 feet contact is maintained between adjacent blocks at contraction joints, and as the heart of the dam has now almost cooled down, this contact is likely to be permanent. It would follow that below this level there would be no tendency

for shrinkage cracks to extend through a dam. Evidently the lateral spread of the concrete due to elastic and plastic movement is sufficient at this depth to counterbalance shrinkage due to cooling.

With a dam 177 feet high from the foundations the contraction in a vertical direction is not negligible, and allowances for shrinkage varying up to a maximum of $\frac{3}{4}$ inch were made in fixing the crest-level at the time of concreting. Provision for shrinkage was also made in the fixing of the cast-iron guides of the scour-culvert roller gate up the face of the dam.

Shrinkage-Cracks.

Cracks were observed on the upstream face of the dam within one or two months of concreting. The dam was divided up into blocks by radial contraction joints at about 45-foot intervals, and the cracks seemed to form at about 15-foot centres. In some cases the cracks could be traced to local causes, such as an abrupt change in section, as at a vertical step in the rock foundation or at the upper corners of the diversion openings. Where a radial contraction joint was terminated above the foundation a crack extended downward therefrom in spite of strong reinforcement across the base of the joint. Similar cracks also appeared in the downstream face, and a few appeared in the radial faces. The cracks on the upstream and the downstream face gradually extended upwards towards the crest of the dam. They do not yet appear to have penetrated to the inspection gallery, a distance of only 11 feet from the face, and in view of the observations previously cited they may not do so.

As has already been stated on p. 29, care had been taken throughout concreting to obtain a well-graded mix and to ensure that there should be no excess of water. The low water-content and the high density would tend to keep shrinkage down to a minimum, and it would be further discouraged by the presence of displacers.

Cracking must be caused by the setting up of an excessive tensile stress in some part of the cross-section of the dam. Such tensile stress may be due to the restraint of shrinkage which would otherwise take place as a result of cooling and drying-out of the concrete. It is a well-known fact that with complete restraint drying-out alone is sufficient to cause tensile failure. The restraint is only complete at the foundations, but as the concrete would remain damp in this position the initiation of cracking here is unlikely, apart from the effect of steps in the rock surface. Where, as at Laggan dam, contraction joints are provided, the restraint of the foundations is of negligible effect at crest-level, so that the tensile stress at the crest will be small and there, also, the initiation of cracking is

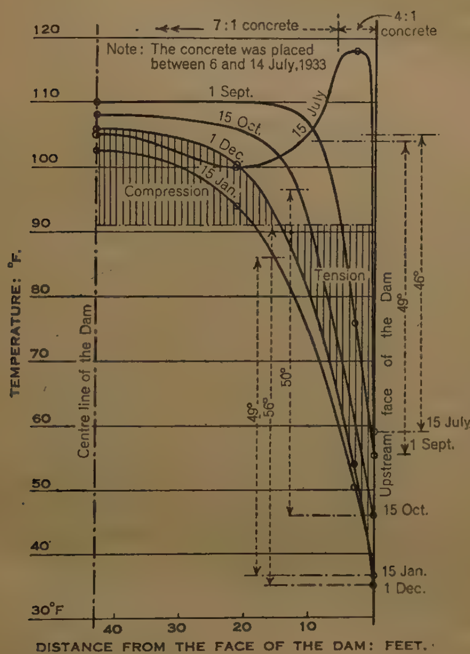
unlikely. At intermediate levels, however, there will be a skin tension due to the cooling and partial drying of the exterior relative to the still hot and moist heart of the dam. Such a tension would act in all directions parallel to the surface, but the vertical component is reduced by the compressive stress due to the weight of the superimposed concrete, and is further relieved by the horizontal construction joints between successive lifts of concrete. The maximum tensile stress is therefore in a horizontal direction, and the resultant cracks will be vertical. By making certain assumptions some idea may be gained of the magnitude of this stress. It is assumed in the first place that the effect on this stress of restraint at the base can be neglected; this will be the case except for a short distance above the foundations. Secondly, that there is no thrust transmitted from adjacent blocks. The temperature of the concrete rises rapidly to the initial maximum temperature. It is restrained from longitudinal expansion, but as it is still quite green and plastic no appreciable longitudinal compressive stress will be set up.. As the second maximum temperature in the heart of the dam is the higher, and as the superimposed concrete will tend to produce horizontal spreading of the underlying concrete, it is probable that there will be some thrust between blocks during the first few months. The effect of this would be to reduce the tensile stresses, but for the purpose of this calculation it will be ignored. It follows, then, that the total compression and total tension at any vertical section will be equal.

Consider now a radial section through the centre of a block and the stresses along a typical line as A, 9, 8, 3, 4, 5, W, Fig, 11, Plate 2. *Fig. 13* gives successive forms of the temperature curves. Let it be assumed that the stresses are proportional to temperature differences; the ordinate of mean temperature will then become the axis of zero stress dividing the diagram into an area of compression at the centre of the dam and an equal area of tension at the faces. It will be seen that within a day or two a stress corresponding to 46° F. was set up, and that this gradually increased to a maximum value of stress corresponding to 56° F. after $4\frac{1}{2}$ months, this increase being due to the falling air temperature. To this must be added the daily fluctuation below mean temperature of the face of the concrete. The maximum tensile stress will, therefore, correspond, in round figures, to a temperature difference of 60° F., or to a strain of $60 \times 0{\cdot}000006 = 0{\cdot}00036$. Taking a low value for the modulus of elasticity of 1,000,000 lbs. per square inch to allow for creep,[1]

[1] Building Research Technical Paper No. 18. Published by H.M. Stationery Office.

and assuming that plane sections remain plane, this gives a tensile stress at the upstream face of 360 lbs. per square inch. Actually, plane sections will not remain plane unless the distance between radial joints is large compared with the thickness of the block or unless contraction of the face is counteracted by cooling of the radial faces. This would tend to reduce the stress, but on the other hand it will be considerably increased by the partial drying-out of the upstream face. As the tensile strength of the 4-to-1 concrete is

Fig. 13.



not likely to exceed 300 lbs. per square inch, this analysis may be said to account satisfactorily for the formation of cracks.

Where a radial face of a block remains exposed for more than a few weeks similar tensile stresses will be set up therein. At Laggan dam some cracks were discovered on those radial faces which had been exposed for several months. There appeared to be a tendency for such cracks to follow the edge of continuous vertical keyways formed in the radial joint. In one case the location of a crack was determined by the longitudinal sides of several widely-separated lifts lying in the same vertical plane. It is of interest to note that similar

cracking was observed in the Bleiloch dam [1] in 1931–2. In view of the deleterious effect of longitudinal cracking on the strength of a dam, it is fortunate that such cracks are unlikely to penetrate to a depth of more than a few feet, and that the tendency for them to extend will be checked as soon as they are covered by the concreting of the adjacent block. This experience prompts the suggestion that the continuous keyways between blocks might with advantage be formed parallel or nearly parallel with the downstream instead of the upstream face. A plane of cracking in this direction would not affect the stability of the dam. It is usually considered advantageous to concrete alternate blocks of a dam first so that as long an interval as possible may elapse before the concreting of the intermediate blocks. In this way the maximum amount of heat is dissipated and the ultimate opening of contraction joints is reduced. In the light of the information obtained at Laggan dam the Author is inclined to think that in large dams it would be better that adjacent blocks should be concreted concurrently with as short a time-lag as considerations of shuttering permit. The maximum temperature in the heart of a block is practically reached within a month, and this is too short a time for this maximum to be sensibly affected by the heat lost at an exposed radial joint some 23 feet away. The tendency for cracks to form at the upstream and downstream faces will therefore not be relieved by concreting alternate blocks first. On the other hand, by exposing radial joints for a short time only longitudinal cracking will be prevented. Moreover, any tendency for relative movement at joints due to differential vertical shrinkage is avoided. This is important as the friction between blocks may prevent vertical contraction of an intermediate block, with consequent opening of its horizontal joints. Although the maximum temperature will remain practically the same, cooling will, naturally, be slower and the ultimate opening of the contraction joints somewhat greater. This is of no moment if the joints are suitably sealed at the upstream face.

The experience at Laggan dam prompts the following observations relative to the prevention of cracking on the faces of a dam. The spacing of contraction joints at from 15 to 20 feet apart (this probably holds over a wide range of sizes of dams) would be effective, but expensive. It is evident from *Fig. 13* that the tendency to crack may be reduced by reducing the temperature difference between the heart of the dam and the faces, by increasing the tensile strength at the faces, or by reducing the shrinkage there. The temperature

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[1] E. Probst, "Principles of Plain and Reinforced Concrete Construction," p. 333. London, 1936.

difference may be reduced by ensuring the dissipation of a greater proportion of the total heat before the concrete is covered by the next lift; this generally involves reducing the rate of concreting. A second method is to ensure the more rapid dissipation of heat after placing the succeeding lifts by the circulation of cooling water through pipes embedded in the concrete. Another method is to reduce the rate of evolution of heat after the concrete has been covered. With this object a coarse-ground cement with a residue of 7 per cent. on the 170-mesh sieve in place of the usual 2 to 3 per cent. was used in the central block of Laggan dam. It has not been possible to assess the value of this method; there is the disadvantage that less heat is dissipated before a lift is covered, leaving more to be dispersed later. Still another method is by the use of low-heat cement. In the case of small masses of concrete there is some value in leaving the timber face-shuttering in place until the temperature at the centre has fallen sufficiently low. This method would have been of no value at Laggan dam unless the shuttering had been retained in place for the best part of a year. The tensile strength at the face may be increased by the use of rich concrete, horizontal reinforcement, or well bonded pre-cast blockwork or ashlar. The reduction or prevention of drying shrinkage is of great importance. If allowed to dry-out at the surface a rich concrete with its greater capacity for shrinkage may be a disadvantage rather than an advantage. A further advantage of the pre-cast facing is the possibility of reducing or even eliminating such shrinkage. Two of these methods might, perhaps, with advantage be combined, such as the use of a low-heat cement in the heart of the dam in conjunction with a rich Portland-cement concrete facing, the surface of which is kept damp until the dam is brought into use. To complete the alternatives, mention should also be made of a method which eliminates the whole problem, namely, the construction of the entire dam in large pre-cast concrete blocks bedded in and grouted with cement mortar.

The Deflexion of Laggan Dam.

In order to obtain information on the behaviour of dams under load, and of Laggan dam in particular, it was decided to install a long plumb-line in the heart of the dam. A 3-inch diameter brass tube was brought up vertically through the dam from the inspection gallery to the scour-culvert roller-gate winch-house. The maintenance of this tube in a truly vertical position through 110 feet of concreting was facilitated by an electrical plumbing device, but unceasing vigilance was also involved. The following are the

measurements, each being the mean of a large number of swings of the plumb-line. They represent the relative deflexion of a point near crest-level to the deflexion at the inspection gallery.

Date.	Water-level.	Deflexion: inch. From measurements.	Deflexion: inch. As calculated.
13.5.34	+ 700 O.D.	Zero datum	0·000
27.8.34	815·6	0·066	0·070
17·9·35	815·4	0·074	0·070
14·9·36	818·8	0·08*	0·079
19·9·36	818·4	0·06*	0·079

* An allowance has been made for obliquity of line of sight.

The total deflexions of the crest corresponding to the calculated relative deflexions of 0·070 inch and 0·079 inch would be 0·083 inch and 0·093 inch respectively. The calculations (see Appendix II) are based on a value of the modulus of elasticity of 4,000,000 lbs. per square inch and of the modulus of shear rigidity of 1,700,000 lbs. per square inch. These values are based on recent information regarding the variation of elasticity with grading, water/cement ratio, age and stress.[1] The mean stress in the dam is low, being of the order of 60 lbs. per square inch, so that the moduli should suffer no appreciable reduction due to magnitude of stress, and creep should be negligible. The agreement between the calculated and observed deflexions is seen to be good. This is probably to some extent fortuitous, for there are certain complicating factors of which no account has been taken. Thus, while the opening of the contraction joints (except at low levels) removes all possibility of horizontal-arch action, the deflexion of the central block should be reduced somewhat by the restraint exerted through the vertical keys by the shallower blocks on either side of it. On the other hand, drying-out of the downstream face, with consequent shrinkage or opening of horizontal joints at that face, should cause an increased deflexion. These effects apparently counterbalance each other. It is evident that the dam is behaving satisfactorily. The provision of some means of ascertaining deflexion in a dam is, in the Author's opinion, justified, if only for the assurance of security which it may give.

Conclusion.

The construction of the Laggan–Treig works occupied 3½ years and was completed in July, 1934. Their cost was approximately £1,000,000, making a total for the civil engineering works of

[1] E. Probst, "Principles of Plain and Reinforced Concrete Construction." London, 1936.

the first two stages of development of about £4,000,000. The Engineers were Messrs. C. S. Meik and Halcrow, Mr. W. T. Halcrow, M. Inst. C.E., being responsible for the design and supervision of the works from their inception. Mr. B. N. Peach, O.B.E., B.Sc., M. Inst. C.E., acted as Resident Engineer, and the Author as Chief Assistant Engineer. The Author also assisted in the design of the works.

With the exception of the road diversions and access roads, the whole of the works were carried out by the principal Contractors, Messrs. Balfour Beatty and Co., Ltd., with Mr. R. A. Waddell, Assoc. M. Inst. C.E., as their Agent and Mr. F. L. Williams, Assoc. M. Inst. C.E., as Chief Engineer. The hydraulic gates and valves at Laggan and Treig dams were supplied by Messrs. Glenfield and Kennedy, Ltd.

The Author wishes to record his thanks to Mr. W. T. Halcrow for permission to write this Paper, and for supplying him with information and drawings for reproduction therein.

The Paper is accompanied by twelve sheets of drawings, from which Plates 1 and 2 and the Figures in the text have been prepared, by two photographs, and by the following two Appendixes.

APPENDIX I.

Derivation of the Temperature Curves.

It is possible to arrive at the approximate form of the temperature curve from theoretical calculation. It is assumed that the temperature along a line W, 5, 4, 3, 8, 9, A (Fig. 11, Plate 2) will correspond very closely with the temperature distribution of a parallel slab of thickness WA. As nearly all the heat evolved in setting is given off during the first month, the general shape of the curve after the first few months will be indistinguishable from the cooling curve for initially hot inert concrete.

It is further assumed that the temperature of the surface of the concrete may be taken as the same as the mean monthly temperature of the atmosphere or the water, as the case may be. The concrete surface temperature will not be able to follow the rapid daily fluctuations of external temperature, but the effect of such rapid fluctuation extends a very short distance only into the concrete.

The temperature curve for any point in the concrete may be considered to be the algebraic sum of the cooling curve from an initially uniform high temperature, with a constant external temperature equal to the mean annual temperature, and the periodic curve of fluctuation corresponding to a variation of temperature at the surface equal to the annual variation. This is not quite correct, for the annual temperature cycles are initially in phase throughout

a hot body of uniform initial temperature, whereas for simplicity of calculation it is assumed that a stable periodic cycle has been attained with a time-lag increasing to the centre of the slab. The error is quite small, for the range of fluctuation in the centre is small and near the surface the lag is small. After the first few months, which in any case do not correspond closely with actual temperatures, the theoretical curves as derived in this way are applicable. The variation of external temperature is assumed to follow a simple sine curve and to be the same at both faces of the dam.

The cooling curve and the curve of periodic fluctuation will now be considered :

Symbols :

Let θ denote the temperature measured above mean atmospheric temperature.
,, θ_0 ,, ,, initial temperature above mean atmospheric temperature.
,, θ_1 ,, ,, temperature at the surface above mean atmospheric temperature.
,, t ,, ,, time.
,, T ,, ,, periodic time.
,, x ,, ,, distance from the centre.
,, r ,, ,, half thickness.
,, h^2 ,, ,, thermal diffusivity $= \dfrac{\text{conductivity}}{\text{specific heat} \times \text{specific gravity}}$.
,, b ,, ,, $\dfrac{2\pi}{T}$.
,, a ,, ,, $\dfrac{1}{h}\sqrt{\dfrac{b}{2}}$.
,, λ ,, ,, lag.
,, n ,, any integer.
,, A and B denote constants.

(i) *Formula for the temperature of a slab initially at a uniform high temperature with constant surface temperature.*—This formula is given in standard textbooks and is as follows :—

$$\frac{\theta}{\theta_0} = \frac{2}{\pi}\sum_{n=1}^{n=\infty}\Big[1-(-1)^n\Big]\frac{1}{n}e^{-n^2\left(\frac{\pi}{2}\right)^2\frac{h^2t}{r^2}}(-1)^{\frac{n-1}{2}}\cos\frac{n\pi x}{2r}$$

Neither θ_0 nor h^2 are known. If log θ/θ_0 from the above equation is plotted against t the magnitude of θ_0 will not affect the shape of the curve. In this way curves can be drawn for various values of h^2 and their shapes compared with the curve obtained by plotting the log of the observed temperatures against the time. The value of h^2 which makes the curves parallel will give the thermal diffusivity, and the value of the vertical interval between the curves will be log θ_0, whence the equivalent initial temperature for the theoretical cooling curve follows. The above expression can then be applied to give the temperature at any point on the section after any interval of time, on the assumption that the external temperature remains constant at its mean value. The results will be applicable with the exception of the first 2 or 3 months, when the effect of the non-instantaneous generation of heat is of importance.

(ii) *Formula for the temperature in a slab subject to an equal periodic fluctuation of temperature on each face.*

It is assumed that a stable temperature-cycle has been attained, the same

distribution of temperature recurring after an interval of time T (1 year in this case).

The general equation of heat conductivity is

$$\frac{d\theta}{dt} = h^2 \frac{d^2\theta}{dx^2}.$$

The surface temperature is assumed to be given by a sine curve: $\theta_1 = \theta_{1_{max}}$ sin bt. As conditions are symmetrical about the centre of the slab, a general periodic solution to this equation symmetrical in x is :—

$$\theta = A[e^{ax} \sin (bt + ax) + e^{-ax} \sin (bt - ax)] + B[e^{ax} \cos (bt + ax) + e^{-ax} \cos (bt - ax)]$$

Putting $c = \sin^{-1}\dfrac{B}{\sqrt{A^2 + B^2}}$,

$$\frac{\theta}{\sqrt{A^2 + B^2}} = \sin (bt + c) \cos ax\, (e^{ax} + e^{-ax}) + \cos (bt + c) \sin ax\, (e^{ax} - e^{-ax})$$

and putting $m = \sin^{-1} \dfrac{\sin ax\, (e^{ax} - e^{-ax})}{\sqrt{e^{2ax} + e^{-2ax} + 2 \cos 2ax}}$,

$$\theta = \sqrt{A^2 + B^2}\, \sqrt{e^{2ax} + e^{-2ax} + 2 \cos 2ax} \,.\, \sin (bt + c + m).$$

Now at the surfaces there is no lag and θ will be proportional to sin bt

$$\text{that is, } c = -\sin^{-1}\frac{\sin ar\, (e^{ar} - e^{-ar})}{\sqrt{e^{2ar} + e^{2ar} + 2 \cos 2ar}}$$

and the lag $\lambda = -(c + m) = \sin^{-1}\sqrt{2}\dfrac{\sin ar \sinh ar}{\sqrt{\cosh 2ar + \cos 2ar}}$

$$- \sin^{-1}\sqrt{2}\frac{\sin ax \sinh ax}{\sqrt{\cosh 2ax + \cos 2ax}}$$

Hence $\dfrac{\theta}{\theta_{1_{max}}} = \sqrt{\dfrac{\cosh 2ax + \cos 2ax}{\cosh 2ar + \cos 2ar}} \sin (bt - \lambda)$

Where $b = \dfrac{2\pi}{T}$

$$a = \frac{1}{h}\sqrt{\frac{b}{2}}$$

In particular, at the centre of the slab :—

One half the temperature-range $= \theta_{1max}\sqrt{\dfrac{2}{\cosh 2ar + \cos 2ar}}$

and the lag $\lambda = \sin^{-1}\sqrt{2}\dfrac{\sin ar \sinh ar}{\sqrt{\cosh 2ar + \cos 2ar}}$.

From the above equations the deviation of the temperature at any point and at any time from its annual mean value can be found. It is found that the variation of external temperature at Laggan dam could be approximated by a sine curve of amplitude $\theta_{1_{max}} = \pm 12°$ F. and that the mean temperature was 45° F.

Combining the results from (i) and (ii) the actual curve of temperature at any point can be found approximately.

APPENDIX II.

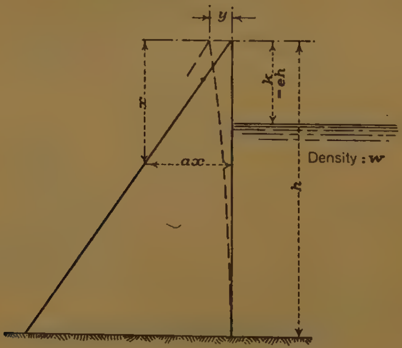
The Deflexion of a Dam.

The rigid solution of the deflexion of a dam is difficult and any additional accuracy obtained thereby would be counterbalanced by the non-elastic behaviour of concrete, distortion due to inequalities of temperature and the possibility of distortion due to unequal drying-shrinkage on the upstream and downstream faces. It is nevertheless of interest to know the order of magnitude of the deflexion due to water-load, and for this the usual elastic-beam theory is employed.

Shear-strain cannot be neglected; for convenience the deflexion is first calculated on the assumption of no shear-strain, and then the deflexion due to shear-strain alone is calculated and added.

A simple triangular cross-section with a vertical upstream face is assumed. The general case of the dam partially filled is considered. It is required to find the deflexion at any level. Consider a unit length of dam.

Fig. 14.



Symbols.

These are as in *Fig. 14,* and in addition

Let p denote the unit pressure at upstream face.
,, S ,, ,, shear.
,, M ,, ,, bending moment.
,, E ,, ,, modulus of elasticity.
,, G ,, ,, rigidity.
,, I ,, ,, moment of inertia $= \dfrac{(ax)^3}{12}$.
,, m ,, ,, shear-strain factor:

It is assumed that the angular shear-strain corresponds to a shear-stress m times the mean shear over a section.

$$\text{Let} \begin{cases} k = eh. \\ x = rh. \\ x_1 = r_1 h. \\ x_2 = r_2 h. \end{cases}$$

(i) *Deflexion due to elastic bending alone.*

From $x = k$ to $x = h$:

$$\frac{d^2M}{dx^2} = p = wh(r - e)$$

Intregrating : $\dfrac{dM}{dx} = S = \dfrac{wh^2}{2}(r - e)^2$ . . . . (1)

Integrating : $M = \dfrac{wh^3}{6}(r - e)^3 = EI\dfrac{d^2y}{dx_2} = E\dfrac{a^3h^3}{12}r^3\dfrac{d^2y}{dx^2}$

$$\therefore \frac{d^2y}{dx^2} = \frac{2w}{Ea^3}\left[1 - 3\frac{e}{r} + 3\frac{e^2}{r^2} - \frac{e^3}{r^3}\right]$$

Integrating and inserting $\dfrac{dy}{dx} = 0$ at $x = h$,

$$\frac{dy}{dx} = \frac{wh}{Ea^3}\left[2r - 6e \log r - 6\frac{e^2}{r} + \frac{e^3}{r^2} - 2 + 6e^2 - e^3\right] \quad . \quad . \quad (2)$$

Integrating and inserting $y = 0$ at $x = h$, then at depth $x_2/ \geqslant k$,

$$y_2 = \frac{wh^2}{Ea^3}\left[1 - 6e - 6e^2 + 2e^3 - \frac{e^3}{r_2} - 6e(e - r_2)\log r_2 \right.$$
$$\left. - r_2(2 - 6e - 6e^2 + e^3) + r_2{}^2\right] \quad . \quad . \quad (3)$$

Above the water-level, that is, $x < k$, $\dfrac{dy}{dx}$ is constant and equal to its value at k,

$\therefore x_1 \leqslant k,\ y_1 = y_k - (k - x_1)\dfrac{dy}{dx_k}$, whence from (2) and (3), putting r and $r_2 = e$,

$$y_1 = \frac{wh^2}{Ea^3}\left[1 - 6e + 3e^2 - 6e^2 \log e + 2e^3 \right.$$
$$\left. - r_1(2 + 3e + 6e \log e - 6e^2 + e^3)\right] \quad . \quad . \quad . \quad (4)$$

The deflexion is given by formula (4) for points above the water-level and by formula (3) for points below the water-level.

(ii) *Deflexion due to elastic shear-strain alone.*

$\dfrac{dy}{dx} = -m\dfrac{S}{axG}$, hence for $x \leqslant k$, since $S = 0$, $\dfrac{dy}{dx} = 0$.

From $x = k$ to $x = h$, substituting for S from (1)

$$\frac{dy}{dx} = -\frac{m}{aGrh} \cdot \frac{wh^2}{2}(r - e)^2 = -\frac{mwh}{2aG}\left(r - 2e + \frac{e^2}{r}\right).$$

Integrating and inserting $y = 0$ at $x = h$,

$$x_2 \geqslant k,\ y_2 = \frac{mwh^2}{4aG}\left[1 - 4e - 2e^2 \log r_2 + 4er_2 - r_2{}^2\right] \quad . \quad . \quad . \quad . \quad (5)$$

$$x_1 \leqslant k,\ y_1 = y_k = \frac{mwh^2}{4aG}\left[1 - 4e + 3e^2 - 2e^2 \log e\right] \quad . \quad . \quad . \quad . \quad (6)$$

(iii) *Total Deflexion.*

Combining (i) and (ii) the total deflexion is :—

For points above the water-level, $x_1 \leqslant k$,

$$y_1 = \frac{wh^2}{Ea^3}\left[1 - 6e + 3e^2 - 6e^2 \log e + 2e^3 - r_1(2 + 3e + 6e \log e - 6e^2 + e^3)\right]$$
$$+ \frac{mwh^2}{4aG}\left[1 - 4e + 3e^2 - 2e^2 \log e\right] \quad . \quad . \quad . \quad . \quad (7)$$

For points below the water-level, $x_2 \geqslant k$,

$$y_2 = \frac{wh^2}{Ea^3}\left[1 - 6e - 6e^2 + 2e^3 - \frac{e^4}{r_2} - 6e(e + r_2)\log r_2 - r_2(2 - 6e - 6e^2 + e^3) + r_2^2\right] + \frac{mwh^2}{4aG}\left[1 - 4e - 2e^2\log r_2 + 4er_2 - r_2^2\right] \quad . \quad . \quad (8)$$

From (7) and (8) the relative deflexion between any two heights can be obtained.

For the total deflexion of a triangular dam when full, that is, when $k = 0$

$$y_0 = \frac{wh^2}{Ea^3} + \frac{mwh^2}{4aG} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Application to Laggan dam.

Units: lbs. and inches except where stated otherwise.

$a = 0{\cdot}7$.

$h = 143 \times 12 = 1716$ (Note: the apex of the equivalent triangle is taken at + 823 O.D.).

$w = 62{\cdot}5 \div 1728 = 0{\cdot}0362$.

$E = 4{,}000{,}000$.

$G = 1{,}700{,}000$.

$m = 1{\cdot}5$ (assumed, as for a rectangular beam of uniform section).

$x_1 = 823$ feet $- 819$ feet $= 4$ feet $\therefore r_1 = 4 \div 143 = 0{\cdot}028$.

$x_2 = 823$ feet $- 706$ feet $= 117$ feet $\therefore r_2 = 117 \div 143 = 0{\cdot}818$.

Case (1): Water-level at + 815·4 O.D. (that is, $k = 7{\cdot}6$ feet or $e = 0{\cdot}053$).

Applying the formulas:—

	Bending: inch.	Shear: inch.	Total deflexion: inch.
y_1	0·055	0·027	0·082
y_2	0·0022	0·0098	0·012
		Relative deflexion	0·070

(*Cf.* measured deflexion is 0·066 inch with water-level at + 815·6 O.D., and 0·074 inch with water-level at + 815·4 O.D.)

Case (2): Water-level at + 818·6 O.D. (that is, $k = 4{\cdot}4$ feet or $e = 0{\cdot}031$).

	Bending: inch.	Shear: inch.	Total deflexion: inch.
y_1	0·062	0·030	0·092
y_2	0·0024	0·0102	0·013
		Relative deflexion	0·079

(*Cf.* from measurements, 0·08 inch with water-level at + 818·8 O.D., and 0·06 inch with water-level at + 818·4 O.D.)

Fig. 15.

LAGGAN DAM.

Fig. 16.

TWO SIPHONS AND SCOUR-CULVERT OF LAGGAN DAM IN ACTION.

Fig. 17.

UPSTREAM FACE OF LAGGAN DAM DURING CONSTRUCTION; MARCH, 1934.

Fig. 18.

TREIG DAM, SHOWING RAILWAY-DIVERSION AND TUNNEL-OUTFALL.

Discussion.

The AUTHOR drew attention to Fig. 1, Plate 1, where the shaded The Author.
ea indicated the total catchment area for the completed scheme, hich consisted of the first, second, and third stages. The works the first stage lay entirely to the west of loch Treig, and the tchment area for those works passed only slightly to the east of at loch. The second-stage development took in the catchment ea eastwards as far as the watershed between the rivers Pattack d Mashie—no dividing-line was shown in *Fig. 1*—and extended the north to the watershed between the river Spey and loch aggan. The third stage would include the remainder of the shaded ea and would consist of the Spey dam and conduits to bring in aters from the river Spey and the river Mashie. The second-stage velopment, therefore, consisted of the dredging of a channel at e west end of loch Laggan, the construction of the Laggan dam (with crest level equal to the high-water level in loch Laggan), and the version of the water through a tunnel into loch Treig, where other dam was necessary to maintain high-water level. He then owed a number of lantern-slides illustrating the work, some of hich were reproduced in *Figs. 15* to *18*.

The PRESIDENT, in proposing a vote of thanks to the Author, The President.
oserved that it was not true to say that the countryside at Lochaber d been spoilt by the works which had been carried out. The orks reflected great credit upon the designers.

Mr. W. T. HALCROW observed that the Paper gave such an excel- Mr. Halcrow.
nt description of the construction of the second stage of the ochaber water-power scheme that there was little that he could ld. Two matters which were, perhaps, of outstanding interest ere the provision of siphons, with their novel features, in the aggan-dam spillway, and the research work which was carried out the concrete of that dam.

Laggan reservoir was not large enough to regulate fully the flow water from its catchment-area, and it was therefore essential to ake the most of the limited range of water-level available. The erflow-spillway allowed the water to rise to the lower limit stipu- ted, namely 6 inches above the spillway-level, without losing uch water. After that limit was reached, the siphons came into eration in pairs and quickly passed off the excess water.

While there was sufficient knowledge of siphons to satisfy him at they would function when required, very little information peared to be available on the problem of causing the siphons to

Mr. Halcrow. prime and break within a narrow range of water-levels. For t reasons given in the Paper, it was desirable that all the sipho should be out of action at the time when the water fell to the low stipulated level, as otherwise they would continue to function un air was drawn in through the inlet, and the available storage wa would be lost. The siphons had worked very smoothly, and, whi the simple air-inlet lip method of breaking the siphons referred on p. 14 did not act within the small range desired, the "Morgan air-valves had functioned admirably. During the $2\frac{1}{2}$ years sin the dam was closed no alterations had been made to the origi adjustment of the siphons. Sufficient experience had been gain of their working for him to decide to make some slight adjustmen to allow of a somewhat larger margin of working to the stipulat levels. It would be realized that the effect of strong winds on reservoir 12 miles in length with an irregular shape at the weste or dam end was difficult to predetermine.

The Author had dealt very fully with the question of the rise temperature of concrete on setting, and Mr. Halcrow did not prop to comment on that matter. The question of low-heat cement w the subject for study of one of the Research Committees of T Institution, working jointly with the Committee on Special Ceme appointed by the International Commission on Large Dams, and was to be hoped that in due course a cement more suited to construction of dams would be obtainable.

The Treig dam spillway had been in operation on several occasio and he had a report on an observation by Mr. B. N. Peach, M. I C.E., who was his resident engineer on the construction of the seco stage works and who was now the British Aluminium Compan maintenance-engineer for the constructional works at Foy Kinlochleven, and Lochaber. The observation was made w there was a depth of 1 foot of water over the spillway. Mr. Pe wrote :—

"Conditions of spill at Treig dam were observed with water le in the loch at 820 feet O.D. The water flowed smoot and evenly over the apron and along the wing walls baffles, gradually becoming more turbulent towards toe. Water was very turbulent over the wave wal the toe of the dam piling up in four places in line w the main flow down the wings and baffles. The app ance was almost identical with that shown in the ph graphs taken of the Treig dam spillway mo Downstream of the dam the turbulence rapidly away and at 120 feet from the toe the flow in the r was smooth and spread fairly evenly over the bed."

thought that that was an interesting record of the effect of the fle walls on the spillway, and it was also another instance of the ue of the hydraulic model. Mr. Halcrow.

He would like to refer to the risk of landslides, to which reservoirs h steep sides, such as loch Treig, were liable. The natural face of the ground where part of the railway diversion was conucted was steeper than $1\frac{1}{2}$ to 1, and the ground consisted of glacial raine overlying the rock. It was not possible to foretell the effect submerging ground of that nature by raising the level of the loch. er the greater length of the banks of the reservoir landslides would unimportant, except for the possibility of causing a wave over the llway should the landslide occur when the reservoir was full. ere was, however, a length of about 2 miles of reservoir on the th-east side where a landslide would be a serious matter, as night endanger the L.N.E. Railway. A considerable area of the hillside below the railway and below the top water-level of the ervoir had been protected with stone. In the section nearer to dam this took the form of hand-placed pitching, but the more icult area lay to the south of the tunnel, and there the protection isisted of loose rubble, which was known to be a satisfactory y of protecting the slopes. The area which had to be treated s situated in probably the most exposed part of the loch, which l a length of some 6 miles with mountains on both sides rising out 2,000 feet. The configuration of the hills was somewhat nel-shaped, with the narrowest part at the southern end of the way diversion. During strong south-westerly gales the velocity the wind in that constricted space was bound to be very high; aused short, sharp waves which travelled along the shore and had onsiderable erosive effect. From his point of view the difficulty l been to know to what extent the protection ought to be carried. at had been largely a matter of judgment. At one time he had isidered a scheme involving the expenditure of from £80,000 to 00,000 on protective works which might or might not be required. e interest and cost of services on a capital expenditure of £100,000 uld be £4,000 or £5,000 a year. He had been convinced that the ual maintenance required would not amount to that sum, and had advised the directors of the company accordingly. The ervoir had now been in use for about 2 years, and the annual cost additional rubble stone to strengthen places which showed signs veakness after storms had been considerably less than the amount ich would have been required for interest.

He would like to take the opportunity of expressing his apprecia- n of the good work done by Mr. Peach, the resident engineer, by Author and by the rest of his staff on the works. He would also

Mr. Halcrow. like to congratulate the contractors, Messrs. Balfour Beatty & C Ltd., on the successful manner in which they organized and carr out the works, which were of a very varied character.

Mr. Gourley. Mr. H. J. F. GOURLEY remarked that, before referring to design of the Treig dam, he would like to say a few words about floods which might have to be passed over the spillway. catchment area was $43\frac{1}{2}$ square miles, or 28,000 acres, and the water-area was 1,790 acres, or 6·4 per cent. of the catchment a Following the Report of the Institution Committee on Floo a normal maximum peak inflow for such an area would be at the of 312 cusecs per 1,000 acres, or 8,750 cusecs; having regard, howe to the unusually rapid, and indeed almost precipitous, slopes and to consequent reduction in the period of concentration, it was prob that the peak inflow would be 50 per cent. higher, or say 13, cusecs, to which had to be added a further quantity for the back- from the Ben Nevis tunnel and the inflow from the Laggan tun He had assumed that to be 2,000 cusecs, which might be on the side. If 3 feet were assumed as the corresponding peak-head of flow, the lag-effect would reduce 15,000 cusecs inflow by 60 per c to 6,000 cusecs, and that meant a little over 3 feet over the spill as compared with 7,600 cusecs, which the Author gave. Thus, e with a flood of catastrophic character, the capacity of the spill would not be exceeded.

He had been very interested to see the results given by the Au on p. 20 of the experiments on a model of the Treig spillway. logarithms were used the results when plotted lay exactly c straight line, giving a formula $Q = 3{\cdot}12bh^{1{\cdot}525}$, which suggested a weir having a rounded crest and upstream and downstream sl tangential to it might have advantages for water measurement.

The Paper described the provisions that had been made in design of the dam to deal with the possible effects of settleme the rock fill; as the work had been completed for over 2 year would ask the Author whether any material settlement had occu

The rock fill of the Shing Mun dam at Hong Kong was buil in comparatively shallow layers, with a slight inclination upstr Labour was cheap there, and as each layer was brought up al interstices were carefully filled. It was believed that in that w very compact portion of the dam was secured so far as the rock fil concerned. Settlement had occurred, but it was difficult to be ce of the amount that was the result of compression of the found and of how much was due to natural consolidation. The maxi height of the rock fill was about 260 feet, and as the fill proce

[1] "Interim Report of the Committee on Floods in Relation to Res Practice." Inst. C.E., 1933.

full height the work steadily settled down. To judge by the latest Mr. Gourley.
servations taken on the completed work, it seemed likely that
ther settlement would not exceed 12 or 18 inches at the most at
e section of greatest height.

The general features of the design of the Treig dam had clearly
en the result of very careful consideration, and the outcome was
composite structure in which local conditions had dictated some
usual features. In particular, reference might be made to the
aptation of the core-wall to serve as the spillway crest, and to the
ssing of the flood water over the rock fill, which was protected by
ncrete panels.

The Pontian Ketchil reservoir, one of the head-works of the Singa-
re water-supply, had been completed in 1931, and was formed by
o earthen embankments, each with a reinforced-concrete core-wall.
e subsidiary dam was a structure about 40 feet high for practically
e full length of 2,800 feet. Part of the flood-water from this
ervoir would pass over 110 feet of the core-wall and thence over a
ies of concrete weirs forming compartments which were partially
ed with rubble, in order to form a stilling basin and thus to enable
e discharge to spread quietly over the swampland downstream.
no rock or other reasonably watertight material was available
the site, the cut-off below ground was formed of interlocking
el sheet-piles, each driven to a set of $\frac{1}{100}$ inch and to a depth of
feet below the bottom of a 15-foot deep trench. The concrete
e-wall rested upon and enveloped the top of the sheet-piling, with
vision for slight relative movement between the concrete and the
es to allow of deflexion of the core-wall without longitudinal
cking.

According to the figures given in the Paper, the flood discharge
the Laggan catchment, measured at Laggan dam, was 12,400
ecs, which was equivalent to 132 cusecs per 1,000 acres; that
npared with 205 cusecs per 1,000 acres inflow, or 18,300 cusecs
m the 94,000 acres (147 square miles) of catchment area, as given
the Floods Committee [1] curve of normal maximum floods, and
vas probable that that would be somewhat higher if the extra
epness of the slopes, compared with those upon which the Com-
tee had worked, was taken into account. In the case in question,
reservoir rise was not to exceed 2 feet for more than 24 hours.
ce the reservoir had an area of 2,545 acres, or 2·7 per cent. of the
chment, the lag effect would be small, and would not exceed 10 per
t., or, at the most, 15 per cent. He felt certain that the limits of
Act could be complied with when dealing with a normal maximum

[1] *Loc. cit.*

Mr. Gourley. flood, but he was not sure whether, in the case of a catastrop flood, it would be possible to reduce the flow over the dam to a 2-f limit above the spillway within the 24 hours prescribed by the A

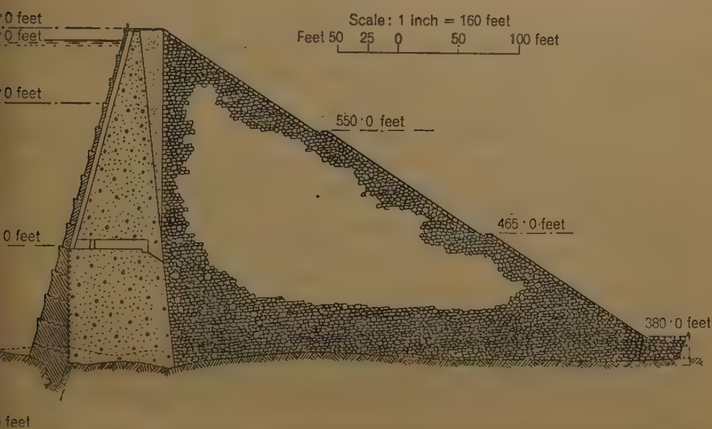
Where an outlet-pipe was relatively short and was provided wit jet-disperser, the coefficient of discharge was about 0·6; a long p leading to the disperser might entail a substantial decrease in overall coefficient which should not be overlooked. In ma cases where a considerable head might operate, however, it essential to disperse the water in order to reduce, and indeed alm eliminate, the possibility of localized erosion. With that object view, the 36-inch scour-pipe of the Shing Mun dam was fitted w a disperser, the centre of which was at 360 feet above datu the top water-level of the reservoir was 625 feet above datum, that the gross head available was 265 feet. At that dam provis was made for the discharge of over 8,000 cusecs by means o bell-mouthed overflow—which had been one of the alternati considered in designing the Treig dam—and of 3,000 cusecs b battery of six siphons, three being designed to come into action about 9 inches and the others at 15 inches above the sill of 74-foot diameter weir.

The Author's account of the flow through the diversion openi was most interesting. The lowering of the roof of the divers passages which the Author suggested was also of advantage in t it made the ultimate concrete filling form a tapered plug. The of 4–to–1 concrete for the water-face of the dam, increasing fr 2 feet thick at the crest to 7 feet at the lowest point, might be co pared with the articulated diaphragm which formed the water-fac the Shing Mun dam above the level 453, where the normal wa pressure was 182 feet. He would like to draw attention to t alternative method by which the effects of cracking might obviated, and a means by which joints might be rendered wa tight, for purposes of comparison with the methods adopted the case of the Laggan dam. The thickness of the diaphragm the Shing Mun dam at the level 453 was 6 feet, and at 52 feet be top water-level it became 3 feet and continued at that thickness the top; it was of concrete with 690 lbs of cement per cubic ya The face was constructed in alternating panels with vertical joi copper-strip stops and asphalt filler on either side at 25-foot inter and in lifts of 20 feet at one pour with horizontal sheet copp strip stops; each panel was reinforced, and rested upon and was f to move on the sloping face of buttresses formed in the thrust-bl (*Fig. 19*). The concrete of the thrust-block contained 300 lbs cement per cubic yard, and no special joints were specified or mad the thrust-block, for, if cracks occurred, they could not be subjec

water-pressure. The peculiar form of nib at the top of each of Mr. Gourley.
e panels was simply a convenient device for supporting the
uttering of the panel above. The shutters were of steel, and
re self-contained units which could be moved about quite
adily.

It might be asked whether that system had been tested. He could
ly say that the reservoir had been filled up to a level of 600 feet,
ich was only 25 feet below top water-level, and the amount of
ter which had been gauged at a point immediately downstream
the dam did not exceed 5,000 gallons a day. As far as was known,
ne of that water passed through the diaphragm; it was probably
ainage from the rock fill and from the hillside. The diaphragm
filled its function admirably.

Fig. 19.



Professor F. C. Lea observed that there were a number of points Professor Lea.
interest in the Paper. The discharge through certain siphons
given in the Paper (p. 13) as 590 cusecs. He wondered whether
Author had any information as to what happened in the top part
he siphon. It was mentioned in the Paper that near the top of
siphon the velocity was about 40 feet per second, the actual
ssure being 10 feet head of water; he had estimated that at
04 O.D. the pressure head was probably rather less than 8 feet.
vould also be of considerable practical as well as of theoretical
rest if the Author could say at what point in the siphon when
as discharging its maximum quantity, or the quantities referred
n the Paper, the siphon itself was really full of water.
hat raised another question. The Author said that from the

Professor Lea. model it was indicated that the siphon would discharge when t depth over the crest was $\frac{1}{3}$ of the throat depth. It would appe that that figure would be an actual value, and he was not qui sure that it was correct to express it as a ratio. He thought t point was of considerable importance for future design, and perha the Author could say whether in his view the dimension ought to expressed as $d/3$ or whether it ought to be expressed as, say, 1 foo In other words, if the top of the siphon were made 4 or 5 feet abo the crest, and everything else below it were unaltered, would t discharge be affected ?

Mr. Halcrow had said that the air-inlets as installed did not a quite as desired, but that the air-valves functioned perfectly. Ha they found from their experience that it was necessary to instal a valves, or could the simpler air-inlets be made to give the desirab control ?

He would like to ask what precautions were taken in time of seve frost, as the air-valves were controlled by water flowing into tank During severe frosts there was not likely to be any considerab flow over the dam but a sudden thaw might take place by the su shining on the mountains, causing a good deal of snow to mel the large masses of water and concrete would, however, probably reta ice in the neighbourhood of the dam far longer than the snow wou remain on the mountain-sides, and trouble might therefore ari Had any such experience occurred ? Had the possibility of fra ice forming at the siphon-inlets been considered ? Mr. A. Beaumont, M. Inst. C.E., had sent him recently some very interesti particulars of ice formed on the reservoirs of the Wakefield wat works, and he thought it would be possible for ice to form bel the surface on the grids of the Laggan siphons if conditions are such as those prevailing at the Wakefield reservoirs.

The Author stated that one of the siphons was estimated to gi a flow of 640 cusecs, whereas the $\frac{1}{8}$th-scale model had given 6 cusecs. The Author had suggested that the difference was due viscosity, but Professor Lea wondered whether it was not due to t ratio of roughness of the surface of the model to the linear dimensio of the model being different from the ratio of the roughness of t surface of the actual siphon to the dimensions of the siphon.

Dr. Glanville. Dr. W. H. Glanville remarked that the Author had describ very fully a considerable amount of experimental work, which h given results which should prove of very great value to enginee faced with similar problems in the future. The Author's work connexion with the temperature-rise and dissipation of heat in t concrete was extremely valuable, and had enabled the theor developed at the Building Research Station from small-sc

eriments to be confirmed and developed in a practical manner. Dr. Glanville.
Glanville proposed, however, to leave that aspect of the Paper to dealt with by his colleague, Dr. Norman Davey, who had been ponsible for carrying out the experimental work at the Building search Station and for analysing the data provided during the struction of the Laggan dam.

There were one or two points in the Paper on which he would like have further information. The first concerned the crushing ength of the concrete used. In the Paper the Author gave figures the 7–to–1 concrete and the 4–to–1 concrete, and those concretes described as having been made with a water/cement ratio by ght of 0·55 and 0·40 respectively. The strength of the concrete gave as 174 and 301 tons per square foot respectively. In the ords of the Building Research Station there was a considerable nber of test results on concretes of a very wide range of water/ nent ratios, and for ordinary Portland cements he would expect m those results a figure of 300 tons per square foot to be associated h a water/cement ratio of 0·55, and a figure of 425 tons per square h to be associated with a ratio of 0·40. The cement used in the n might have been of a slow-hardening type, but he was inclined think that the water/cement ratios used were not the values en. The water/cement ratios which would be required to produce strengths given in the Paper for ordinary cements were of the er of 0·75 for the 7–to–1 concrete and 0·55 for the 4–to–1 con- te. He would like the Author to comment on that point, because felt it had a very important bearing on the shrinkage and the t developed in the concrete. When it was possible on works to trol the water/cement ratio carefully it would be wrong, if re were some error in the information given in the Paper, attempt to control the water/cement ratio according to that ormation.

The method used for bridging cracks in the concrete was of interest; l the method proved successful? Assuming the most favourable ditions, the amount of adjustment which could be expected from h a length would be of the order of 0·006 inch, and he felt that t would have but a small influence in preventing the formation a crack, although it might mean that several smaller cracks were ned instead of one larger crack.

Another point which had interested him was that the cracks reased in width as they extended upward from the base of the n. That would normally be attributed to the restriction to inkage afforded by the foundation. He had examined the thor's suggestion that it was due to the lateral spread of the crete from elastic and plastic movement counterbalancing the

Dr. Glanville. strain, but unless those movements in the case in question wer an exceptional order he did not think that that explanation c be accepted. It had been established by experiments that lateral creep in concrete was extremely small, and was, in f practically negligible. Estimating, however, for the purpose obtaining an outside figure, that it was of the same order as the lat elastic movement (namely, that it corresponded to a value of Poiss ratio of 0·15) the maximum movement that could take place a 45-foot length would be about 0·01 inch, which was extrem small compared with the probable total thermal movements shrinkage.

It was clear that the stress-conditions in massive concrete da were extremely complex, and that high tensile stresses on external face were practically unavoidable unless very spe precautions, such as internal cooling by means of pipes carry water or a refrigerant, were used. The only other way of eliminat the trouble involved such a slow rate of placing of the concrete t it was really ruled out. He did not feel that the use of a rich concr or of horizontal reinforcement in the face would prevent cracki The rich concrete might in fact increase it, since it was well esta lished that the tensile strength of concrete did not increase to very great extent as the cement-content was increased, and order to eliminate cracking a very large increase in strength wou be necessary. Horizontal reinforcement would tend to distribu the cracks so that instead of a large crack a number of smaller o might be formed, but the quantity required to produce any appre able effect would probably be so great as to make the cost prohibiti He could not help feeling that if it was necessary to eliminate cracki altogether, the only way was to use pre-cast blocks which had be allowed to cool down and to shrink for some considerable peri before being placed in the structure. Mr. Gourley had put forwa a very ingenious alternative which allowed of movement witho cracking.

In conclusion, there was one small correction which he wou suggest on p. 42. The Author stated that "The mean stress the dam is low, of the order of 60 lbs. per square inch, so that t moduli should suffer no appreciable reduction due to magnitude stress, and creep should be negligible." It was an experiment fact that creep was for all practical purposes proportional to stre so that it would be expected that the moduli would be reduced ve considerably from the initial values. After about 1 year their effe tive values might reasonably be expected to be one-quarter or eve one-fifth of the initial values. That had an important bearing on t calculated deflexion.

Mr. Powys Davies observed that, although the construction of hon spillways had been developed on the Continent, in America, l elsewhere for many years—perhaps for a quarter of a century— thought that it was true to say that there had been a considerable ount of opposition to their introduction in Great Britain. At any e, it had been done only very tentatively, and he thought that thanks of the profession were due to Mr. Halcrow and his staff the first demonstration that the siphon spillway was a practical position and one which possessed many advantages. He had ently carried out certain tests of a siphon spillway in Indore, ntral India, and he had obtained a short cinematograph film of various operations involved, such as the priming and the dis- ation of energy. Mr. Davies.

Mr. Davies then showed the film, a copy of which he presented to e Institution.

Lieut.-General Sir Ronald Charles observed that interference h natural conditions always necessitated having to pay for it in way or another. For example, hill roads and many railways India were subject to tremendous breaches and wash-outs, owing the sides of mountains having been disturbed. His Company l been faced with similar occurrences on the shores of loch Treig; uble had started when the water had been first let through from n Laggan, coming out of the tunnel outfall into a very small sidiary lake known as Idir loch. The bed of that loch was on a minal moraine, consisting of sand and very fine gravel, and it had n been discovered that the natural hydraulic gradient did not ncide with that which the engineers had introduced, with the ult that excessive erosion had been met with. Considerable enditure and much ingenuity had been necessary to enable the ter to flow into loch Treig and to raise its level without danger the engineering works in the vicinity. Lieut.-General Sir Ronald Charles.

ttacks by wave action on ground which was either made, or else for centuries never been touched by anything heavier than inary rain water, had produced a good deal of erosion, particularly places where the hillside consisted of similar terminal moraine, such slopes had to be protected.

Ie desired to acknowledge, on behalf of the British Aluminium npany, the debt which it owed to Mr. Halcrow and his staff and the contractors for the design and execution of what was one of premier hydro-electric undertakings in Great Britain.

Ir. J. S. Wilson remarked that the design of the cross-section eared to have received considerable study. It was stated, for ance, that 50 per cent. of the water-pressure uplift had been wed for, and that "the maximum vertical compressive stress Mr. Wilson.

Mr. Wilson. at the upstream face . . . will be about 11 tons per square foot.' That corresponded to a "maximum principal stress at the down stream toe of under 15 tons per square foot." Further details were not given, and unfortunately the cross-section (Plate 1, Fig. 4 was at the sluices, and the general cross-section was not quite clear The usual course in such cases appeared, however, to have been followed and all the preliminary calculations had been used by th engineer responsible merely as a guide in deciding on a simpl triangular section with 70-per-cent. batter on the downstream fac and a sufficient width for a roadway across the top, and apparentl a perfectly sound dam was the result. He had not made any cal culations, but the relation of thickness to height appeared norma for an economical design.

The points in the design which had attracted his attention carrie the whole problem of masonry-dam design back 30 years. In 190 the late Professor Karl Pearson with his research student, Mr. L. W Atcherley, had greatly surprised all those interested in the desig of dams by stating that a dam might fail by tension in the down stream toe.[1] The Author on p. 9 stated that in the Laggan dan as designed there was no likelihood of tension in the downstrean toe, whilst in Appendix II, where he calculated the deflexion h assumed the distribution of the shearing stress to be parabolic as i a beam. One result of the Paper by Professor Pearson and M Atcherley had been the contribution to The Institution 28 year ago of three Papers,[2] proving that the suggestion of tensile stress i the downstream toe was completely fallacious.

The possibility of getting a tensile stress at the toe depended o the shear stress there. A beam had parabolic distribution of stress and a rectangular dam-section like that shown in *Fig. 20* also ha that distribution; there was no shear on the vertical downstrean face and therefore no shear on the horizontal section there. Wit a sloping downstream face (*Fig. 21*) there could be a high intensit of shear stress on the horizontal section at the face. The idea c tension at the downstream toe had seemed so preposterous, i spite of Professor Pearson's high authority, that Gore and himse had set to work to disprove it with their model in 1905.

A rubber model had been made representing a slice of a triangula

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[1] "On some Disregarded Points in the Stability of Masonry Dams. [*Draper's Company Research Memoir.*] London, 1904.

[2] Sir J. W. Ottley and A. W. Brightmore, "Experimental Investigations the Stresses in Masonry Dams subjected to Water-Pressure." Minutes Proceedings Inst. C.E., vol. clxxii (1907–8, part II), p. 89.

J. S. Wilson and W. Gore, "Stresses in Dams: An Experimental Investig tion by means of India-Rubber Models." *Ibid.*, p. 107.

E. P. Hill, "Stresses in Masonry Dams." *Ibid.*, p. 134.

am. Water-pressure had been reproduced by pressure-plates Mr. Wilson. nd weights, and the masonry had been given the right proportionate eight by weights suspended from the centres of the various sections

Fig. 20.

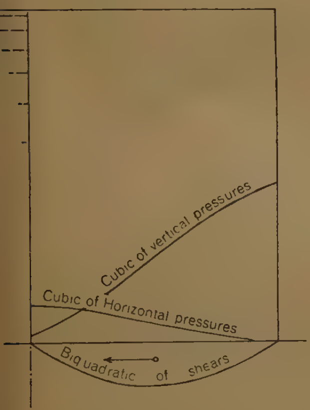


Fig. 21.

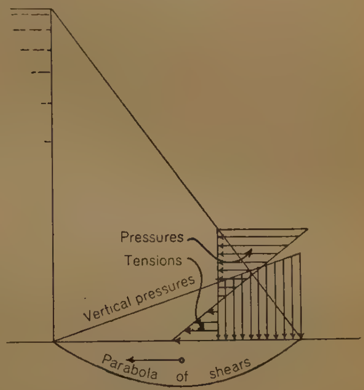
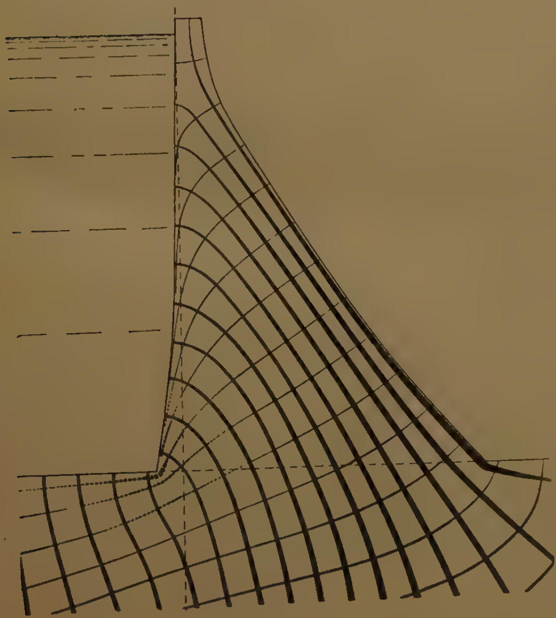


Fig. 22.



Mr. Wilson.

ruled on the rubber. The experiments had been to determine th shear distribution, so the model had been supported on a layer c balls and the shear had been taken up by loads applied at variou points along the base, which could be arranged as desired, to repro duce maximum shear at the upstream face or at the downstrean face. By comparing measurements on the face of the rubber whe loaded and unloaded, it had been possible to obtain the strains fror point to point and to deduce the stresses. They had definitel established the fact that the shear was represented by a triangle wit a maximum intensity at the downstream face, and there was n sign of tension anywhere near the downstream toe. Their late experiments, described in their Paper,[1] gave results for a dam c rational design. The ellipses of stress (all derived from the measure strains) gave the complete record of the stress conditions. Th stress conditions were shown in another way, although not s completely, in *Fig. 22* (p. 61). The lines showed the directions of th principal stresses and their thicknesses showed their intensitie The dotted portions showed where there was tension around th sharp angle at the upstream toe in the model. He felt sure that had the Author realized how completely the possibility of ther being tensile stress at the downstream toe had been disproved i 1908, he would not have mentioned it.

Mr. Stanger.

Mr. R. H. H. Stanger observed that a few remarks about th coarser-ground cement might be of interest. The cement for th works described by the Author had been drawn mainly from on factory and had had to conform to the usual British Standar Specification. The average fineness before the change-over to th coarser grindings mentioned in the Paper had been well below 4 pe cent. on a 170-mesh sieve, and the average tensile strength of th mortar had been 460 lbs. and 540 lbs. per square inch at 3 and days respectively. After the change-over to the 7-per-cent. fines the mortar-strengths had averaged 450 lbs. and 530 lbs. per squar inch respectively, so that the coarser grinding had had but littl effect on the strength in tension.

With regard to the strength of the concrete, Mr. Halcrow ha been good enough to let him have the list which he had prepare with all the strengths, and he (Mr. Stanger) found that th concrete strength for the 4-to-1 mix before the change-over to th coarser cement was 4,800 lbs. per square inch (about 310 tons pe square foot), whilst after the change-over it was 4,650 lbs. per squar inch (say 300 tons per square foot). For the 7-to-1 mix th strengths were about 2,800 lbs. and 2,900 lbs. per square inc

[1] Footnote 2, p. 60.

espectively for the fine and for the coarser cement; that was quivalent to 182 tons or 189 tons per square foot. Changing to ie coarser-ground cement had had practically no effect on the rength of the concretes which were being used. Mr. Stanger.

There was one point to which he desired to draw attention. When ngineers spoke of compression-strengths of concrete he wished that ey would all speak in terms of one unit. The Author spoke of tons per square foot" in the Paper, but the majority of engineers ealing with concrete constructions would use "pounds per square ch." Seeing that the Code of Practice for reinforced-concrete ructures, and the important building regulations as issued by the ondon County Council, all made reference to "pounds per square ch," he suggested that some authoritative statement should be ade by The Institution as to which unit engineers should use.

Mr. H. D. MORGAN remarked that with regard to the Laggan dam, r. Wilson had already expressed the wish for some further informa- on about the design of the dam. A large number of stability- agrams had been prepared for every conceivable condition of head, lift, and so on, and he felt that it would have been of interest if e Author could have included some of them in his Paper. Mr. Morgan.

With regard to the spillways, it had been laid down in the Act at a certain level in loch Laggan proper was not to be exceeded. the other hand, it was necessary to take as much advantage as ssible of the storage capacity, which had meant the desirability making the dam as high as possible, and there had therefore been narrow range within which to work. In the original spillway it d been proposed to instal eight siphons, and also tilting gates. ore detailed calculations had then been made and it had been und that, by taking full advantage of the storage capacity of the ervoir above water-level, the whole of the tilting gates could be minated and the number of siphons reduced to six. In the Laggan ervoir the flow was dependent upon the difference in water-level tween the two parts of the reservoir; that was to say, between h Laggan proper and the reservoir which was immediately above dam. In the double-reservoir problem with which the Floods mmittee[1] dealt, however, the flow discharged over a dam in the per reservoir and was a function of the head above that dam.

He had assisted in the tests of the siphons, and he did not think t the Author had mentioned all the models which had been tested. ere had been the scale model $\frac{1}{8}$th full size, which had given a charge for the siphon with the higher head of 613 cusecs as com- ed with the 640 and 590 cusecs for the siphon as designed and as

[1] *Loc. cit.*

Mr. Morgan. mentioned on p. 13. He agreed with the Author that the sma difference was probably due to viscosity. There had, however, bee another model which was intended to show whether the water goir over the crest of the siphon would, in fact, travel across to tl opposite side of the downstream leg and expel the air in the throa There had also been another model (which had consisted of a pip of the full depth of the siphon in order to establish the actual vacuu which would take place, and in order to determine how much a would be required to break that vacuum. That had been rath important, because very little had been published about that subje and it was difficult to find any figures. Actually, after those exper ments, the size of the air-pipes going to the throats of the sipho had been increased in order to make quite certain that there woul be sufficient air to break the vacuum.

The design of the siphons had been somewhat modified due t the experiments, particularly with regard to the downstream le and in order to avoid any possible spalling of the concrete due to lo pressures the siphons had been provided with steel linings in tl vicinity of the throat.

One criticism which he would offer was in connexion with tl Author's remark that the curve at the bottom of the Treig spillwa would assist the formation of a hydraulic jump. The one place which a hydraulic jump could not possibly take place was at a poi where the high-velocity water was rising vertically. The momentu of high-velocity water in the jump was taken up in establishing tl high-level flow; what the curve did was to ensure that the jun should take place at some point upstream of that toe, and not belo it on the river-bed.

He questioned the correctness of the assumptions on which tl calculations on p. 46 were based. The Author had dealt with tl elementary cantilever of unit width situated at the deep part of t dam, standing on its own without any restraint, but on either si there were a number of other cantilevers of various heights wi which it was bonded, and they were bound to exercise considerat restraint. He was of the opinion, based on the experience he h had with arch dams, that the restraint would alter the results of t Author's calculations to a considerable extent.

He was not at all sure that it was possible to measure deflexio of the order of 0·07 inch by means of a plumb bob. He felt that was one of those measurements in which it was a great advanta to know beforehand in which way the movement was expected take place. The most that could be said was that a movement h taken place (as might be expected) downstream and that the deflexi was somewhere of the order of 0·1 inch. The fact that there w

reement between the calculation and the recorded deflexion was Mr. Morgan.
obably nothing more than an accident; he noticed that the ithor said that he thought it might be "to some extent fortuitous." was possible that the Author was being a little modest, because little was known about the modulus of elasticity of those large isses of concrete, and there was such a wide range of values to oose from, that he knew from his own experience that it was ficult to maintain an attitude of strict scientific detachment in e face of a very great temptation to pick out the most suitable lue.

He desired to say a word about his air-valve. Any kind of valve iich was operated by a simple float had considerable disadvantages. r one thing, the range of water-level within which it had to erate—that was to say, to make and break the vacuum—was clearly proximately equal to the stroke of the valve, which could not be ide so small as the desired 3 inches. Again, the water-level rose nost imperceptibly at times, and if the float was gradually moving the valve might be partially open for days. It had to have a inite and rapid stroke at a predetermined water-level for making breaking the vacuum. That had been the idea of the differential w. The Author said that the flow began to rise when the inflow ghtly exceeded the capacity of the drain. That was not so. It gan to rise when the inflow slightly exceeded the capacity of the fice at the bottom of the float tank, which was partially throttled a taper plug attached to the valve-spindle. As soon as the float k a slight movement upwards, the taper plug immediately began throttle the orifice and the movement upwards was progressively id; so that, however slightly the level of the reservoir might e, the stroke of the valve took place in about $1\frac{1}{4}$ minute, and ilarly when it was downwards the initial movement of the float reased the effective area of the orifice, and it went faster and ter downwards, thus rapidly opening the valve.

Mr. JAMES WILLIAMSON said the figures given for the flow over the Mr. Williamson.
cial type of weir at the Treig dam were very interesting. They re lower than the figures for flow over a sharp-edged weir, but in usual spillway of a concrete dam, where the upper edge of the st was suitably rounded, the actual figures for spilling were more n the coefficient for a sharp-edged weir. He thought the Flood nmittee's Report[1] had been based in all cases on taking the ires for a sharp-edged weir. He thought that that was a point ich was worthy of some further investigation. With a suitable nding of the upstream edge, it was possible to get coefficients of

[1] *Loc. cit.*

Mr. Williamson. 3·5, 3·6, 3·7, 3·8, or 3·9 for various flows, but the coefficients depende somewhat on the depth of flow over the weir and on the radius of th upstream curve. He had not seen any very definite figures whic would give the true coefficients over a range, and he thought i would be quite a simple and useful investigation. He had recentl asked the engineers of the Safe Harbour Water Power Corporatio if they had made any tests of the coefficient over the round-edge weirs of their flood-gate spillways, which were very deep. He ha been told that the coefficient was almost 4. If, by suitably arrangin the crest, it was possible to get a coefficient of 3·7, 3·8 or 3·9, instea of 3·3, that was a very useful addition to the spillway capacity.

The Author had mentioned that he would be inclined to put th keyway, or grooving, next the downstream face parallel to the slope He was of the opinion that that was wrong, and he thought that th diagram of the lines of maximum stresses which Mr. Wilson ha given (*Fig. 22*, p. 61) supported his opinion. The lines of on principal stress (the minimum) were necessarily at right angles t the downstream face and ran almost asymptotic to the upstrean face. The lines of the other stress (the maximum) were at righ angles to the upstream face and ran asymptotic to the downstrean face. A joint at right angles to the lines of maximum pressur would be satisfactory, but it was going against sound principle to put in a joint or groove which would induce a crack paralle to the lines of pressure, that is, in a direction across which th stress sometimes tends to be tension.

Mr. Halcrow had designed Laggan dam to be constructed c separate blocks with straight joints between them. That method c construction had been very common for the last 15 or 20 year particularly in America. The alternate blocks were brought up wit shuttering around all the four faces, and then, when the shuttering o the ends was stripped, the intermediate blocks were filled in. A rather different method had been adopted on the Galloway dam constructed by Sir Alexander Gibb's firm. Blocks had been buil up with spaces about 5 feet wide between them. The ends of th blocks had been grooved, and the 5-foot spaces had been filled i with concrete at a later stage after as much time as possible ha been allowed for all the blocks to cool and shrink. It was too earl to make any comparison as to the results; it would be sufficien in 5 or 10 years' time to look at the dams and to see which metho might be best.

The Author had mentioned that he thought that there was no sig of shrinkage below 100 feet from the top. Three months ago Mr Williamson had been inside the inspection tunnel of the Norris dan in Tennessee, and he had seen the joints open at 150 feet below th

op of the dam; that dam had only been finished in August 1936. Mr. Williamson.
[H]e had also had the privilege of visiting a series of dams which were [v]ery comparable to the Laggan dam in that they were on a hydro-[e]lectric scheme for the purpose of producing aluminium. There [w]ere two dams across a narrow gorge, comparable with the width [o]f the Laggan gorge. One was 230 feet high and 900 feet long on the [t]op, and was an arch dam. The design had been tested by means of [a]n indiarubber model before the works had been built, and the [in]teresting point about it was that it had been necessary to build the [d]am in order to test the model! That dam was of particular interest. [T]he catchment area was 1,800 square miles, but provision had been [m]ade for a flood of 200,000 cusecs. There was no fixed spillway [a]nd no siphons; almost the whole length of the crest of the dam [w]as occupied with large roller gates of the Stoney type. Those [g]ates were 20 feet deep. There was another dam on the same system, [c]ontrolling the flow from a catchment area of 175 square miles, [w]hich was approximately that of the Laggan dam. There also there [w]as no fixed spillway; the whole of the flood discharge was passed [b]y Taintor gates, and the flood spillway on that dam was designed [fo]r 40,000 cusecs, as compared with 14,000 cusecs on the Laggan dam.

Mr. B. D. Richards said that the siphons worked on a head Mr. Richards. [co]nsiderably greater than the atmospheric head, and that the siphonic [ac]tion was maintained by tapering the down-take pipe and by [th]rottling the outlet. All the siphons were of the same design [ex]cept that two of them had their outlet level 10 feet higher than the [ot]hers; those two showed a discharge of some 8 per cent. less than [th]e remainder. Would it not be possible to increase the discharge [of] the two upper siphons to equal that of the lower ones by reducing [th]e throttling? The siphons were designed for an assumed vacuum [h]ead of 24 feet; it seemed possible that they might work efficiently [o]n a larger vacuum head than that, although it might be unwise to [as]sume that they would do so unless that vacuum head could be [co]ntrolled in some way. Could that not be done by replacing the [fi]xed throttling of the outlet pipe by a variable throttle by means of [a]n adjustable baffle in the outlet? With the siphon in operation, [th]e baffle would be gradually opened until a point was reached when [th]e water column commenced to break. The baffle would then be [tu]rned back slightly and after re-testing would be permanently [fi]xed. In that way it should be possible to get the maximum dis-[ch]arge without incurring any risk of the siphon failing to function. [It] seemed quite possible that Mr. Morgan's ingenious air-valve [w]ould show to even greater advantage in a siphon working under a [so]mewhat larger vacuum head.

On pp. 40 and 41 the Author cited various methods by which

Mr. Richards. shrinkage-cracks in dams could be obviated, but it would appear tha only two of those methods entirely eliminated the problem. Those were, firstly, the construction of the dam in pre-cast concrete blocks and, secondly, the introduction of vertical sections at much close intervals than usual, say every 15 feet. The first method wa rather attractive, but might be somewhat costly. The second metho would also be costly if it involved constructing the dam in shor vertical sections of only 15 feet, but he thought a modification o that was possible which would be equally effective and very muc cheaper; that was to say, at 15-foot intervals vertical partition could be erected radially across the dam at the same time a the shuttering, the concrete being deposited simultaneously o each side of those partitions. The partitions, which would consis of thin stiff sheets, would be carried up continuously to the top o the dam and left permanently in position. The joints would b staunched by copper strips. Contraction joints built in that wa would interfere very little with the continuous placing of the concret in the dam and should therefore be very cheap.

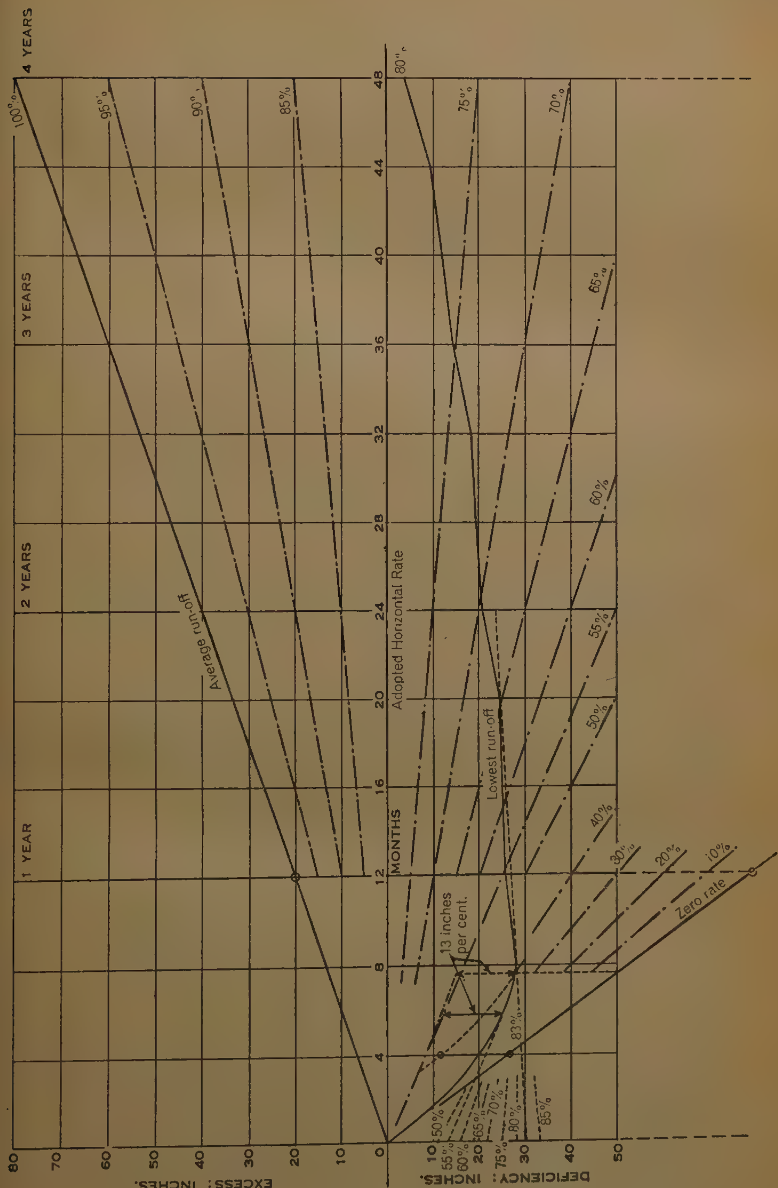
Mr. McClean. Mr. W. N. McClean said that the Author had pointed out th interesting difference between the storage required for a unifor water-supply and the lesser storage required for water-power, whe the power could be reduced during specially dry periods and b raised when the reservoirs were high in the wet season.

Fig. 23 was a diagram which was built up from the records o lowest run-off on the river Garry, which was in the same neighbou hood as the Lochaber scheme; the longer periods were derive from rainfall records reduced by the estimated loss. It was draw on a percentage basis (the annual run-off being taken as 100 inche so as to be applicable to similar areas of high rainfall such as that o Lochaber.

If the power-water averaged 80 per cent., and in the dangerou dry periods was reduced to 55 per cent. of the annual run-off, th saving in storage at the critical period of 8 months after depletio commenced would be 14 inches per cent. of the average run-off— a very big reduction—if the reduced power were applied in goo time. To test the economic value of different storages, it was onl necessary to take points on the deficiency ordinate and to dra tangents to the curve of lowest run-off. In the present case (*Fig. 2* the balanced run-off should be 83 per cent. of the average, showi that 30 inches per cent. would be the storage required in the case water-supply. With a temporary reduction to 55 per cent. the would be a saving of storage at 8 months of two-thirds multipli by 28 inches per cent., or about 18 inches per cent.

On the Lochaber scheme, if the average run-off were assumed

Mr. McClean.


4 YEARS
3 YEARS
2 YEARS
1 YEAR
100%
95%
90%
85%
80%
75%
70%
65%
60%
55%
50%
40%
30%
20%
10%
Zero rate
Average run-off
Adopted Horizontal Rate
Lowest run-off
MONTHS
13 inches per cent.
83%
EXCESS : INCHES.
DEFICIENCY : INCHES.
80
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
36
40
44
48

Mr. McClean. be 70 inches per annum, and if the most economical storage for 83 per cent. (namely, 58 inches) of power-water were required, the full water-supply storage would be 30 per cent. of 70 inches, namely 21 inches. The storage of lochs Laggan and Treig was 13 inches for 300 square miles, thus showing a difference of 21–13 inches, or 8 inches of saved storage. That would mean that the power-water had to be reduced to 13 ÷ 21 (namely, 67 per cent.) of 70 inches, or 47 inches per annum.

It was stated in the Paper that records of the flow of the river Spean and of the overflow of loch Treig had been kept since the commencement of the first-stage works. Presumably the power-water passing through the tunnel was also recorded, but it seemed unlikely that the inflow from the many intakes into the tunnel was gauged. Without such gauging it was, of course, impossible to make a balance-sheet of water for the loch Treig area, since those intakes swelled the storage of loch Treig. He believed that it was true that in Switzerland, Sweden, and other countries all the flows contributing to water-power installations were gauged, but that in Great Britain there was no obligation on the user to record the consumption. The British Aluminium Company had done much to assist the records of run-off on the Foyers and the Spey, and he hoped that as the scheme progressed it might form an example of complete water records of the large area of 300 square miles. The records would not be quite complete without some attention to rainfall, although no absolute daily value of rainfall could be obtained, but there was much to be said for good meteorological stations on loch Laggan and at Fort William, together with one or two high gauges for snow measurement; it was certainly not complete without actual gauging of the siphon-flows in a channel below the dam.

The Author had referred to some physical features in the area which were of interest from the hydrological aspect. He had come across recently a note on the parallel roads of Lochaber, which were conspicuous in the Glen Roy valley; they were glacier terraces left as the glacier receded when the climate became less severe. He travelled past loch Treig during its first lowering below its natural level. Around the loch was a level shelf of sand just below wave level effect; it was, in fact, a parallel road, and the only breaks were where the streams had already broken through the shelf. That shelf might have remained as a parallel road had the water-level continued to subside, but the subsequent rise of the loch swept the whole terrace away. The lowering of lochs created an engineering problem which had been very evident in loch Laggan, where the principal streams had extensive deltas very little above the level of the loch. All those streams were naturally finding a lower bed

[H]e found the stream which came in at the head of loch Laggan of [p]articular interest. For the first time he had witnessed what [a]ppeared to have been the effect of a catastrophic flood on a stream [o]f small slope in alluvial ground. For perhaps 2 miles the banks [h]ad been strewn with the debris of banks and bushes. It had [o]nly been quite a small flood, but the loch had been lowered to the [e]xtreme limit and the velocities had been consequently greatly [i]ncreased; the regime-flow had been upset by the change in control [a]t the lower end. It would appear that the artificial channel [s]everal miles long between the old loch Laggan outlet and the tunnel [t]o loch Treig had to be proof against every sort of variation in flow, [a]nd that it should have either artificial control-points or a bed of [b]oulders somewhat similar to that which occurred in nature when the [b]ed-slope was steep, as otherwise the flood occurring at low levels [w]as bound to produce catastrophic velocities of flow. Mr. McClean.

Mr. J. K. HUNTER referred to the question of the sealing of dam [fo]undations, and described the method adopted during the con[st]ruction of the Tongland dam of the Galloway water-power scheme. [T]hat consisted of drilling and grouting alternate holes to a depth of [.]0 feet, and then drilling and grouting the shallow intermediate [ho]les. The grout used during the initial stages of injection had [co]nsisted of 8 per cent. of cement by weight, and the consistency [w]as gradually thickened to a 60-per-cent. mixture for finishing off [th]e work. Mr. Hunter.

Dr. W. L. LOWE-BROWN remarked that the information given [a]bout the rise of temperature of the Laggan dam concrete in setting [w]as very instructive, and it interested him particularly because [hi]s first job had been connected with that question. In 1895 when [S]ir Benjamin Baker had been designing the Aswân dam he had [w]anted to know the modulus of elasticity of cement mortar to enable [h]im to calculate the stresses due to temperature-differences. Dr. [L]owe-Brown had therefore spent 18 months making observations [o]n the bending of beams made of cement mortar. Dr. Lowe-Brown.

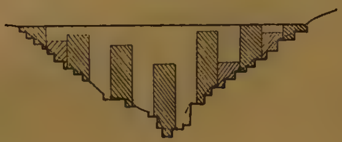
The correlation between the observed temperature at the Laggan [d]am and the laboratory tests had already been fully dealt with by [D]r. Norman Davey, Assoc. M. Inst. C.E., who had given some simple [ru]les for calculating the rise of temperature which might be expected.[1] [H]e thought, however, that Dr. Davey would agree that his rules [w]ere only a first approximation, and that when more information [w]as available they might be modified appreciably. The outstanding [le]ssons to be learnt were, firstly, that whatever kind of cement was

[1] Building Research Technical Paper No. 18. Published by H.M. Stationery [O]ffice.

Dr. Lowe-Brown.

used hydration took place very quickly. On the Laggan dam, whe the lifts were from 3 feet to 3 feet 6 inches, the first temperatur maximum was reached in from 1 to 3 days, and after that th temperature fell gradually until the next lift was placed. Th laboratory tests showed that under adiabatic conditions the rapi hardening cement gave off its heat much quicker in the first 24 hou than the coarse ordinary Portland cement, but after 3 days the

Fig. 24.



temperatures were equal, and at the end of a week the ordina cement had reached a higher temperature. The second point w that so long as the upper surface of a lift was left exposed the dis pation of heat was very rapid. Until those facts had been reveal by the careful records at the Laggan dam and elsewhere it had be assumed that the heat of hydration took a long time to develo and that dams should be built in series of isolated vertical sectio with gaps between, as shown in *Fig. 24*, which represented the co

Fig. 25.

struction of an imaginary dam. That practice, which might called the perpendicular or " skyscraper " method of constructi had been introduced about 30 years ago. It had been followed many parts of the world until quite recently, but in view of results given in the Paper it appeared to have been wrong and have tended to increase temperature-stresses instead of reduc them. In fact, it would seem that the old English method building dams in horizontal layers without any gaps (*Fig. 25*) wo be preferable, so long as expansion-joints were provided at sh

tervals—the shorter the better. A modified form of that method as used in the more recent dams built in America. Dr. Davey ad calculated that, using lifts of 3 feet 6 inches the rise of temperature ith two lifts per week would be 45° F., whereas with one lift per eek it would only be 25° F. It had been argued that one lift er week was too slow. That was true so long as the "skyscraper" ethod was followed, but if a reversion were made to the old English ethod of building in horizontal layers without any gaps the progress ould be just as fast, and he believed that a contractor could ganize his work to build the dam just as cheaply. By the use of eel shuttering and low-heat cement the rise of temperature might limited still further. From the results given in Building Research aper No. 18 it might be concluded that rapid-hardening cement as better than normal Portland cement for dam-construction, but e question was far too complicated to be resolved so easily. The bject was extremely difficult, but no doubt the strong Joint ommittee which was studying it, would be able to find a much tter cement than those at present in use. Research had already own that there was one constituent which generated a large nount of heat and added very little to the final strength; the anufacturers would, therefore, doubtless reduce it to a minimum. ost of the other constituents were useful, although they generated eat, some quickly and some slowly. Their proper proportioning as, he understood, still a matter of opinion, and probably depended the way in which the cement was to be used. He wondered hether it would be possible to reduce the rise of temperature by me more intensive method of pre-hydration than had yet been ied. Mr. R. V. Allin, M. Inst. C.E., in the discussion on a recent per,[1] had shown that concrete made of normal cement could, th certain precautions, be mixed and remixed for 5 hours before acing without losing strength. It would not be possible to mix ncrete for 5 hours on the work, but if it could be shown in the boratory that the rise of temperature could be very greatly reduced prolonged mixing, it might be possible to find a way of pre-drating the cement immediately before delivering it to the mixer. e above remarks applied only to a dam built of concrete cast place. The alternative, referred to by the Author, of using e-cast blocks was the most certain way of reducing temperature-esses.

Dr. Lowe-Brown.

Many engineers thought that siphon spillways were new, but had recently found an interesting early reference to them in

D. Kennedy and H. E. Aldington, "Royal Docks Approaches Improvement, ndon." Journal Inst. C.E., vol. 2 (1935–6), p. 39 (February, 1936).

Dr. Lowe-Brown.

an article [1] dated 1843 in which the theory and advantages of sipho spillways were given clearly and in some detail. The Author that article, Mr. Robert Mallett, was as enthusiastic on the subje as the Author of the present Paper, although the siphons he describ were smaller than those used by the Author.

Dr. Davey.

Dr. NORMAN DAVEY observed that the Author had dealt ve fully with the valuable series of observations of thermal and volumetr changes occurring in Laggan dam. The records of those observatio had been placed at the disposal of the Building Research Station Mr. Halcrow, but he felt that there was little that he could useful add to the conclusions that the Author had already drawn from t investigation.

The deleterious effects of high internal temperatures and of ste temperature-gradients in large masses of concrete could not be ove estimated, and the Author had shown from his analysis of t problem that the cracking which had occurred in the dam mig be accounted for by the temperature-difference between the insi of the mass of concrete and its surface. It was therefore clear desirable that, in structures of the type dealt with, more serio attempts should be made to minimize the risk of steep temperatu gradients, either by reducing the speed of placing or by using cement which generated less heat. The tensile stress at the face the dam, due to temperature-conditions alone, had been estimat at over 300 lbs. per square inch; it would be safer to restrict th value to 150 lbs. per square inch. To achieve that, however, t difference between the inside of the mass and its surface should n be allowed to exceed 25° F., and that in turn would mean in the ca of the Laggan dam a reduction in the equivalent rate of placing the concrete from approximately 0·9 foot per day to about 0·6 fo per day. If a cement could have been used which evolved say, on from 50 to 55 calories per gram during the first 7 days, inste of the one actually used (which evolved 65 calories per gram the desired condition could have been obtained by adopting a ra of placing of 0·75 foot per day.

It would seem, therefore, that the use of low-heat cements of t type developed in America and elsewhere would assist very co siderably in overcoming the risks involved by too-rapid placin Manufacturers in Great Britain had been exploring the possibil of producing that type of cement, and the Joint Sub-Committ on Special Cements of The Institution had been engaged in drawi up suitable tests for the selection of such products.

Table A showed the wide variation in the heat-characterist

[1] Weale's "Engineering Papers of 1849." vol. vi, p. 51.

various types of cement tested at the Building Research Station. Dr. Davey.
ne first two types, classified by the makers as normal Portland ment and rapid-hardening portland cement, all conformed to the itish Standard Specification for Portland cement, yet it was en that one cement might generate twice the amount of heat in a ven time than another. A point which was even more striking s that some consignments of rapid-hardening Portland cement nerated less heat than some consignments of normal Portland ment. The strength developed in a mass of concrete was closely ated to the heat generated; thus, where a temperature-gradient isted, a strength-gradient would also occur. Since both shrinkage d creep varied with strength, it was reasonable to suppose that ey also would vary throughout the mass of concrete.

TABLE A.—HEAT EVOLVED BY DIFFERENT TYPES OF CEMENT.

Type of cement.	Number of consignments tested.	Heat evolved per gram of cement: calories.		
		At end of 1 day.	At end of 2 days.	At end of 3 days.
rmal Portland cement .	13	23–46	42–65	47–75
pid-hardening Portland cement . . .	13	35–71	45–89	51–94
tland blast-furnace cement . . .	6	18–28	30–51	33–67
h-alumina cement .	3	77–93	78–94	78–95

A typical series of records from Laggan dam had been compared h the temperature-rise obtained in a sample of the same concrete [1] red under adiabatic conditions (that was, under conditions of nplete insulation). In the early stages the temperature-rise urring in the structure exceeded that obtained in the insulated nple, the reason being that in practice some heat was received m the concrete already placed. Two maxima had been observed the temperature-time curve, and it had been shown from the ervations made on the Laggan dam, and from observations carried by Sir Alexander Gibb on the Tongland and Clatteringshaws ns in Galloway, that in placing concrete in lifts of equal thickness l of the same proportions of mix, the first peak of temperature in particular lift was dependent upon the age of the preceding lift l the second peak of temperature was dependent upon the time rval before the succeeding lift was placed.

he observations on the Laggan dam and on similar structures had

Building Research Technical Paper No. 18, p. 16. Published by H.M. ionery Office.

Dr. Davey. been of very great assistance in the study of the thermal properties concrete, and the maximum temperature-rise likely to occur in large mass of concrete could now be forecasted with a fair degree accuracy. To do that it was necessary to know the cement-conte of the mix, the heat generated by the cement up to, say, 7 da (determined in the laboratory), and the speed of placing to adopted on the work.

Mr. Gibb. Mr. H. M. Gibb considered that the Author's observations on t temperature-rise in the concrete of the Laggan dam formed a valual contribution on the subject. A point worthy of notice was t fine appearance of Laggan dam, which was obtained by an app priate treatment of the work above the spillway at what must ha been a trifling increase in cost. It would have been better if t disperser outlets to the siphons had been housed inside the da but he understood that those had been added as an afterthought.

It would have been of interest to have had a cross-section showi the distribution of pressure over the base and the hydrostatic upl assumed, including the weight per cubic foot used for the concrete.

He had been associated with some of the constructional work, a had a few points to raise on the suitability of the design from t point of view of construction. As the design and construction w two steps in the same process they should be correlated so that far as possible nothing in the design was directly antagonistic economical construction.

In a dam of that height and across such a narrow valley it wou have been better if the 6-foot diameter drainage-culvert had be placed well below the level of the temporary river-diversion openi instead of 18 feet above them, and that culvert would have been bet if it had been made big enough to take a reasonable flow in the ri with the water-level still below the inverts of the temporary divers openings. In that way the closing of the diversion openings wo have been a comparatively simple operation, as the river would diverted through the drainage-culvert. Due, however, to it be too high and of too small capacity, the water-level would rise v quickly when the diversion openings were closed; and if those w not closed successfully the first time, considerable expense would involved in rectifying any unsuccessful attempt. The tempor damming of the river upstream was an expedient used to compens for the inadequate capacity of the culvert. He did not like a nee valve on the end of the drainage-culvert. That type of valve small water-passages when compared with a sluice-gate. Th passages were curved and were liable to catch and to hold sm pieces of timber and other material, so that it was essential befo valve of that nature was closed that the roller gate on the upstre

d of the drainage culvert should be closed and the needle-valve Mr. Gibb.
pected for debris. Otherwise, there was a risk, if something lodged
it, of failing to close the needle-valve and cracking the casing.
knew of a case abroad where a needle-valve of about the size
ed at Laggan dam had failed to close and had cracked the casing
e to the end of a concrete-hoe becoming lodged in the valve,
hough the tunnel had previously been thoroughly inspected.
her debris, such as submerged pieces of timber, would give similar
uble in a valve of that nature.

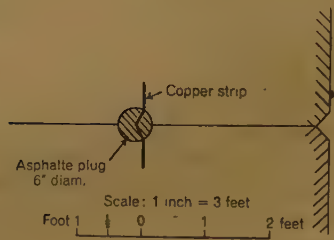
It would be seen (Figs. 3, Plate 1) that the inspection gallery ran
aight through the middle of the diversion openings. There did
t appear to be any reason against running that gallery above or
ow those openings, and if it were considered that the gallery
uld be useful to inspect the diversion openings either during
icreting or after, that could quite as well have been done by
erting the gallery over the openings. With the gallery running
ough the openings the concrete filling was complicated by the
ed for shuttering them, with no corresponding advantage.

On p. 10 the Author had stated that Laggan dam represented the
t instance in Great Britain of the embodiment of a large siphon
llway in the design. On the Tummel Development of the Gram-
n Scheme four siphons had been placed in the Dunalastair dam
pass floods, the total capacity having been 2,500 cusecs. Two of
se siphons were bigger than those in the Laggan dam, their throat
iensions being 8 feet wide by 4 feet high, the other two being
eet wide by 3 feet high. They were in operation in November,
3, whereas the Laggan dam had not been completed until July,
4. The siphons in the Dunalastair dam had about the same
ative capacity as those in the Laggan dam, namely, about one-
rter of the maximum flood-flow.

A brief description was given on p. 9 of a construction joint used
ween the 45-foot blocks of which the dam had been constructed.
joint consisted of a 6-inch diameter cylindrical space across which
ggled copper strip passed ; the copper strip was embodied in the
acent blocks, and the cylindrical space was filled with a bituminous
ture (*Fig. 26*, p. 78). The position of the copper strip appeared
iim to be bad for its own security against water-pressure, and it
de construction difficult. Pieces of timber had to be introduced
each side of the strip, and as they had to contain some easing
ice to allow withdrawal after the concrete was set, the construction
he joint was complicated and could be considerably simplified.
vas also an unsound design for the hydraulic conditions, as the
er-pressure near the bottom of the dam was bound to be in the
hbourhood of 3 tons per square foot ; that pressure acted

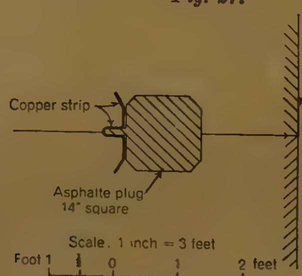
Mr. Gibb. directly on the bitumen in front of the copper strip, which consequently subjected to the full pressure, and there was a dan of the strip rupturing. An alternative design (*Fig. 27*) which had used on the Ericht dam, and which was free from those we nesses, was to place the strip across the downstream side of

Fig. 26.



opening where the water-pressure against it was directly convey into the concrete backing, and the joggle had only to carry it acr an opening of a fraction of an inch instead of one of 6 inches. Th joint was also comparatively easy to construct. Further, a 6-in diameter cylindrical hole was too small for a joint over 100 f high for simple construction and also for cleaning out; it was a too small if, due to any cause, the bitumen tended to flow out of t

Fig. 27.



joint, as there was not a sufficient quantity of bitumen per foot compensate for such leakage.

The Author stated on p. 8 that the side slopes of the dredg channel were formed at $1\frac{1}{2}$ to 1. For the material at the site, whi was subject to water and to wave-action, a much flatter slope w necessary if it were going to be permanent. The design of the tunn outlet made no provision for scour when the water-level of Lo Treig was below the invert of the tunnel. With the tunnel runni

l there would be a jet of water 14 feet in diameter, the surface Mr. Gibb. which would fall approximately 5 feet at the exit, giving a velocity probably from 12 to 15 feet per second. That would not be pable of spreading the full width between the diverging wing-alls, and the latter could have been placed closer together. To nimize scour below the structure the invert should have been pressed or baffles should have been provided to dissipate the jet-ect of the flow.

The design of the cross-section of Treig dam (Fig. 9, Plate 2) peared to follow too closely the details of an earth dam with a ay-puddle core-wall. The core-wall in the present case was 10 feet ick at a depth of 46 feet below the spillway-level and was heavily nforced, and it hardly appeared necessary to introduce on the stream face selected clayey fill unless there was a doubt about the ter-tightness of the core-wall; the face of the core-wall below at level was not, however, similarly protected in gravelly material ich freely conveyed water, nor was the core-wall reinforced below at level. It would appear, therefore, that the factor of safety of ter-tightness had been very considerably increased above ground-el to that which existed below ground-level.

Mr. H. G. Lloyd remarked that on p. 29 the water/cement ratio for Mr. Lloyd. 7-to-1 concrete was given as 0·55, and the average crushing ength as 174 tons per square foot, equivalent to 2,706 lb. per are inch at 28 days. Instead of those figures Dr. Glanville uld have expected a water/cement ratio of 0·75, or alternatively trength of 300 tons per square foot. In the 4-to-1 concrete the ter/cement ratio was given as 0·40 and the crushing strength as . tons per square foot, equivalent to 4,681 lb. per square inch, as npared with Dr. Glanville's figures of 0·55, or alternatively 425 s per square foot, equivalent to 6,609 lb. per square inch.

To enable him to determine if the water/cement ratios of 0·55 l 0·40 were reasonable quantities for the aggregates used, typical dings had been supplied to him by the Author, from which he l calculated the quantity of water required to damp them. The st damping value (L.D.V.) was obtained by wetting the aggregate l then moving it to and fro on a large plate-glass or metal plate impervious container, until the aggregate just failed to wet the face and only left small streaks of moisture upon it when so ved. The percentage of water by weight was then termed the .V. That method of estimating the quantity of water for crete had been fully described by him in 1920, and had the cordial roval of the late Dr. W. C. Unwin, Past-President Inst. C.E. had gradually developed the method and checked it in practice, was now able to give the percentage of water required to damp

Mr. Lloyd. aggregates in that manner. The L.D.V. could be found for an grading of cuboid or spheroid aggregates from the followi formulas :—

For sizes from 6-inch to $\frac{3}{16}$-inch square mesh : Thames ballas $0{\cdot}22 + 0{\cdot}67\ 1/d$; granite, $0{\cdot}05 + 0{\cdot}29\ 1/d$.

For fine aggregates below $\frac{3}{16}$-inch square mesh : Tham sand, $3{\cdot}65 + 0{\cdot}0354\ 1/d$; granite, $1{\cdot}6 + 0{\cdot}0354\ 1/d$.

The water required to flux the Portland cement had been tak as 22 per cent. by weight.

These damping-values had been used to obtain the water/ceme ratio, which for the 7-to-1 concrete was 0·40 compared with 0· given by the Author, showing an additional 37½ per cent. of wat above that required to obtain the greatest strength. The consisten would be fairly suitable for the class of work, where displace were used. The calculated figure for 4-to-1 concrete was 0·3 compared with 0·40 given by the Author. That was equivalent 25 per cent. excess of water by weight, and should have proved suitable consistency for the facing work. It appeared from t gradings that in all cases some of the coarse aggregates contain fine aggregate, and the fine aggregates also contained coarse aggrega Making allowance for that, and of necessity reducing the proportio to weights, the 7-to-1 by volume mix became 5·8 : 3·4 : 1, or 9·2 to by weight. The 4-to-1 by volume mix, on the same basis, beca 3·07 : 1·64 : 1, or 4·71 : 1, by weight.

Those estimations seemed to support the water/cement rat given by the Author. The proportions when stated by weight, a making allowance for the difficulty of moulding concrete containi large aggregate in 6-inch moulds, accounted to some extent for t strengths not being as high as Dr. Glanville would expect from laboratory tests.

The discussion had drawn attention to the unsuitability of t water/cement ratio as a criterion of the consistency of concrete different materials, different gradings or different proportions aggregates. Moreover, the water/cement ratio made no allowa for the varying quantity of water required to flux differe cements. It therefore did not appear to be a sound standard general use.

The Author. The AUTHOR, before replying to the discussion, wished to tha Mr. Halcrow for the loan of lantern-slides, and Mr. J. A. Johnst his chief of staff, for his helpfulness on all occasions.

On behalf of the Engineers he wished to thank Lieut.-Gene Sir Ronald Charles for his appreciation of the design and execut of the works, and, at the same time, to pay a tribute to the cour

d foresight of the British Aluminium Company in carrying forward The Author.
e second-stage development throughout the period of economic
pression.

There had been and was much criticism of works such as those of
e Lochaber scheme from the æsthetic standpoint, and the remarks
the President were welcome. Mr. H. M. Gibb had commented
the fine appearance of Laggan dam, and the Author felt that in
njunction with the winding reservoir above it had not only con-
buted to the interest but had enhanced the beauty of the
intryside.

The figures derived by Mr. McClean from *Fig. 23* emphasized
smaller storage required with suitable regulation, but they
atly underestimated the economic output. As in the diagram
dry periods were listed in order of severity, without regard to
ir natural sequence, it was probable that they represented severer
ditions than would ever occur, and that they would even be an
luly severe criterion for water-supply, where a minimum output
l to be guaranteed.

Mr. Gourley had shown that the Treig dam spillway was adequate
deal with "catastrophic" floods, but had expressed a doubt
to the adequacy of the provision therefor at Laggan dam. In
Interim Report of the Floods Committee [1] a "catastrophic"
od was imperfectly defined, its peak intensity being given but not
duration and variation with time. Moreover, information on
chment-areas as large as that of Laggan was very meagre. Whilst,
suggested by Mr. Gourley, the water-level at peak discharge
ild be above +822 O.D., rough calculations based on an assumed
ve of peak run-off indicated that it would not exceed that level
more than the 24 hours permitted in the Act. The dam had been
igned before the appearance of the Interim Report, and it was
eresting to find that the assumptions made conformed so closely
h the "catastrophic" flood considered in it.

Ir. McClean had suggested that the dredged channel of the river
an should be lined with boulders to resist erosion, or should be
vided with artificial control-points. Such control was in fact
rded at low water-levels by the gradient of the channel-bed,
igher levels by the power-consumption, and during floods by the
lway-level at Laggan dam, the result being that at no stage could
excessive velocity be set up in the channel. It would not matter
gradual general erosion were to occur, as the present minimum
l of loch Laggan would still be maintained by a rock bar.
H. M. Gibb had criticized the side-slopes of 1½ to 1 as being

[1] *Loc. cit.*

The Author. unstable if the material were subject to wind and wave action. the Author's opinion no practicable side-slopes would be flat enou to be immune from erosion under such action. Fortunately the ste side-slopes afforded protection against all but longitudinal win and it was very doubtful whether flatter side-slopes would offer a advantage in that respect.

Whilst the stress-diagrams for Laggan dam had not been included the Paper, the essential results had been given. The remarks of J. S. Wilson suggested that a simple triangular section with a 70-p cent. batter, to conform with the usual practice, had been decid upon without attention being given to refinements of curves. Actual however, both downstream and upstream faces were of varyi batter, the section being designed to keep the resultant force at levels at about two-thirds the width from the upstream face. W concrete weighing 150 lbs. per cubic foot, and assuming a triangu distribution of uplift-pressure on the base, increasing unifor from zero uplift at the toe to an uplift equal to 50 per cent. of hydrostatic pressure at the heel of the dam, the resultant thrust w the reservoir full passed approximately through the second third-po of the base. Mr. Wilson had stated that a parabolic distribut of shear-stress had been assumed. That, of course, was not case, and did not follow from the assumption of an approxim value of 1·5 for the coefficient m in the calculation for deflex (Appendix II). Mr. Wilson had shown diagrams illustrat the distribution of stress in a perfectly elastic dam monolit and continuous with foundations of the same material. S conditions never existed in practice, and in the Author's opin reliance on such a theoretical distribution of stress could be danger At Laggan dam the surface rock, though sound, was intersected vertical fissures. With the heel carried down to close-jointed r and the toe near the rock-surface, tension would be set up in base of the dam owing to the rock at the toe being less rigid i horizontal direction. A similar effect might be produced wit slender curved toe inadequately anchored at the end. Th considerations would explain why it was stated in the Paper t as the curved downstream toe was carried well down into so rock, and was of considerable thickness, there was no likelihoo tensile stress being set up in the concrete of that toe.

Mr. H. M. Gibb's criticisms were valuable as representing Contractors' point of view. He had suggested that the drain culvert might have been situated below the diversion-openings made sufficiently large to take the flow of the river without water-level rising to the diversion-openings. As, however, inverts of the latter were at about river-bed level, that would

ve been possible without a considerable raising of the openings, The Author.
ich would have added greatly to the expense and difficulty of
nstructing the cofferdams. The temporary damming of the
er upstream during closure of the openings had been a compara-
ely inexpensive matter. His criticism of a needle valve with small
ssages situated below a larger sluice gate was in general sound,
t the danger he envisaged was absent at Laggan dam since the
vert was situated well above the bed of the river and well below
e lowest possible water-level. It was therefore immune from
th floating and waterlogged debris. Any constructional difficulty
e to the inspection-gallery passing through the diversion-openings
d been very small. Whilst conceding the constructional advantage
a larger cross-section for the bituminous plug around the joggled
pper sealing strip at contraction-joints, the Author did not agree
t there was an unbalanced pressure on the copper strip. Bitumen
ng heavier than water, the pressure on each side of it was merely
t due to the head of bitumen. It was quite possible that the
ernative design of *Fig. 27* (p. 78) would be durable, but he pre-
red a design where the copper was completely free from unilateral
ssure and was protected on both sides.

Dr. Glanville had suggested that the water/cement ratios given
the Paper were much lower than the true values. The figures
en by Mr. Lloyd, on the other hand, would indicate an error on
high side. The Author could not claim close accuracy for the
res given, as the arrangements for gauging the water were not
al and the amount of water contained in the coarse aggregate
s not accurately known; nevertheless, the maximum error could
be more than about 0·05. Dr. Glanville based his belief on the
ure of the test cubes to attain the stresses proper to the
ter/cement ratios used. That was readily explained. The making
6-inch test cubes with 2-inch or 1-inch aggregate and such a low
er/cement ratio was a matter of extreme difficulty. In the
oratory adequate means were taken to ensure thorough consolida-
n, but on the works it was an exhausting and prolonged manual
rt to attain reasonable consolidation. The different quality of
cubes was also evidenced by the relatively wide variation in the
ngths obtained, in comparison with the consistent results of the
oratory. The Contractors would support the Author when he
ed that the amount of water used was the minimum consistent
h adequate consolidation. For that reason he felt that the
er/cement ratios for greatest strength worked out by Mr. Lloyd
the Laggan aggregates, namely 0·40 and 0·32 for the 7 : 1 and 4 : 1
cretes respectively, would entail the use of vibrators for the
per consolidation of the concrete. He would like to thank

The Author. Mr. Stanger for his figures showing that the use of coarse-grou cement in the central block of the dam did not appreciably aff the strength of the concrete.

By destroying the adhesion of the gunite for a total width of 2 f at cracks on the upstream face of the Laggan dam it had been hop to prevent the formation of visible cracks. Dr. Glanville's rema suggested that the formation of one or more definite cracks w probable. In any case the efficacy of the seal was not impair as it was unlikely that cracks in the gunite and the concrete wo coincide. No inspection had been possible.

It would appear from Professor Lea's remarks that he h erroneously assumed that the mean velocity at the throat of t siphons was about 40 feet per second; that figure, however, was t maximum velocity, which would be at the crest surface on t assumption of a free-vortex distribution of velocity across the thro It was the basis of the design that a maximum vacuum of 24 f head of water or 10 feet absolute pressure should nowhere be e ceeded. The pressure at +804 O.D. would be about 19 feet absol pressure, instead of less than 8 feet as suggested.[1] The operati of the siphons was perfectly steady, and there was nothing to sugg that they are not running full. He agreed that the effect of viscos on the discharge of the model would be small. As, however, t discharge of 640 cusecs was only obtained by calculation, a clo agreement with the model experiment could hardly be expected.

Professor Lea had questioned the advisability of expressing primi depth as a fraction of the depth at the throat, for example $\frac{d}{3}$, and h asked whether it should not be expressed simply in feet. If t depth at the throat were increased to 4 or 5 feet without decreasi the width, it was possible that priming would take place at t same height above the crest, but it would be a bad and uneconomi design. If, however, the width were reduced in about the sa proportion, the capacity of the siphon remaining unchanged, greater priming-depth would be required. The requisite depth priming was not a constant, but in general increased with the s of the siphon-passages. It did not increase in direct proport to the scale, a larger siphon being able to prime with a smaller p portionate depth of overflow, but it was more nearly proportio to depth than constant. Thus, whilst the model siphon had indica $\frac{d}{3}$, the Laggan siphons primed naturally at about $\frac{d}{7}$. The quest

[1] The full calculations are given in Author's book, "Siphon Spillwa London, 1935.

priming was so involved that a chapter had been devoted to the ɔject in the Author's book.[1] The Author.

Although the simple air-inlet lips as designed did not allow siphonic ion to cease as early as had been hoped, it did not follow that se regulation could not be obtained without air-valves. The air-et lips had a wide flat undersurface in order to ensure priming the level of the lips. It was now evident that that same priming-el could have been ensured with a sharp-edged or rounded lip level of which was adjusted by trial. Such modification of the isting air-inlet lips could readily be effected at Laggan dam when need for full utilization of the available storage rendered it sirable. By reducing the thickness and increasing the length the air-inlet lip and the horizontal area enclosed it should be ssible to reduce the range between priming and breaking to as all a value as desired.

No trouble had been experienced in connexion with the operation the air-valves as a result of frost, nor was there any likelihood of ch trouble. In the first place, the air-valve floats and cisterns re totally enclosed. Although surrounded by snow-capped untains, the temperature-conditions at Laggan dam were not ere owing to its low altitude, and a thaw would take effect at that ver level before affecting the catchment generally. Moreover, was very unlikely that the reservoir would be full during frost, d in any case the surface-area of the reservoir would ensure a delay the rise of water-level at the dam. Until the reservoir commenced overflow, the water at the siphon-mouths, which were at a con-erable depth below the spillway-level, was stagnant. Under such nditions frazil ice could not form, as any spicules of ice would ve floated up to the surface before reaching the dam.

Mr. Richards had suggested that the discharge of the two upper hons should have been increased. The resulting increase of charge would have been small and would have entailed special ulds for the outlet castings. It was possible that a vacuum 24 feet of water might be exceeded by a foot or two with impunity, t as the discharge was in proportion to the square root of the head gain would be small and would not justify running the risk of itation.

The coefficient of discharge of the jet-dispersers was much higher n was suggested by the value of 0·6 given by Mr. Gourley for a rt outlet-pipe with disperser. It was perhaps more accurate to er to the loss of head due to the disperser. According to Mr. Bruce ll, of Messrs. Glenfield and Kennedy, Ltd., that loss was about

[1] Footnote 1, p. 84.

The Author. $0{\cdot}06 \frac{v^2}{2g}$ for the siphon-dispersers, whilst the loss at the combin needle-valve and disperser of the scour-culvert was about $0{\cdot}25 \frac{v^2}{2g}$.

Mr. Gibb had pointed out that the Dunalastair siphons were t first large siphons to be brought into operation in Great Britai The Laggan siphons were the first to be designed, but owing to t magnitude of Laggan dam they were completed at a later dat they were the first large siphons to be used in conjunction with large dam in Britain.

It had been stated by Mr. Williamson that a higher coefficie could have been attained for the discharge of Treig dam spillwa The original intention had been to design it to give a coefficie of 3·9, but the decision to provide a control-gate at the Lagga Treig tunnel inlet rendered that unnecessary, and a more robu design with a large-radius crest had therefore been adopted.

Mr. Gibb had criticized the provision of clayey fill on the upstrea side of the core-wall when there was porous ground against its low portion. There was no possibility of drying-out, with conseque shrinkage-cracking, below ground-level, but that was conceivab though rendered unlikely with a clayey fill, in the upper part. T bending-moment on the core-wall diminished below ground, and was not considered necessary to continue the reinforcement to t foundation. The Laggan–Treig tunnel would never be allow to run full when the water-level in loch Treig was low, so that t scouring action feared by Mr. Gibb would not take place. The out wing-walls were splayed out to their present extent before it w known that a concrete-lined channel would be required.

He wished to thank Dr. Davey for the information regarding t tests carried out in connexion with the concrete of Laggan da which supported the calculations given in the Paper. Accordi to Dr. Glanville, the elastic and plastic movement at contractio joints might be about 0·01 inch at a depth of 100 feet. Neverthele the reduction in the annual movement observed in the inspectio gallery from 0·035 inch at the highest level to 0·005 inch at the low could, in the Author's opinion, only be accounted for by conta over some portion of the lower part of the joints, the foundatio being too distant to exercise appreciable restraint. He wou suggest that the excess water imprisoned in the concrete at t heart of a large dam might possibly have the effect of increas the plasticity and Poisson's ratio above the values found by labo tory experiments. A further possibility was that the continua of contact between blocks at the heart of the dam might be the res of initial compression due to the evolution of heat during harden

he concrete. The joints were open at the inspection-gallery in The Author.
gan dam as, according to Mr. Williamson, they were at the
rris dam. Since in both cases the gallery was near the upstream
e it did not follow that there was no contact at the centre of
dam. He could not agree with Mr. Williamson's criticism of
suggestion that contraction-joint keyways should be formed
allel to the downstream instead of the upstream face. If there
e the possibility of considerable tension being set up either normal
the downstream face or horizontally across the dam, whether
elastic deformation or shrinkage-stress or both, it was surely
ter that the direction of consequent cracking should be determined
as to have the minimum weakening effect on the dam.
Dr. Glanville's statement that the tensile strength of concrete
reased to no great extent with an increase of cement-content
eared to condemn the normal practice of using rich concrete facing.
ere would still, however, appear to be some possible advantage
the use of a not-too-rich Portland-cement facing with low-heat-
ent hearting. Mr. Gourley's interesting diaphragm construction
uld, he imagined, on account of expense be limited in its applica-
to dams of the largest size. Mr. Williamson had mentioned the
of 5-foot spaces left between blocks, to be filled in with concrete
later stage when as much time as was possible had been allowed
cooling and shrinkage. That was the obvious method of con-
iction in the case of arch dams, but the Author could not see
great advantage in its use in the case of a gravity dam, and it
certainly more expensive. He agreed with the advantages of
struction in horizontal layers as shown by Dr. Lowe-Brown.
Richards' suggestion for the sub-dividing of concrete lifts into
rt lengths by thin sheeting appeared to merit serious consideration.
he values chosen for the moduli of elasticity in calculating the
exion of the dam conformed with recent information on the
ject.[1] Those values had not, however, been reduced to allow
the effect of creep, as the Author had not realized that, as pointed
by Dr. Glanville, creep was proportional to stress, however low
stress, so that over a long period it could not be ignored. Mr.
gan's emphasis on the fortuitous nature of the results would,
refore, appear to be justified, but his contention that deflexion-
lings could not be taken to an accuracy closer than 0·1 inch was
borne out by the regularity of the observations, which indicated
order of accuracy of about 0·02 inch. Whilst some support might
eceived from the shallower blocks on either side since there could

E. Probst, "Principles of Plain and Reinforced Concrete Construction,"
don, 1936.

The Author. be no horizontal arch action, such restraint could not apprecia affect the relative deflexion between the level of the inspect gallery and the crest. There were, however, many complica factors which could affect the observed deflexion. Perhaps on the most important was the distortion which was certain to h taken place during the cooling of the interior of the dam. The l of agreement between observed and calculated deflexions appea to indicate the need rather than the uselessness of further experim

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ORDINARY MEETING.

22 December, 1936.

Sir ALEXANDER GIBB, G.B.E., C.B., F.R.S., President, in the Chair.

The discussion on the Paper by Mr. Naylor on "The Seco Stage Development of the Lochaber Water-Power Scheme" continued and concluded.

*** The Correspondence on the foregoing Paper will be publis in the Institution Journal for October, 1937.—SEC. INST. C.E.

THE SECOND-STAGE DEVELOPMENT OF THE LOCHABER WATER-POWER SCHEME.

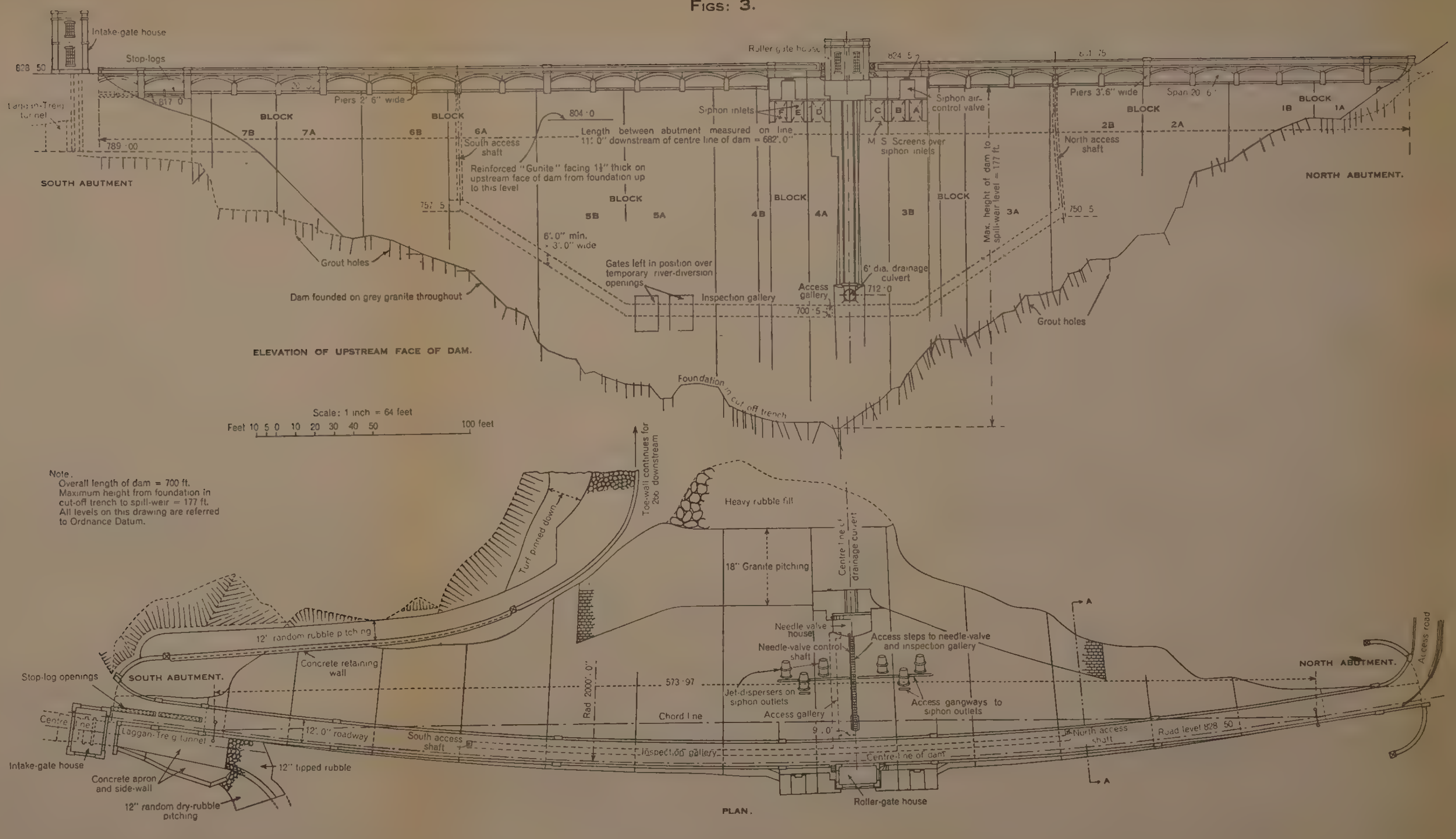
Fig: 1.

SITE MAP.

Fig: 2.

LONGITUDINAL SECTION RIVER SPEY TO RIVER LOCHY.

Figs: 3.



LAGGAN DAM.

Fig: 4.

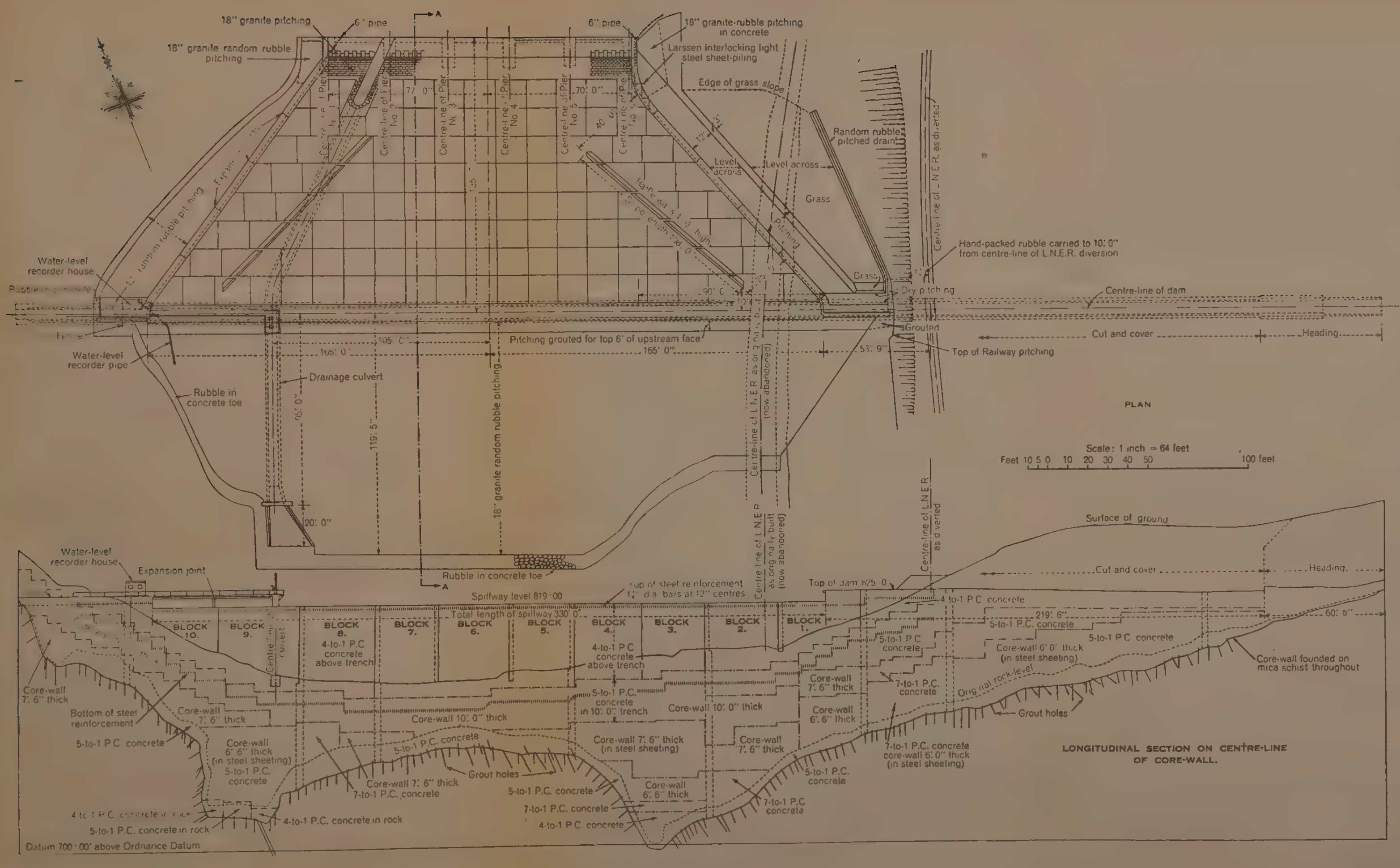
LAGGAN DAM: CROSS-SECTION ON CENTRE-LINE OF DRAINAGE CULVERT.

A. H. NAYLOR

William Clowes & Sons, Limited: London.

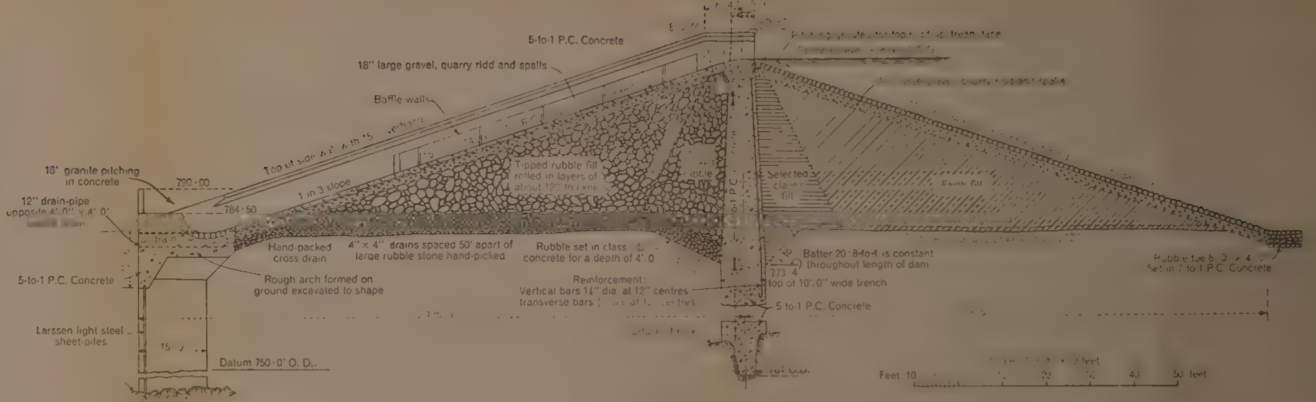
THE SECOND-STAGE DEVELOPMENT OF THE LOCHABER WATER-POWER SCHEME.

FIGS: 8.



TREIG DAM.

FIG: 9.



TREIG DAM: SECTION A A.

FIG: 11.

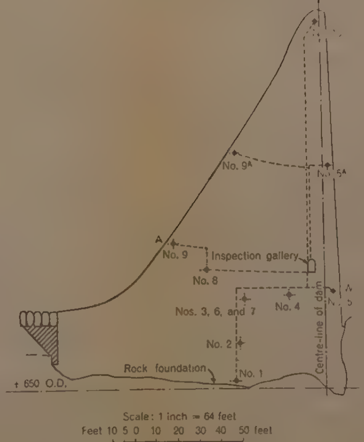
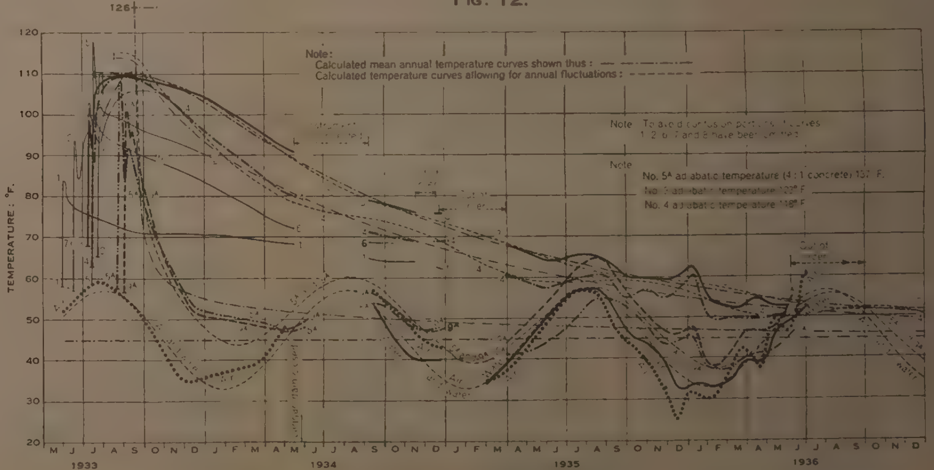


FIG: 12.



LAGGAN DAM TEMPERATURE RECORDS.

A. H. NAYLOR.

WILLIAM CLOWES & SONS, LIMITED: LONDON.

ORDINARY MEETING.

12 January, 1937.

Sir CLEMENT DANIEL MAGGS HINDLEY, K.C.I.E., M.A., Vice-President, in the Chair.

The Scrutineers reported that the following had been duly elected as

Members.

Ernest Kinnear Adamson.
Ralph Strick, B.Sc. (Eng.) (*Lond.*).

Associate Members.

Bertram George Fletcher Adlington, B.Eng. (*Sheffield*).
Alfred Thompson Alkin.
James Kenneth Anderson, M.A. (*Cantab.*), Stud. Inst. C.E.
Harry Armistead, Stud. Inst. C.E.
Mahmoud Hasan Bakir, B.Sc., Ph.D. (*Birmingham*).
Francis Edward West Barnes, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.
George Gordon Bennett, B.C.E. (*Melbourne*).
John Brierley, Stud. Inst. C.E.
William Basil Bryan.
Henry Caird, Jun.
Leslie Charles Hadland Cole, B.Sc. (*Edin.*).
Ronald George Cox, Stud. Inst. C.E.
Roy Cozens.
Leslie John Culligan, M.Eng. (*Liverpool*).
Charles Garton Cumming, Stud. Inst. C.E.
John Justice Cumming, M.Eng. (*Sheffield*).
Alfred Thomas Davies, Stud. Inst. C.E.
Rhydwyn Harding Evans, M.Sc. (*Manchester*), Ph.D. (*Leeds*).
John Macdonald Gordon Forsyth, B.Sc. (Eng.) (*Lond.*).
George Fraser.
Leslie Malcolm Gardiner.
Francis Edward Griggs, M.A. (*Cantab.*), Stud. Inst. C.E.
Guy Montague Wiles Hagger.
Lionel Jack Paret Hay, Stud. Inst. C.E.
William Edwin Hayward, Stud. Inst. C.E.
Guy Phillip Searle Hodge, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.
Cooper Hodgson.
Edward Carlisle Hodgson.
James William Horner, B.Sc. (*Leeds*).
William Leigh Houlbrook, *Flight-Lieut. R.A.F.*
Reginald Ralph Cheshire Johnson, Stud. Inst. C.E.
Clifford Brynmor Jones.
Philip William Kennedy, Stud. Inst. C.E.
John William Lovatt, Stud. Inst. C.E.
John Alexander McGregor, B.Sc. (*Edin.*), Stud. Inst. C.E.
Henry Campbell Macready.
Ralph Chevalier Marc, M.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.
Lawrence Bertram Mitchell, Stud. Inst. C.E.
Ronald Francis Norman.
Albert Arthur Osborne, Stud. Inst. C.E.
Bernard Boswell Pfeil, B.Sc. (Eng.) (*Lond.*).

John Frederick Stanmore Phillips, B.Sc. (Eng.) (*Lond.*).
Geoffrey Poole, Stud. Inst. C.E.
Hubert Pryce-Jones, B.Eng. (*Liverpool*).
Ernest Ronald Raworth.
Leonard Vinson Relph, Stud. Inst. C.E.
Richard Charles Rippon, B.Sc. (Eng.) (*Lond.*).
Stanley Albert Rossiter, B.Sc. Tech. (*Manchester*).
David John George Shennan, B.Sc. (*Aberdeen*), Stud. Inst. C.E.
Walter Paul Shewell.
Robert Reid Shiach.
William James Sivewright, M.A. (*Cantab.*).
Harold Cuthbert Smith.
Stanley Robert Sparkes, M.Sc. (*Bristol*), Stud. Inst. C.E.
George Winstanley Spencer, Stud. Inst. C.E.
William Gerald Maunsell Ternan, B.A., B.A.I. (*Dubl.*).
George Alan Thompson, B.Sc. (*Leeds*), Stud. Inst. C.E.
Maung Hla Thwin.
Harold Edwin Westwood, B.Sc. (*Leeds*), Stud. Inst. C.E.
William Stanley Whitehead, B.Sc. (Eng.) (*Lond.*), Stud. Inst. C.E.
Thomas Alfred Kenneth Wilson, Stud. Inst. C.E.
Frederick Henry Woodrow, B.Sc. (Eng.) (*Lond.*).
Ralph Walter Charles Yeo, B.A. (*Cantab.*).

The following Paper was submitted for discussion, and, on the motion of the Chairman, the thanks of The Institution were accorded to the Author.

Paper No. 5076.

"Pre-Stressing Bridge Girders."

By Herbert John Nichols, B.Sc., M. Inst. C.E.

TABLE OF CONTENTS.

Introduction.

In 1926 a decision was reached by the Bombay, Baroda and Central India Railway to rebuild the present bridge over the Nerbudda river at Broach, but it was not until 1931 that the main features of the layout had been decided upon and the span length had been fixed. For various reasons this was made 288 feet from centre to centre of piers and 282 feet between rocker bearings. At that time it was proposed by Messrs. Rendel, Palmer & Tritton, Consulting Engineers to the railway, to raise the working stress for structural steel from 8 to 9 tons per square inch, and they addressed the Railway Board in India for permission to proceed with the design of the girders on this basis, on the understanding that steps would be taken to design the girders in such a manner that deformation stresses would be kept at a minimum, or would be eliminated. With this proviso, and in view of the fact that the British Standards Institution were at that time contemplating a similar increase of working stress, the proposal was agreed to and the girders were designed with this object in view.

The girders were required to carry two tracks designed for the Indian Heavy Mineral Standard of Loading with 2–10–2 engines, having maximum axle-loads of 28 tons and weighing in all 267½ tons. The equivalent uniformly-distributed load per track for a span of 282 feet amounted to 863 tons. The weight of steel in one span as

designed amounted to 680 tons, and it is estimated that the saving in the weight of steel per span on account of the increase in stress amounted to about 9 per cent., or say 60 tons.

In view, therefore, of the substantial saving in weight, it became a matter of some interest to determine to what extent the method followed to reduce deformation stresses had been successful, and whether the increase in working stress from 8 to 9 tons per square inch had been adequately covered by a corresponding decrease in deformation stresses.

Under the Indian Railway Board Bridge Rules of 1926 it was specified that the sum of the stresses caused by dead load, live load, impact, curvature of track, secondary stresses, wind load, longitudinal forces, and temperature, should not exceed the normal working stresses (8 tons per square inch in tension) by more than 25 per cent. (that is, they were permitted to amount to 10 tons per square inch in tension). Under the revised rules of 1933 stresses from the above causes were permitted to amount to 10·5 tons per square inch in tension.

Deformation stresses were defined as " bending stresses, caused by the vertical deflection of the girder combined with rigidity of the joints." Secondary stresses were defined as arising from " the eccentricity of connection, or the lateral loading of members generally such as occurs in the case of a beam." It will be seen, therefore, that deformation stresses were not to be specifically calculated and it was merely necessary to design girders so as " to minimize deformation stresses as far as possible."

Before the apparent looseness of this requirement is criticized, it would be well to remember that the calculation of deformation stresses is a very lengthy business, except in the simplest of frames, and it will be shown that the results obtained by ordinary methods, that is, methods depending on rigid-body statics such as that given by Johnson, Bryan, and Turneaure,[1] are liable to give results very far from the truth. The calculation of deformation stresses by " least-work " methods would be still longer although the results would probably be of greater value.

It had, therefore, been usual to accept deformation stresses as a necessary evil and to assume that they were not likely to be very large; in any case experience had shown that the factor of safety was capable of covering them. A method of design which had for its object the reduction or elimination of deformation stresses carried with it every justification for a simultaneous increase in primary

[1] "The Theory and Practice of Modern Framed Structures," Part II. Tenth Edition, New York and London, 1929.

stresses. It is of interest to compare this development in steel-work-design with the accepted procedure in the case of the design of structures in reinforced concrete. In the case of the reinforced-concrete structures the design can be proceeded with on the fairly safe assumption that intersections are rigid, whilst in the case of steelwork there may be doubt as to the degree of fixity of many connexions. Plastic yield in the case of reinforced concrete must eliminate undesirable dead-load deformation stresses, whilst local overstress under full load may produce the same effect in the case of steel structures.

Design.

The principal details of one of the 282-foot span trusses are shown in Fig. 1, Plate 1. The effective depth is 40 feet and, contrary to the usual practice for spans of this order, the top chord is straight. The reason for this is that it was intended that the steelwork should be made on an interchangeable basis and jig-drilled, and it was found that the saving in shop-work resulting from this simplification in jig-work more than compensated for the slight increase in the weight of steel. There are eight panels sub-divided by sub-diagonals and sub-verticals into sixteen sub-panels, each of 17 feet 7½ inches. The positions of the splices in relation to the panel-points are shown; those details were subsequently found to be of some importance in the distribution of stresses. The top chord is provided with single channel-section lacing-bars as stiffening for the lower flange, whilst the end rakers have double lacing-bars of similar section. The top flange of the lower chord is stiffened with batten-plates at intervals. The web-members have single channel-section or tube lacing-bars.

The condition that deformation stresses should be reduced to a minimum under the full design load implies that in this state members should be straight, or very nearly so. As more usually erected, the members of a frame are straight when on the camber-jacks and the joints are riveted and the intersection angles fixed by this profile. Later, when a load is imposed on the frame, the axial elastic strains of the various members would require corresponding changes in the intersection angles. The rigidity of the joints, if no slip takes place, retains the intersection angles at their original values and in consequence the members must bend. If the members, during erection, could be bent in the reverse sense, then the application of the designed load would exactly remove this bending and the members would finish straight when under full load.

This result could be obtained theoretically if each member in the frame were lengthened or shortened by an amount equivalent to its calculated strain under the designed load, whilst the drilling of the

gussets was such that the intersection angles of the uncambered frame were preserved.

It is clear that under these conditions of design the shape of the cambered profile under no load would be incidental, and it could be modified without sacrifice of the accuracy of the pre-stressing only by altering the section of certain members so as to change their stress and elastic strain. It is usual to specify that the loaded chord of bridge girders shall have zero camber under full load, but the absolute necessity for this requirement is not clear. This convention is, however, of convenience in putting into effect the scheme of pre-stressing mentioned above.

The various steps in the procedure are to calculate the strain in each member under dead load plus full live load, and to draw a Williott diagram using these strains as the changes in length of the members.

If the loaded chord, say the bottom chord, of the truss is to be straight under full load, the above diagram will then give the shape of the bottom chord when erected on camber-jacks.

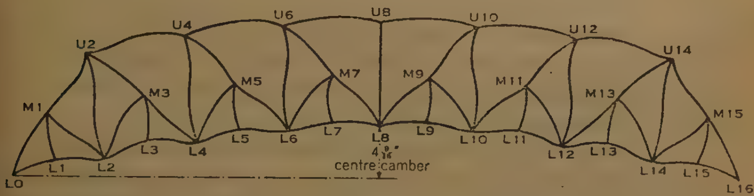
There are several points, however, to be noted before the "cambered" lengths of the members of a truss can be given. The strains as calculated would show increases for the bottom chord and web-members in tension, and decreases for the top chord and web-members in compression. Instead of increasing the length of the top chord and decreasing the length of the bottom chord in order to balance the above strains, the same results may be obtained if, say, the bottom chord and all the floor details (in the case of a through truss) are left unchanged, and if double the provision is made in the top chord. An adjustment of this nature can readily be made in the Williott diagram without changing the cambered shape by moving the bottom chord points horizontally until they lie on the same vertical, indicating no horizontal movement. The corresponding web-member strains may thus be obtained by completing the diagram from the bottom chord points so obtained. In the case of the Nerbudda spans the web-member strains were so adjusted that they were all additive. The results as finally arrived at are given in Fig. 2, Plate 1, to the nearest $\frac{1}{64}$ inch.

It was specified that the whole bottom chord was to be drifted and riveted perfectly straight and then cambered by lowering the ends; the web-members were then to be erected and drifted, and lastly the top chord was to be erected by working symmetrically from the centre outwards, each panel-length being cambered in turn as soon as the splices were made. Lastly the end rakers were to go in, the final closure being made at the L_0 joints. The shape of the truss so erected is shown in *Fig. 3*, with greatly exaggerated deformations. Theoretically it is then clear that when the design load

covered the whole span, the camber increment in each member should have been exactly neutralized by the elastic strain, and the truss camber should have been exactly taken out, leaving all members straight and free from all but axial stresses.

Web-members will carry their maximum axial stresses when the prescribed load covers only part of the span, whereas the above system of pre-stressing is designed to eliminate deformation stresses when a load covers the whole span. It might be objected that it is more important in the case of such members that the deformation stresses should be zero for maximum axial stresses; this objection is valid, but to effect this, however, would, with the present methods, require a great deal of calculation and, as pointed out above, an adjustment in the length of a web-member to bring this about would lead to an irregular cambered shape of the bottom chord.

Fig. 3.



Further, if web-members are free of deformation stress under conditions of partial load they will not be free under full-load conditions, whilst the existence of bending stresses in them puts bending stresses also in the chords to which they are attached, and these when the chords are carrying their maximum axial stresses. The conditions of each case must decide whether it is advantageous to give preference to the web-members or to the chords. Sub-members and hangers, however, offer a real difficulty, as they do not necessarily share in the stress-distribution which goes to cause deflexion of a girder as a whole, and they may remain unstressed while other non-redundant members are fully deformed. They may thereby give rise to heavy bending stresses in the members to which they are attached. This point will be considered further.

This short description will serve to indicate the characteristics desired of these trusses and the steps taken in the design to achieve this result. The contract for the manufacture of the steelwork was in due course let to a firm of contractors in India [1] who have specialized in the use of hard steel bushed jigs and who have developed this method of interchangeable manufacture to a high state of excellence.

Since there was every assurance that the accuracy of manufacture

[1] Messrs. Braithwaite & Co. (India), Ltd.

would represent the highest standard that was likely to be obtained at present, there appeared to be an excellent opportunity during the erection of the steelwork to carry out a careful investigation into the stresses which actually occurred in a steel frame, and preparations were made accordingly.

It should be emphasized at the outset that the reason for the investigation was to investigate the stress-distribution, not under ideal or laboratory conditions, but under normal conditions of careful field erection, since these are the conditions for which it is necessary to legislate.

The objects in view were :—

(1) To determine how the calculated deformation stresses compared with the actual stresses.

(2) To determine to what extent the special measures which had been adopted in the design and erection of the trusses to reduce deformation stresses had been effective.

(3) To determine to what extent reliance could be placed upon the deflexions given by a Williott diagram, Young's modulus being taken as 13,500 tons per square inch.

(4) To determine the reason for the loss of camber invariably noticed after the first application of a full design load.

(5) To determine if possible a more rapid and accurate method of dealing with the whole question of pre-stressing, bearing in mind the practical limitations of steelwork manufacture.

Instruments.

In order to obtain the information required it was necessary to measure the stresses in each member of the truss at the four corners of cross-sections taken at each end of the member and as close to the intersection points as joint-details would permit. For each condition of loading eight readings were therefore taken on each of the sixty-one members. The total number of readings was about 40,000.

Sets of readings were taken under the following loading conditions :—

(*a*) Zero readings were taken with all members resting on supports before assembling into the truss.

(*b*) After assembling and drifting to form a truss lying prone and virtually on frictionless supports.

(*c*) After erecting to form a truss and carrying dead load only. The span was oscillated vertically before these readings were taken.

(*d*) When carrying the full design live load (without impact).

(*e*) Under various stages of partial loading, the load being removed by panel-lengths.

(*f*) Under dead-load conditions again after removing the live load, when a certain amount of camber had been permanently lost.

(*g*) Series (*f*) was repeated after freely oscillating the span by a locomotive with its wheels slipping.

Certain additional stress-readings were also taken at various points, together with a full series of camber-readings. All stress-readings were taken at night during a period of 6 months and while the steelwork-erection was proceeding elsewhere on the construction.

The first set of readings was started in April, 1934, using two Fereday-Palmer stress-recorders and working on the first span to be erected. The procedure followed was to obtain the zero readings (while the members were on the ground) at the eight points on each member by comparison with a reference-bar, which was allowed to come to the temperature of the members before a reading was taken. The hard-steel points of these instruments are 20 inches apart and it was therefore necessary to select positions on the members which were free from obstructions, such as rivets or structural details which would influence the readings. This was by no means easy, and many compromises had to be accepted. It was considered that satisfactory results would be obtained if the 20-inch bases were marked on the member merely by moderately heavy punch-marks made with a sharp punch, whilst the points on the instruments were slightly dulled. There is every reason to believe that the results obtained from these direct markings were satisfactory, but they were very liable to damage and special steps were taken to keep them prominently in view whilst handling members or riveting.

Considerable difficulty was experienced at first in supporting members in such a manner as to eliminate stresses arising from their own weight and local deformation, as it was found that after turning a member over, slight variations were always noticeable. The plan which appeared to be the most suitable was to support members as nearly as could be judged in the same manner as they would be supported in the structure after erection. The worst calculated stress due to the weight of the member occurred in the top chord, where a stress for fixed-end conditions of about 0·3 ton per square inch appeared probable. It was estimated that errors from this source were reduced to about 0·1 ton per square inch, and they are therefore unlikely to affect materially the assessment of deformation stresses.

Having marked and obtained the zero differences of all members, they were then assembled and the joints drifted and riveted. While the span was still on camber-jacks, a second set of readings was taken. A third set was taken with the camber-jacks removed and the span carrying its own dead load.

It was by this time realized that there were a great number of disturbing influences which were masking the true distribution of stresses around the various joints. The camber-jacks were placed in pairs under the inside and outside webs of the bottom chord, and it was found that an extremely small difference in level between them led to a derangement of the stresses in the members above them. It was also found that the presence of the jacks led to a reversal of stresses in some members, and it was, moreover, impossible to ensure that each pair of jacks was carrying precisely its due share of the load. Further, the floor-system and the laterals introduced distortions and relief of stress to an unknown extent. In short, while on camber-jacks adjusted to the prescribed camber, the truss was far from conforming to its "free" outline, and a measurement of the deformation stresses induced by the special methods of erection could not be expected to give much useful information.

This series of readings was therefore abandoned and arrangements were made to start a fresh series. It was clear that the bottom-chord stresses were extremely sensitive to variations in the heights of the camber-jacks, a variation of $\frac{1}{20}$ inch only in one pair resulting in bending-stresses of as much as 2·5 tons per square inch. No satisfactory results were therefore likely to be forthcoming from readings taken while the span was subjected to the unknown upward loadings imposed on it by the camber-jacks.

A fresh scheme was evolved whereby a truss would be erected flat on one side, thus enabling it to assume its natural shape and camber with no disturbing influences in the plane of the truss. For this purpose steel packings were laid out and levelled so that the truss when assembled would be about 2 feet clear of the ground and freedom of access would be obtained to take the necessary readings at all points. A fresh difficulty then arose in connexion with the bending-stresses induced by the weights of the individual members. It was out of the question, under the limitations imposed by both cost and time, to remove each member in turn to a specially-prepared surface upon which zero markings could be set out, and it was therefore necessary to arrange the packings as far as possible to keep such bending-stresses at a minimum. Each member, however, when being marked, was turned over into the position it would finally occupy when in the span. When turned back through 90 degrees into the prone position slight bending-stresses were

undoubtedly caused, but these are nowhere considered to be greater than 0·25 ton per square inch.

Discrepancies were apt to arise in the readings taken on the top flanges of the bottom chord and the bottom flanges of the top chord. These flanges consist of angles riveted to the webs and cross-connected either by lacing-bars or by occasional batten-plates. When members were turned over into the "prone" position these angles had little lateral support and were apt to show unwanted stresses due to local distortions. These were eventually avoided by keeping the angles clear of the supports.

At this time it was also decided to obtain two 10-inch Whittemore strain-gauges reading direct to $\frac{1}{7}$ ton per square inch. While it was recognized that accuracy might be lost by substituting a 10-inch base for the 20-inch base of the Fereday-Palmer stress-recorders, it was at the same time possible to fit in the 10-inch base markings with much greater facility. The stress-recorders had proved very tiring to handle, especially when working overhead for long, and the punch-marks were apt to deteriorate rapidly.

For the purpose of establishing the zero marks for the strain-gauges a substantial punch-mark was first made with a 90-degree punch; a hole $\frac{1}{32}$ inch diameter and $\frac{1}{8}$ inch deep was then drilled, and lastly a 60-degree countersinking tool was lightly used. The instrument had 60-degree points, and these found a bearing on the countersunk surface below the general surface of the metal. When one hole was completed in this way the second was jig-punched and dealt with in the same manner. The reference-bars were provided with similar holes in hard steel bushes. The holes, of which about five hundred pairs were made, were kept constantly greased, and were cleaned out with petrol before a reading was taken.

When using the instruments it was found that slight variations in readings were likely to occur during the first few applications in any pair of holes, and it was usually necessary to apply and remove the instrument from six to eight times before a steady reading was obtained. With familiarity, however, quite rapid and accurate work could be done.

Procedure.

Variations in temperature made accurate readings very uncertain by day, temperature-differences of up to 10° F. having been found between different parts of the structure, and conditions were most favourable between 10 p.m. and 6 a.m. when the rate of change of temperature was inconsiderable. All readings were therefore confined to this period. It should, however, be mentioned that

the reference-bar was always placed on the member close to the measuring points and it was referred to between readings. The reference-bar was periodically checked against a master bar which was carefully preserved.

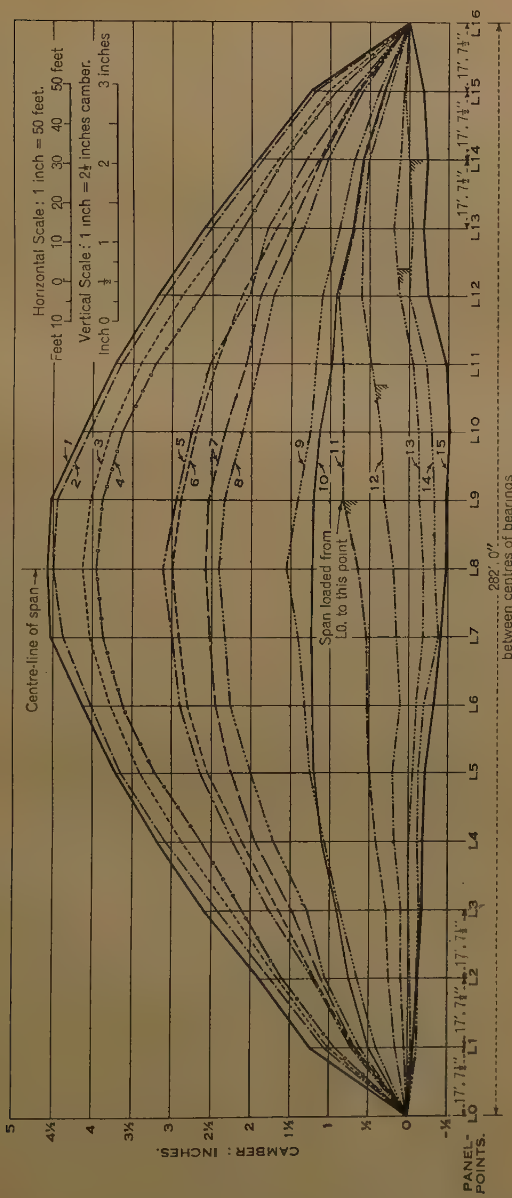
In one truss there were sixty-one members and on each member eight zero bases were marked, that is, four at each end and as close to the end as structural details would allow. These four marks were disposed approximately at the four " corners " of the cross-section, and they therefore permitted the axial stress to be determined together with the vertical and lateral bending stresses. In certain cases the true position of the neutral axis appeared doubtful on account of the proximity of gussets, batten- or splice-plates, and in such members additional bases were marked somewhere near the centre of the member where there were no disturbing details. It was then possible to compare the primary or axial stresses as obtained near the ends with those obtained at " clear " sections, and in cases of doubt to investigate further.

When all members had been marked the zero differences were checked by three completely independent sets of readings made by different observers. The members were then assembled (prone) in the sequence followed for the field erection. All joints were fully drifted and bolted. Each joint was then lifted slightly to ease the friction of the supports and to enable the truss to assume its unrestrained profile. The centre camber was then found to be 4·125 inches as compared with the designed camber of 4·56 inches, indicating a " spring-back " of 0·43 inch (camber-lines 1 and 2, *Fig. 4*).

TABLE I.—EXPLANATION OF CURVES IN *Fig. 4*.

Curve No.	Description.
1.	Camber by Williott diagram.
2.	Camber of truss on camber-jacks immediately before striking.
3.	Camber of truss assembled prone.
4.	Camber given by model.
5.	Camber under dead load immediately after striking.
6.	Camber under dead load, but after oscillation by slipping engine.
7.	Camber under dead load, after applying and removing full designed live load; air-temperature 110° F.
8.	Camber under dead load, after applying and removing full designed live load; air-temperature 78° F.
9.	Camber under dead load plus 1 ton per foot run on both tracks.
10.	Camber under dead load plus full live load on near track only.
11., 12., 13., 14.	Camber under dead load plus partial live load on both tracks.
15.	Camber under dead load plus full design live load, both tracks heavy mineral standard loading, 3·06 tons per foot of track.

Fig. 4 (see Table I).



A set of "no-load" readings was then taken and checked. From these the extent of the pre-stressing, axial, and deformation stresses was obtained, and these stresses have been plotted in the various diagrams appended. Each member, gusset- and cover-plate was then carefully marked for position. The truss was dismantled and dispatched to the site for re-erection as the west girder of span No. 4, and the sixth to be erected at the site.

Profiting by the experience obtained on span No. 16 no readings were attempted while the span was on camber-jacks. It would have been a matter of considerable interest to repeat the "no-load" series after attaching the lateral and floor systems, but the unknown effect of the jacks would have rendered such a series practically valueless. The next series was therefore not taken until the span had been swung and the track had been laid. In order to obtain a normal distribution of stress the span was first oscillated by means of a 120-ton engine with its wheels slipping, and in this process 0·11 inch of camber was lost (curves 5 and 6, *Fig. 4*). The unloaded frequency was found to be 3·22 periods per second. The series of readings for "dead load" was then taken and these also are shown in the accompanying diagrams.

A loading equivalent to the design live load was then applied. The equivalent uniformly-distributed load required to develop the bending moments due to the Heavy Mineral Standard of loading on a span of 282 feet amounted to 3·06 tons per foot of track or to 6·12 tons per foot of bridge. The loading required to develop maximum shear amounted to 3·236 tons per foot of track, or 5·6 per cent. more. It was decided to use the loading required for bending-moments, and it should be noted that the web-members were thereby loaded to only about 94 per cent. of their design loading. These loadings apply, however, only when the span is completely covered, and they increase in intensity as the loaded length decreases.

The heaviest rolling-stock available, when normally loaded, gave an intensity of loading of only 1·2 ton per foot run; by overloading this could be increased to 2·06 tons per foot run, giving 27-ton axle-loads, but by this time the springs were bearing heavily on the frames and it was considered inadvisable to add more weight. The balance of 1 ton per foot run had therefore to be made up by rails laid directly on the cross sleepers. This was unfortunate, since facilities for measuring stresses under conditions of partial loading were greatly curtailed.

This load was applied in the first place on one track only, covering the whole span, and the distribution of load between the two girders was measured. It was found that only 86 per cent. of the total load was accounted for by vertical deflexions, the balance being carried

by the top and bottom lateral systems. The deflexions of the two girders were 1·76 inch and 0·76 inch respectively, and after obtaining the deflexions when both tracks were loaded it was found that the distribution was in the ratio of 68·5 per cent. to 31·5 per cent. By calculation, taking simple moments about one of the girders, the distribution was 72·4 per cent. and 27·6 per cent. respectively. The load carried by the near girder was therefore 68·5 per cent. × 84 per cent. = 57·6 per cent. of the total load, instead of 72·4 per cent., whilst the load carried by the far girder was 31·5 per cent. × 84 per cent. = 26·5 per cent., instead of 27·6 per cent. It is, therefore, clear in this case that, when considering impact effects, if full impact is assumed to be effective on only one track at a time the proportion carried by the near girder will be less rather than more than that obtained by simple calculation—perhaps only 57·6/72·4 = 79 per cent. of that amount. The live load was then placed on the second track and a series of readings under full-load conditions was obtained. These stresses have been plotted in the accompanying diagrams (Figs. 5 to 10, Plate 1). The stress was also measured in certain of the top and bottom lateral bracings in order to estimate the measure of relief afforded by them.

The completion of the above series gave most of the data required to enable an estimate to be formed of the effectiveness of methods adopted to pre-stress the members. A few more days were, however, available for experimental work without interfering with the general progress of erection. The live load on both tracks was therefore removed equally from one end (one 30-foot rail-length at a time, approximating to one full panel-length), and deformation and primary stresses were measured at each stage in certain members. It was intended thereby to compare the variation of actual stresses, primary and deformation, for a load advancing across a span, with those calculated. A load entirely on wheels would have been of far greater convenience for this purpose. These readings will be referred to later.

When the live load was fully removed from the span, having been in place in varying degrees for about a month, the span nominally returned to the "dead-load" condition. A further series of stress-readings was then taken, together with camber-readings, for purposes of comparison with those taken before the live load was imposed.

The camber was found to be reduced permanently by 0·42 inch (curve 7, *Fig. 4*), and it was hoped that the further stress-readings would show the reason for this. It was found that the deformation stresses showed in general little change, but that the primary tensile stresses showed a permanent increase commensurate with the loss of camber. These results pointed to a substantial permanent set in all

members except the top chord, which remained practically unaltered; they appeared to be so unlikely that before they were accepted the span was thoroughly oscillated by an engine with 17-ton axle-loads with its wheels slipping, and a completely fresh series of readings was taken. This further set, whilst differing slightly in the direction of recovery from the earlier set, substantially confirmed the indications of the first series. The camber showed no change as a result of the oscillation and remained at 2·62 inches. The readings obtained have been plotted in Figs. 5 to 10 (Plate 1) inclusive.

This series of readings concluded the experimental work, and before proceeding to examine the results in detail it is proposed to refer to the theoretical work which had been proceeding simultaneously.

In drawing out Williott diagrams it was assumed that Young's modulus (E) would be 13,500 tons per square inch for the members of the truss. There was no doubt that this value would be too high for the first application of load, but for subsequent applications it was anticipated that it would be approximately correct. At the same time, an allowance was made, when calculating the elastic strains of members, for the presence of gussets and details which would reduce the effective length of members. Since the shop camber-lengths of members were rounded off to the nearest $\frac{1}{64}$ inch, the full design-load deflexion would not necessarily exactly balance the camber-provision. The difference at the centre of the span as given by diagram was, however, within $\frac{1}{32}$ inch, a negligible difference.

In comparing the actual no-load camber of the truss with that obtained theoretically by rigid-body statics, certain discrepancies were to be expected. In the first place the bending of the members to obtain the " no-camber " angles of intersection involved all members of the truss in primary stresses resulting from a slight loss of camber (curve 3, *Fig. 4*, p. 101). The loss of camber on this account was measured and was found to be 0·43 inch, a reduction of 9·6 per cent., involving a corresponding decrease in the initial deformation stresses and a corresponding imposition of primary stresses. The corresponding figure by model (curve 4, *Fig. 4*) was 0·6 inch, or 13 per cent. reduction.

Since this initial primary stress was imposed on all members when the truss was lying prone, it was of some interest to determine how much of it was released when the truss was carrying its full load. With regard to the full-load deflexion, extraneous factors were introduced when the truss became part of a girder, and top and bottom laterals and the floor-system were added. The value of these was difficult to assess theoretically, but their effect should have reduced deflexions. Actual measurements of stress in the laterals

indicated that the relief of stress to the chords was of the order of 5 per cent. in the two centre panels of the top chord, 4·8 per cent. in the centre panels of the bottom chord, and 10 per cent. in the end panels of the bottom chord. It may be assumed, therefore, that an overall relief of about 5 per cent. was to be expected in chord strains and about 3 per cent. in centre deflexions.

The centre deflexion under full load could therefore be expected to be, say, 97 per cent. of 4·56 inches, that is 4·43 inches. The actual deflexion (curve 15, *Fig. 4*), was found to be 4·98 inches, that is, 0·42 inch below the horizontal. It would, therefore, appear that the average modulus of elasticity during the first application of load was $\frac{4 \cdot 43}{4 \cdot 98} \times 13{,}500$, that is, 89 per cent. of 13,500. On removing the live load the elastic recovery suggests that the mean modulus was $\frac{4 \cdot 43}{4 \cdot 98 - 0 \cdot 42 - 0 \cdot 10} = 99 \cdot 4$ per cent. of 13,500.

In order to compare the measured deformation stresses with those predicted by theory, it was necessary to carry out a complete analysis of the truss. The "approximate" method (the modified Manderla method), given very clearly in "The Theory and Practice of Modern Framed Structures," Part II*, was followed. This method is based on the assumption that the changes in length of the members of a frame due to external loads are as given by rigid-body statics. This assumption, however, is liable to large errors, particularly in the case of frames possessing sub-members or hangers which are not essential to the stability of the frame as a whole (for example, a load applied at the base of a hanger is partly carried by the hanger and partly by the stiffness of the bottom chord acting as a beam). To take factors of this nature into account would require the employment of "least-work" methods, and the work would become prohibitively long. The object in view was to compare actual stresses with stresses given by practical methods of calculation, and it is considered that a method involving the introduction of "least-work" equations would be definitely impracticable.

In the trusses in question there were thirty-two "unknowns." A complete solution for a load placed at any one panel-point therefore involved the formation of thirty-two simultaneous equations, each equation containing from seven to eleven terms. Each of the intermediate fourteen panel-points required the formation of a fresh set of equations. The work as carried out was, therefore, very long, and it is questionable whether the standard of accuracy attainable could often be considered a justification for the time spent.

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* Footnote 1, p. 92.

Some of the results obtained from these calculations are shown in Figs. 11 to 13 (Plate 2). It will be seen that the dead-load and live-load primary stresses, together with the dead-load and live-load deformation stresses, have been plotted for each end of each member as a live load of the prescribed standard advances across the span. Two curves of total stress have been plotted, one for the top and one for the bottom flange of the member in question.

Stresses as measured have been added for purposes of comparison, and it will be seen that, although differences in actual intensities arise, there is a general similarity in the shape of the calculated and measured curves. The measured curves for practical reasons were for a load of constant intensity advancing across the span, whilst those calculated were adjusted for a load of increasing intensity as the loaded length shortened. The difference in intensity per foot run between half- and full-span load is not very great, and therefore only those portions of the measured curves have been plotted.

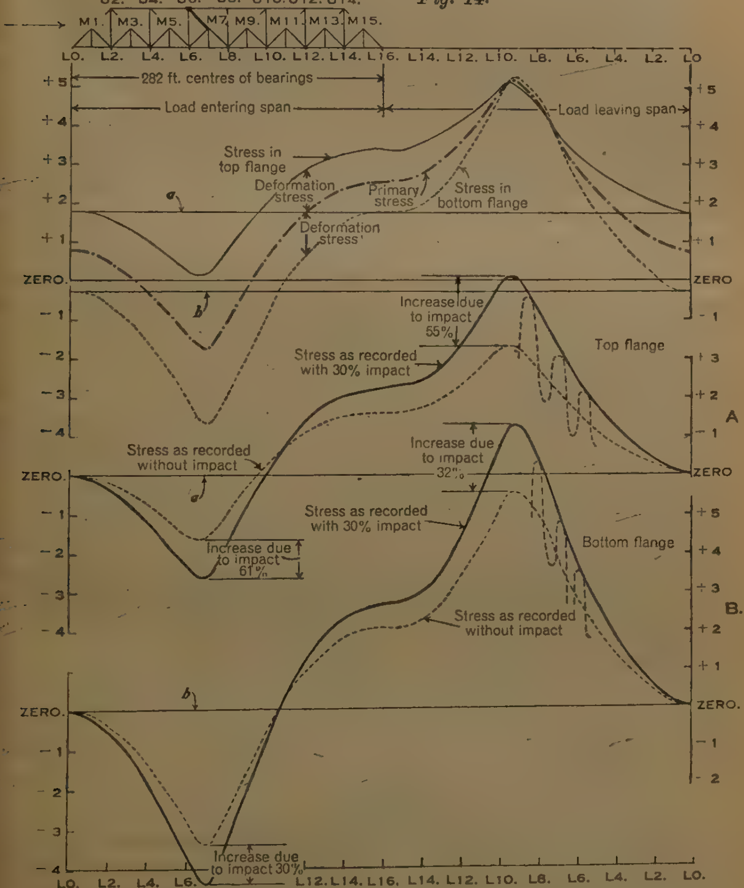
While considering these curves it is of interest to note, particularly in the case of those web-members which are liable to reversals of stress (for example, member U_6M_7, *Fig. 14*), that if stress-recorders were attached to the top and bottom flanges respectively and a "crawl" load passed over the span, the shape of the stress curves would be as shown by the dotted lines in *Fig. 14*, (A) top flange, and (B) bottom flange. In the case of the top flange the worst stress shown on the record would be $-$ 1·65 and $+$ 3·30 tons per square inch, and in the bottom flange $-$ 3·4 and $+$ 5·55 tons per square inch. The true maximum live-load stresses would be, however, from the diagram $-$ 4·4 and $+$ 4·5 tons per square inch. If, therefore, this member were subjected to an increment of stress from impact effects amounting to $\pm$ 30 per cent., the records would show increments as shown by the full lines in (A) and (B) of $-$ 1·0 ton per square inch and $+$ 1·80 ton per square inch, respectively. The increments on (A) would be 61 per cent. compressive and 55 per cent. tensile, and on (B) 30 per cent. compressive and 32 per cent. tensile.

Unless, therefore, the records were carefully synchronized and compared when measuring the impact increments, it would be easy to obtain the impression that the impact effects in such members were greater than they actually were. It is possible that the high impact sometimes recorded in web-members may be in part explained in this way.

It is clear that, since deformation stresses arise from a change in the geometrical shape of a frame, such stresses will occur whether the frame is deformed by a static or by a dynamic load. The deformation stresses will remain a function of the primary stress

and their intensity will, therefore, be subject to the impact increments in the same way as are primary stresses. In short spans, therefore, when the increment due to impact is high, the saving in metal by the elimination of deformation stresses may still be appreciable.

Fig. 14.



Before the theoretical investigation had proceeded very far it was realized that many of the assumptions which it was necessary to make with regard to the distribution of stress, notably in connexion with loads at the sub-panel points, could not yield results which could be of very great value. Other and more convenient means were therefore sought. Determination by model appeared to offer the most promising line of attack, particularly if the model could

be made to exaggerate the axial strains and bending of members sufficiently for them to be measured without the aid of a microscope.

A model was in due course evolved (*Fig. 15*), which magnified primary- and deformation-stress effects 100 times. It was made $\frac{1}{50}$th full size and was cut out of sheet celluloid $\frac{1}{16}$ inch thick and was placed between sheets of glass to prevent buckling during deformation.

In order to preserve the scale it was necessary that the width of each member should be proportional to the moment of inertia of the member it represented. The width, however, together with the thickness of the sheet, fixed the sectional area, which would not necessarily represent to scale the sectional area of the full-size member. This difficulty was overcome by cutting the member near its centre and rejoining the two parts through the medium of four transverse beams, such that the stiffness of these beams against bending represented to scale the longitudinal stiffness of the full-size member. The stiffness of the beams could be varied either by altering their length or their thickness. A means was therefore afforded of obtaining a magnification, and, provided the stiffness of the beams was related to the width of the member to which they were attached, the scale relationship between the longitudinal and transverse stiffness of each full-size member was preserved. Further, the employment of two beams enabled a longitudinal magnification to be obtained without sacrifice of lateral stiffness, and the use of "fixed ends" eliminated mechanical joints. The derivation of the proportions required in designing a model is given in the Appendix. *Fig. 15* is a photograph of the model made to $\frac{1}{50}$th linear scale of one of the 282-foot Nerbudda trusses. The magnification in this case was 100, but it would appear that 50 would be a more serviceable scale to adopt.

In this model the cambered lengths of all members as given in Fig. 2 (Plate 1) were incorporated, and the model therefore represented the cambered shape of the truss with the camber magnified 100 times; that is, since the scale was $\frac{1}{50}$th, the camber of the model was twice the actual camber of the full-size truss. A scale load applied to any panel-point caused the various members to change in length by an amount that was twice that occurring in the corresponding full-size member. In this figure the bending of the various members under this pre-stressing method of erection can be clearly seen, and on the model itself it was a simple matter to measure the deflexion-angles at the ends of various members, and therefore to calculate the stresses which such deflexions, divided by 100, would produce in the full-size members. It will be noted that actual loads were applied to the model; the distribution of stress between redundant members therefore followed the distribution occurring in the full-size structure,

Fig. 15.

NERBUDDA BRIDGE-TRUSS MODEL ($\frac{1}{50}$th LINEAR SCALE) WITH CAMBER EXAGGERATED 100 TIMES.

5076.

and reproduced the complicated interchanges of bending and direct stresses. Further, should, under certain conditions of loading, the deformation stress in any particular member be found to be in need of improvement, it would be a simple matter to test the effect of modifying the camber allowances or sectional area of any member with a view to obtaining an improvement.

The deformation stresses as measured at the end of each member of the model and produced by the full design load have been plotted in Figs. 5 to 10 (Plate 1), for purposes of comparison with similar stresses actually measured on the full-sized structure. The same stresses, as calculated by Johnson, Bryan, and Turneaure's method (the modified Manderla method), have also been plotted on the same figures. It will be seen that the figures given by the model afford a much closer guide to the true stresses than do calculations. As already noted, the deformation stresses given by the model have been reduced by 13 per cent. by the "spring-back," and those of the truss by 9·6 per cent. The time required to make this model was only a fraction of that required for the calculations. The model was, in fact, cut out on a fretwork machine during the spare time of 6 days, and each part was subsequently trued up with a file. It should be mentioned that if a model is cut out of celluloid it will take up a plastic set if kept for any length of time in a deformed shape. It is therefore necessary to complete the experimental work before this occurs. The model had been subjected to a good deal of rough usage when the photograph (*Fig 15*) was taken.

In Figs. 5 to 10 (Plate 1) inclusive, the primary and deformation stresses in each member in one half truss have been set out for various loadings. In these diagrams the stresses shown as "measured" are the average strains measured on 10-inch base-lines on the corresponding member in the right- and left-hand ends of the truss, and converted to stress on the basis of $E = 13{,}500$ tons per square inch. Although the figure of 13,500 tons per square inch may not be the correct one for the members as a whole, yet it must hold good for the metal between the 10-inch gauge-points. In no case has the effect of rivet-holes in reducing effective areas been taken into account.

Primary Stresses.

In Fig. 5, Plate 1, (bottom-chord primary stresses) it will be seen that when the truss was erected prone the special condition resulting from the method of pre-stressing imposed a small tensile stress throughout the chord. A corresponding compressive stress will be seen in the top chord (Fig. 6, Plate 1), and this initial stress is, in fact, traceable in all members (Figs. 5 to 10, Plate 1). *Fig. 4*

(curve No. 3) shows the camber of the truss when erected prone, and the difference between curves 1 and 3, namely 0·43 inch, may be considered to be the "spring-back" resulting from the deformation of members when drifting them together. In Fig. 16 (B) and (D) (Plate 2) it will be seen that the initial stresses mentioned above in the bottom and top chords are approximately proportional to the loss of camber. This was to be expected, since, excepting the slipping of joints, the camber—stress relationship must hold regardless of the manner in which changes of camber are brought about.

Reverting to Fig. 5 (Plate 1), three primary-stress lines have been shown for dead load after the same truss had been dismantled in its prone position and re-erected as the west girder of span No. 4. These lines have been numbered 1, 2, and 3 :—

(1) representing initial dead-load stresses after oscillating the span,
(2) representing dead-load stresses after applying and removing the full design load,
(3) as for (2), but after oscillating the span.

The same group of lines has been shown for all members on Figs. 5 to 10 (Plate 1) inclusive. On *Fig. 4* the corresponding camber lines have been numbered 6 and 7, the oscillation having no effect on the camber at this stage. The drop of 0·11 inch between curves 5 and 6, however, was produced by oscillation before a live load was applied to the span. The effect of applying and removing the live load and subsequently oscillating the span is shown in Fig. 16 (A) to (E) inclusive (Plate 2).

In considering Fig. 16 (Plate 2), the stresses as plotted are really the strains measured by the Whittemore gauges on 10-inch base-lines, and have been converted into stresses on the basis of $E = 13{,}500$ tons per square inch. Figs. 16 (A) and (C) therefore represent the stress-strain relationships, while Figs. 16 (B) and (D) express the geometrical relationship between changes in length between the 10-inch gauge-points and the centre camber of the truss.

Referring to Fig. 16 (A), member L_7L_9 under dead load only, point 1 shows an initial strain represented by 3·5 on the stress scale before applying the live load, and by 4·25 after applying and removing the live load and oscillating ; that is, under the stress produced by the dead load it showed a permanent increase in strain represented by 0·75 ton per square inch. In the case of member L_6L_7 the corresponding increase was also 0·75 ton per square inch, indicating a permanent elongation of the member. This elongation would be accounted for if the modulus of the member during the first loading was only 85 per cent. of that which would apply to a small specimen.

Turning to Fig. 16 (C), the change in strain due to adding and removing the live load is negligible, and the difference in position of points 1 and 3 may be accepted as being within the limits of instrumental error.

In Fig. 16 (B), bottom chord, it will be seen that the strain in member L_7L_9 (and in L_6L_7), is greater after loading than it was before loading, but only because the camber does not return to its initial value. The relationship between the 10-inch gauge-length and the centre camber is maintained during loading and unloading. In Fig. 16 (D), top chord, the strain in the 10-inch bases of member U_6U_{10} (and in U_4U_6), is the same after loading as before, in spite of the fact that the camber has not returned to its initial value. It would, therefore, appear clear that something has happened to upset the geometry of the figure which has altered the effective length of the top-chord members between panel-points and which has also accounted for a permanent loss of camber. A slip in the top chord joints of about $\frac{1}{70}$ inch per joint would explain both these features.

The effect of such a slip is not evident in Fig. 16 (C), which expresses the relationship between the load on the span and the mean length of the 10-inch base-lines. During both the loading and unloading periods the elastic modulus of the member within the 10-inch base does not appear to have varied appreciably from 13,500 tons per square inch. In Fig. 16 (D) the effect of such slip was to reduce the apparent elastic modulus to a value of about 79 per cent. of the normal in the case of U_6U_{10}, and to 86 per cent. in the case of U_4U_6.

Further "load—stress" (stress—strain) and "camber—stress" (deflection—strain) diagrams are shown in Figs. 17 (Plate 3), in which similar characteristics can be traced. A summary of these results is given in Table II.

TABLE II.

Member.	First loading.	Unloading.
Top chord U_4U_{12} . . .	Elastic modulus 93 per cent. of *E*, 13,500 tons per square inch. Joints slipped.	Elastic modulus 99 per cent. of *E*, 13,500 tons per square inch.
End raker L_0U_2 . . .	Elastic modulus 110 per cent. of *E*. Joints slipped.	Elastic modulus 104 per cent. of *E*.
Web vertical U_4L_4. . .	Elastic modulus 90 per cent. of *E*. Little or no slip in joints.	Elastic modulus 100 per cent. of *E*.
Bottom chord L_6L_{10} . .	Elastic modulus 96 per cent. of *E*. No slip in joints.	Elastic modulus 112 per cent. of *E*.
Web diagonal U_2L_4 . .	Elastic modulus 94 per cent. of *E*. No slip in joints.	Elastic modulus 116 per cent. of *E*.

The explanation suggested for the apparent anomaly of the tension members exhibiting such a high elastic modulus during the unloading period is that the slip in the top-chord and end-raker splices had the effect of permanently reducing the camber, which would prevent the bottom chord from returning to its initial dead-load length. A slight reduction in stress in the top chord seen in Fig. 6 (Plate 1) could be expected to result from the same cause.

A further point to be considered is that during erection initial axial stresses were imposed by the pre-stressing, and, as shown in Fig. 16 (E) (Plate 2), the effect of these stresses was to reduce the initial camber. It was to be expected that when the full load was applied to a truss the pre-stressing would be neutralized and that the final deflexion would be that given by calculation. From Fig. 16 (E) it will be seen that these effects had apparently largely been shaken out as soon as the span was oscillated under dead load. As will be seen later when examining deformation stresses, the pre-stressing was also largely neutralized by the application of the dead load. The loading line (Fig. 16 (E)) is then straight, representing a truss 93 per cent. as stiff as that calculated on a basis of $E = 13{,}500$ tons per square inch. The unloading line would appear to be correct for $E = 13{,}500$ tons per square inch. The initial loss of camber by pre-stressing, namely 0·43 inch, was almost exactly equal to the excess of the actual deflection over the theoretical, namely 0·42 inch.

It is clear, therefore, that the effect of pre-stressing must be to reduce the apparent modulus of elasticity of a truss as a whole, and also that in individual members the dead- and full-load stresses may be influenced to some extent by the initial no-load stresses. The estimation of the elastic modulus during the loading period therefore becomes rather uncertain. During the unloading period the initial stresses return in certain members to a greater or less extent, causing uncertainty in this period also.

Deformation Stresses.

Turning to the deformation stresses plotted on Figs. 5 to 10 (Plate 1), it will be seen that under no-load conditions three lines of stress have been plotted :—

(1) representing calculated stresses based on rigid-body statics,
(2) representing those obtained by direct measurement on the model after a "spring-back" of 13 per cent.,
(3) representing stresses obtained from the truss erected prone after a "spring-back" of 9·6 per cent.

On these diagrams have also been plotted points representing the amount of deformation stresses resulting from the application of a full

design load to the truss after assembly as part of span No. 4. For the complete elimination of deformation stresses under full load, lines (3) (above) should have passed through these points. It will be noted, however, that these point stresses correspond to a full-load central deflexion 0·42 inch below the horizontal, or 9·2 per cent. greater than that originally estimated in the design.

On these diagrams stresses in the upper flanges have been shown throughout, and in the case of the vertical members stresses on the flange nearer to the centre of the truss. In the case of the member symmetrical in section, the same stresses, but of opposite sign, apply to the other flanges. In the case of the chords and end rakers the stress is proportional to the distance from the neutral axis. Thus, in the case of the top chord (*Fig. 3*, p. 95), the stresses in the lower flange would be about 1·61 times those plotted, and opposite in sign. In the lower flanges of the end rakers high deformation stresses would occur in the absence of pre-stressing, and would have reached about + 3·22 tons per square inch.

In the vicinity of the M joints the stresses in the bottom and top flanges would then have been :—

Stress (raker).	Bottom flange : tons per square inch.	Top flange : tons per square inch.
Primary dead load and live load	− 6·33	− 6·33
Impact	− 0·32	− 0·32
Deformation	+ 3·22	− 2·0

The relative importance of deformation and impact stresses is illustrated by this example, and the economic possibilities of pre-stressing are emphasized.

Comparing Fig. 5, Plate 1, (bottom chord) and Fig. 6, Plate 1, (top chord) it will be seen that, owing to the presence of sub-members attached to the bottom chord, the calculated deformation stresses in this chord differ widely from those measured, whilst in the case of the top chord the agreement is much closer. The explanation for this is undoubtedly that the calculations assume that members do not change in length when assembled into a truss. In practice, however, the light sub-members which are attached to the lower chord are required to deform the heavy chord to the extent shown at points L_1, L_3, etc., (*Fig. 4* (p. 101), curve 3), and in doing so suffer considerable strain. In the case of the M_1L_2 sub-struts this was particularly pronounced; on insertion and drifting one end, only half holes were obtained at the other end, and there is no question that, in attempting to fair these up by drifting without the use of straining devices, the thin

scantlings of these members would be damaged and the holes elongated. All members in a truss undergo some strain, however, as is indicated by the "no-load" primary stresses plotted on these diagrams. Such changes of length are automatically taken into account by the model, and this is one reason why the stresses obtained agree far more closely with the stresses as read than do those calculated.

The stresses measured in the truss assembled prone are, in general, still less than those given by the model, and this must be put down to the effect of practical manufacturing tolerances. It is estimated that in order to control deformation stresses in normal members of a truss to within 0·2 ton per square inch, matching holes in members and gussets should be correctly positioned and assembled to within 0·005 inch. If this standard of accuracy could be obtained in the drilling of holes there would still remain the difficulty of fairing-up holes at the ends of members to the necessary degree of precision, against the stiffness of the member opposing the deformation which it is required to impose. There is also the question of obtaining the required accuracy in the distance between the groups of rivet-holes at opposite ends of a member; for example, an error of as little as 0·010 inch in the length of a member 40 feet long would result in an undesired axial stress of 0·28 ton per square inch, and errors considerably larger than this are likely to occur.

During the manufacture and erection of the spans in question the tolerances permitted in positioning and fairing-up holes were briefly as shown in Table III.

Practical allowances in setting out were required to provide for variations in rolling margin, gathering of thicknesses, amount of draw of field joints, variations in tension of marking-out tape, shop clearances, and growth during riveting. As a general check on the overall accuracy of the work it may be stated that the average centre camber of thirty trusses when carrying dead load was 2·997 inches. The maximum individual camber was 5 per cent. greater than this, while the minimum was 10 per cent. less. Since the camber was determined entirely by fairing-up holes already drilled and not by adjusting the lengths of members to suit the camber-jacks, the result must be considered as extremely satisfactory. It is clear, however, that in the case of individual members the aggregate tolerances are large enough to explain the differences between the deformation stresses imposed during erection when putting in camber and those resulting, after the joints have been made, in taking out camber.

With regard to the top-chord deformation stresses imposed during erection, certain discrepancies are undoubtedly due to the positioning of the chord-splices directly over the panel-points. The erection of

TABLE III.

Item.	Tolerance: inch.	Limit of error in position of centre of hole: inch.
Fabrication.		
Setting out templates	To nearest $\frac{1}{64}$	$\frac{8}{1,000}$
Hard steel bushes—		
Concentricity	$\pm\frac{5}{1,000}$	$\frac{5}{1,000}$
Diameter of hole (new)	$+\frac{5}{1,000}$	$\frac{5}{1,000}$
Increase of diameter by wear . . .	$+\frac{5}{1,000}$	
Diameter of drill	$-\frac{3}{1,000}$	$\frac{1\frac{1}{2}}{1,000}$
		$\frac{19\frac{1}{2}}{1,000}$
Erection. (Excluding deformation of holes.)		
Hard steel drifts, diameter, new . . .	$-\frac{5}{1,000}$	$\frac{2\frac{1}{2}}{1,000}$
Loss of diameter by wear	$-\frac{10}{1,000}$	$\frac{5}{1,000}$
		$\frac{7\frac{1}{2}}{1,000}$
Total variation of centre of hole . . .		$\frac{27}{1,000}$
Total variation of matching holes . . .		$\frac{54}{1,000}$
Total maximum variation in distance between rivet-groups after drifting		$\frac{108}{1,000}$ or say $\frac{3}{32}$
Machined butt-joints.		
Maximum feeler at any point not greater than	$\frac{8}{1,000}$	

the chord was started from U_8 and proceeded symmetrically towards each end. The U_6 and U_{10} splices could not, however, be fully made until the chord between these points had been cambered. Panels U_6U_4 and $U_{10}U_{12}$ were next added and cambered piecemeal. The resulting shape of the chord, therefore, differed from the smooth curve assumed in the calculations and given by the model, but was bent to an excess curvature at U_8 and was almost straight over U_6 and U_{10}. If splices were to be positioned clear of the panel-points the

whole chord could be supported on brackets and fully riveted straight before cambering, and the desired shape could thereby be obtained. It is common practice to rivet on batten-plates at splices across the open sides of chord sections. If too near the panel-points, however, such plates must be left off to permit the subsequent entry of the web-members, and it becomes necessary to delay cambering the chord until these plates are in place and the splices are properly made. In the present case initial stress was lost in the bottom chord in panels L_2L_3, L_5L_6, and L_7L_8 from this cause (Fig. 5, Plate 1).

Turning to the no-load deformation stresses in the verticals (Figs. 7, Plate 1), and in the diagonals (Figs. 8, Plate 1), it will be seen that there is a general tendency for the stresses imposed on erection to be substantially below the stresses required, and this deficiency constitutes the chief problem in this method of pre-stressing. The web-members in this case were of substantial section and were evidently too stiff to deform to the required extent by operating on rivet-groups at their ends.

In the case of members deformed in single curvature such as U_2L_2, U_2M_3, etc., the difficulty is more easy to overcome, but in the case of members deformed in double curvature such as U_4L_4, U_6L_6, U_4M_5, the deforming end-moments are high, and, as will be seen, these members show the greatest discrepancies. Discrepancies in the web-members are unfortunately not confined to themselves but produce similar disagreements in the chords to which they are attached.

An improvement might be effected by employing special straining devices during erection, but it is argued that a structure whose efficiency depends upon the extent to which such devices have been used during erection would be of uncertain value, and it would be preferable to devise a method whereby the desired results could be obtained by straightforward erection.

So far as can be seen from these investigations, the riveted intersections once closed remain substantially fixed and the usual assumption concerning the fixity of such joints appears correct. In order, however, to obtain satisfactory results for any system of pre-stressing it is essential that the fixing at the joints should be of great accuracy.

As already stated, it is estimated that the maximum allowable error in rivet-holes should not exceed $\frac{1}{200}$ inch if deformation stresses are to be controlled to within about 0·2 ton per square inch. With work of the greatest accuracy, if members are assembled under stress (that is, if the joints are made while the members are bent by drifting), the error cannot well be less than about $\frac{1}{32}$ inch per rivet-hole, and may be more. If, on the other hand, members are under little or no distortion when joints are made, it is probable that the resulting

position will be the mean of all the rivets in the group, and a high degree of accuracy may be expected.

In the erection of a truss by normal pre-stressing methods, the chords are riveted when straight and the deformation stress is put in by allowing the complete chords to fall into the cambered position under their own weight. A high degree of accuracy may therefore be expected in these members. This method cannot, however, be adopted for the web-members, and since it is essential that such members should definitely be pre-stressed during erection, such stressing must be done by methods other than that of attempting to bend the members by driving the drifts in compact groups of holes at their ends.

For dealing with these members it is therefore suggested that their stiffness should be materially reduced until their ends have been drifted and riveted. This result could be attained if the members were rotated through 90 degrees about their own axes to bring the lacing-bars into the plane of the truss. During erection lacing-bars, or the equivalent in batten-plates, would be loosely attached by small bolts, and each member would be assembled into the truss as two virtually separate flanges of very small lateral stiffness.

After making good the end attachments under little or no stress, the flanges would be connected by drifting up the "lacing" systems, pre-drilled as for straight members. It is clear that by driving drifts in this way along the whole length of the member a very much more powerful deforming agency would be obtained than that of attempting to operate on a compact group of rivets at each end. Alternatively, the riveting of the lacing-systems could be left until a full design load had been applied to the span. The members, including the chords, must then be free of deformation stresses, and the operation is simple to supervise. The above alteration need not necessarily involve a reduction in the areas at the ends available for rivet-holes, or a reduction in the lateral stiffness of web members.

A few experiments were carried out on typical laced members to assess their rigidity when deformed into single or double curvature. The members were supported in a horizontal position and loaded to produce the type of deformation required. Similar tests were also carried out on a scale model of one of the Nerbudda web-diagonals (actually U_2L_4, which is the heaviest in the truss), and were compared with a model of equivalent theoretical moment of inertia but with a solid web.

The ratios of stiffness of the members with laced webs to the stiffness usually assumed in calculation were found to be as given in Table IV.

The loss of stiffness due to lacing is considerable, and it seems clear

TABLE IV.

Member. (Single-laced webs.)			Ratio— $\frac{\text{Stiffness in plane of lacing}}{\text{Stiffness normally assumed}}$: per cent.	Remarks.
No.	Flange area: square inches.	Effective depth: inches.		
1	2·88	18·00	82	Single curvature.
2	2·88	16·34	69	,, ,,
3	8·69	10·49	90 twenty panels lacing	,, ,,
			Loading 78, Unloading 83 } twenty panels lacing	Double curvature.
			First loading 45, Second loading 75 } ten panels lacing	,, ,,
			First unloading 45, Second unloading 49 } ten panels lacing	,, ,,
4	Model of web-member U_2L_4 37·5	22·06	72 } Five panels lacing	Single curvature.
			61 }	Double curvature.

that to rotate web-members in a truss would serve the double purpose of increasing the transverse stiffness of the span whilst facilitating the pre-stressing.

In *Fig. 18* are shown the cross-section of a typical web-member (*a*) as normally designed and erected, and (*b*) as proposed for future work. The calculated moments of inertia about the axis at right angles to the plane of the lacing-bars have been multiplied by a factor as suggested by the experiments referred to above. The true I_{xx} of (*b*) is then considerably higher than that of (*a*), while the I_{yy} for each flange separately in (*b*)—which is the measure of the resistance to be overcome in pre-stressing—has been reduced to a negligible figure.

As an example, let it be assumed that a member is to be pre-stressed to bend it into uniform curvature in order to produce stresses of $\pm$ 2 tons per square inch in the flanges. Then if the two flanges are erected free of each other or only loosely attached, the ends will be drifted to the required slopes with little resistance, and in order to avoid axial stress the flanges will adopt a shape somewhat as shown in full lines in Fig. 19 (Plate 3). In order to impose the desired stress of 2 tons per square inch in the flanges it will be necessary for the lacing-bars, when inserted, to load the flanges radially so as to produce the required hoop stress. The mean load per foot of member will be quite small. Assuming that the sectional area of

each flange is 40 square inches and that the member is 24 inches deep in the plane of the paper, then the hoop load would be about $2 \times 40 = 80$ tons, and the mean radial load about $\frac{1}{75}$ ton per foot of length. Under full-load conditions the member would be straight and the lacing-system would be nominally free of stress.

In the case of a member bent in double curvature the form of

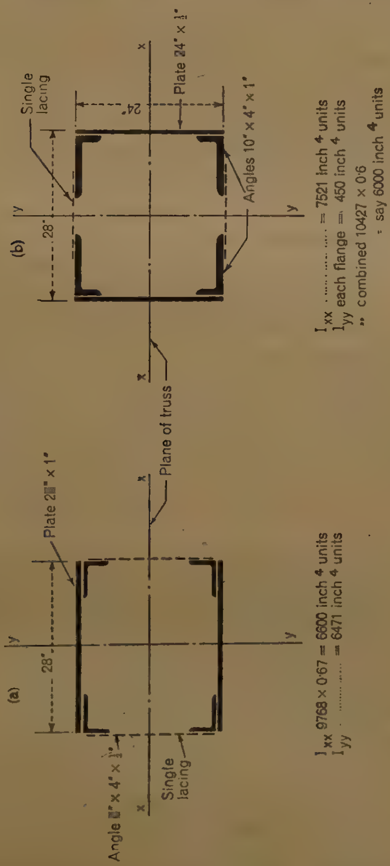


Fig. 18.

deformation in the ideal case would be as represented in Fig. 20 (Plate 3). Both flanges would remain free of axial stress until the lacing-system was inserted. Then if the stress in the flanges is required to be $\pm$ 2 tons per square inch, the stress in the lacing-bars would be $2 \times 2 \times \sin^2\phi$ tons per square inch, where ϕ is the inclination of the lacing-bars to the normal. If the angle ϕ were 30 degrees, as is common, the stress in the lacing-bars would be 1 ton per square inch.

It is clear that in short members, where the lacing-bars were few, they would impose under this form of deformation only small flange stresses. Further, the strain or stress in them would not vary appreciably with changes in their sectional area. As the length of the member increased and additional lacing-bars came into operation, the flange stresses under bending would increase and the stiffness of the member as a whole would approximate more closely to the theoretical value.

With regard to the initial deformation stresses in the sub-verticals (Figs. 9, Plate 1), and in the sub-diagonals (Figs. 10, Plate 1), it will be seen that the stresses imposed during erection varied considerably from those either calculated or given by the model, and it is clear that the imposition of the initial axial stresses—1·42 ton per square inch in the case of M_1L_2—into these members composed of thin scantlings damaged the rivet holes and destroyed the accuracy of the riveted attachments. In these members the gross stresses are low and the results are of little actual importance, but they serve to indicate the difficulty of combining light and heavy members in the same truss, and the extent to which the light members must conform to the requirements of the heavier members. All the sub-diagonals, and all but one of the sub-verticals, deform actually in double curvature, whereas theoretically nearly all should deform in single curvature. The discrepancy must be due to the departure of the bottom chord from the straight under full-load conditions.

Turning again to Figs. 5 to 10 (Plate 1), it will be seen that three lines have again been plotted for dead-load deformation stresses. These lines are of some interest since they show the effect on the dead-load deformation stresses of applying and removing the full design load with and without oscillating the span.

Since the loss of camber during the first application of the full load would suggest a joint-slip at certain panel-points, it would be expected that a slip in an axial direction would be accompanied by a rotational slip, which must have materially changed the bending stresses in the members concerned. Further, if such slip occurred under full load and the pre-stressing was in defect to begin with—as these readings indicate that it was—any rotational slip must have tended to increase the deformation stresses when the span returned to dead-load conditions. Since camber was permanently lost to the extent of 0·42 inch the span did not return to its initial dead-load camber, a factor operating to reduce the dead-load deformation effects. Plastic strain under full-load conditions would tend to operate in a manner similar to rotational slip and would show an increase of deformation stress under dead-load conditions. The

differences shown on Figs. 5 to 10 (Plate 1) are, however, in all cases small and probably within the limits of instrumental accuracy, and it would appear that no rotational slip took place.

The effect of oscillation was, in general, to tend to restore members to their initial condition. The extent of the re-adjustment was appreciable, although a period of about 10 days elapsed between the full-load readings and the oscillation, while the load was being gradually removed. It would appear that internal friction, such as that responsible for vibrational damping, would account for this difference.

On Figs. 5 to 10 (Plate 1) the deformation stresses remaining in the various members of the truss under full design load have also been shown. For perfect pre-stressing these would have been very close to zero under a uniformly-distributed load. Actually, under conditions of dead load only (say one-third total load), a very fair approximation to zero deformation stresses was obtained in the web-members and top chord. This standard is, however, of considerable value as indicating that the desired results are within reach. It is clear that there must always be some "lost motion" when fairing up rivet-holes in members under stress, and the solution must be either to relieve the member temporarily of stress or to estimate suitable allowances to be made when positioning holes—by no means an easy thing to do.

Secondary Stresses.

To distinguish them from the deformation stresses in the truss mem bers which result from vertical deflexions, the stresses arising from the attachment of the lateral systems are here referred to as secondary stresses. These stresses, as measured under full-load conditions, in the top and bottom chords have been plotted in Figs. 21 (Plate 3). The load was distributed uniformly across the whole span and increments due to wind, etc., were absent. The readings plotted represent the means of the right- and left-hand ends of the west truss of span No. 4.

The diagram for the top chord suggests little induced stress. Panel-point 6 appears to have been pushed out slightly, owing probably to the continuity of the K-type lateral bracing over the two centre panels. The relief of stress afforded in these two panels, as stated earlier, amounted to about 5 per cent., and the movement at U_6 is probably due to the resultant of the diagonal thrust in the bracings. The distortion at U_2 is more pronounced, and there is no feature in the lateral system and portal which could account for it. It must be assumed that the additional metal in the substantial

lateral gussets attached to the inside face of the chord at this intersection was responsible for a reduction of compressive stress on the inside face, and possibly, too, imperfections in the butting of the mitred hip-joint may have contributed to the distortion.

In the case of the bottom chord a double-diagonal lateral system has been provided, connected to the lower flanges of alternate cross-girders. In alternate sub-panels and at each main panel-point expansion joints have been provided in the stringers. The bottom chord therefore obtains a certain relief of primary stress from the laterals (about 4·8 per cent. in the centre of the span and about 10 per cent. near the ends), and a certain amount of outward secondary bending at each sub-panel point.

The continuity of the stringers is broken by the expansion joints, but 0, 1 and 2 ; 3 and 4 ; 5 and 6 ; 7, 8 and 9 are rigidly connected. The effect of the small lateral deflexions resulting from these connexions upon the lateral bending stresses on the bottom chord is shown in Figs. 21 (Plate 3) ; for example, the centres of the cross-girders at L_0 and L_2 have been deflected towards each other, inducing the stresses shown.

The movements recorded at the expansion joints of the stringers when a full design live load was applied to the span were as follows, the figures being averages of four readings :—

L_2	. . .	$\frac{1}{16}$ inch.
L_4	. . .	$\frac{3}{32}$,,
L_6	. . .	$\frac{7}{64}$,,

These represent about two-thirds of the lower-chord strains in the panels in question and make the lateral bending of individual cross-girders from one-half to one-quarter of these amounts. In Figs. 22 (Plate 2) are shown the lateral bending stresses induced in the web-members by the cross-girder attachments for a full design load covering the whole span.

In the case of the sub-verticals the inside flange carries practically the whole of the cross-girder load at the bottom, but at the top the stress-distribution becomes evened out, partly by a transfer of stress through the lacing-bars, and partly by a twisting of the diagonal to which it is attached. Indications of twisting are given at the top of sub-diagonal M_7L_6, where a compressive stress has been induced on the inside flange by the corresponding tensile stress on the inside flange of the sub-vertical. The same applies at M_3.

In the main verticals the inequality of stress is much less pronounced, but this is due to the much heavier sections employed, and to the fact that these members are stressed chiefly by the shear loads on the truss as a whole. The fact remains, however, that these

members as struts do not carry the truly axial load which is assumed in theory.

It would appear that a definite measure of relief could be obtained in such cases if the ends of the cross-girders were finished with a slight outward inclination when under no load, such that they would become vertical when the girder deflected under a full load. The lower ends of the web-members would then be vertical in a transverse plane under full-load conditions and the rivets in the connexion would be relieved of tension.

The distribution of stress across certain cross-sections is shown in more detail in Fig. 23 (Plate 3) for sub-members M_1L_1 and M_1L_2. The points at which the stress has been measured are shown by dots. The indication is that, even in the simplest sections, the distribution of stress is far from uniform, and this must apply to an even greater extent near the end connexions of more complex members. In the examples shown one set of readings was taken under dead load only, a second set under a live load approaching the design load and a third set under dead load again after removing the live load.

Under the applied live load considerable local deformations can be seen in the flange angles resulting from the lacing-bar attachments, and after the removal of the load a substantial part of such deformations in many cases remains. This result appears rather surprising, since the general stress is considerably less than the elastic limit of the metal. It is probable that similar local deformations are at least in part responsible for the enhanced stresses recorded in other members of the truss under dead-load conditions after the application and removal of the full design load.

Conclusions.

A summary of the stress-reductions from deformation effects attained by the methods of pre-stressing on the Nerbudda spans is given in Table V for a few of the more highly-deformed members, together with the allowances made for impact in the design of these members.

For non-symmetrical members the above figures refer to the deformation stresses, which are of the same sign as the working primary stresses.

The displacement of the neutral axes in the top and bottom chords by the U-shape section in both cases operates to reduce the effective deformation stress. The bending of the chords to the general camber-curve produces in itself very small stresses. The value given above for the bottom chord is due chiefly to the sub-member attachments. Deformation stresses in the top-chord member

TABLE V.

Member.	Deformation stresses (measured).				Provision for impact :
	Without pre-stressing :		With pre-stressing :		
	tons per square inch.	per cent. of gross primary stress.	tons per square inch.	per cent. of gross primary stress.	tons per square inch
Top chord U_2U_4 . .	$-2\cdot96$	42	$-1\cdot5$	21	0·36
End raker . . .	$-2\cdot0$	31	$-1\cdot5$	24	0·36
Bottom chord L_5L_6	$+1\cdot17$	16	$+0\cdot85$	11	0·38
Web-vertical U_4L_4 .	$-2\cdot4$	55	$-1\cdot70$	39	0·42
Web-diagonal U_4L_6 .	$+3\cdot0$	42	$+1\cdot70$	24	0·49
Sub-vertical M_3L_3 .	$+2\cdot15$	50	$+1\cdot30$	30	1·84
Sub-diagonal . . .	$-1\cdot85$	175	$-1\cdot60$	153	1·28

U_2U_4 have been greatly increased by the contraflexure caused in this panel by the hip-joint. In the end raker the M_1 connexion is chiefly responsible for the high stresses recorded.

A substantial reduction of deformation stresses has been obtained in members throughout the truss, with the exception of the sub-members (in which the working stresses are low). The relative importance of the stresses still remaining is indicated by a comparison with the provision made in each member for impact effects. It is not unusual to devote considerable time in arriving at a reasonable provision for the latter, whilst the very much higher deformation stresses are ignored. As span-lengths or working stresses increase the percentage effect of impact must decrease, whilst for high working stresses the absolute value of deformation stresses must increase. The importance of pre-stressing must therefore be accentuated by the introduction of high-tensile steels. A problem will then arise as to how to distinguish a span which has been pre-stressed from one that has not.

Lateral bending-stresses which arise in the chords from the attachments of the floor and top lateral systems may be considerable, and it would appear that no better method of examining, and if possible of improving upon, present designs could be found than that based on the use of scale models.

PROPOSED MODIFICATIONS FOR PRE-STRESSING.

(a) *Non-Redundant Frames.*

Rigid-body statics afford sufficiently accurate prediction of stresses in members from which Williott diagrams can be drawn, and in

order to secure the required effect for pre-stressing the only modification proposed is to turn web-members so that lacing-bars lie in the plane of the truss. These lacing-bars in each web-member would only be riveted after the ends of each member had been riveted at intersections. In the case of light members, merely turning without leaving the lacing-bars loose should be all that is necessary. Further, a splice might be provided in the end rakers to close at the point of contraflexure. Pre-stressing should provide for dead plus live plus impact loads.

(*b*) *Frames with Redundant Members.*

Primary stresses cannot be obtained with any certainty from rigid-body statics. In these cases the use of models is recommended to determine primary stresses. When the primary stresses have been obtained, Williott diagrams can be drawn and the procedure would be as for non-redundant frames.

In order to obtain primary stresses from a model, it would appear that the following would be the most satisfactory procedure :—

(1) The model should be designed for a magnification of 50.
(2) The model should be constructed to geometrical shape ; that is, without camber.
(3) The model should be loaded upwards at the panel-points to produce the central camber as given by the Williott diagram, on the usual assumptions of stress distribution under full load. "Full load" should include a provision for impact.
(4) The actual change in length of each member should be scaled off and applied for pre-stressing.

(*c*) *Special Points.*

The top chord should be designed so that it can be assembled and riveted complete and straight before cambering. The bottom chord should be fully riveted before cambering. Batten-plates at splices, if too near the panel-points, must ordinarily be left off to enable the gussets to be jacked out to admit the web-members. If finally riveted after cambering, stress is inevitably lost ; the bottoms of web-members should therefore be inserted into the chord and riveted before the chord is cambered.

By turning the web-members so that their "flange" plates are in a transverse plane, it will become necessary to design the lacing-system for the method of erection in view. Lacing-systems permit far greater elastic deflexions than do web-plates, and for a given lateral deflexion the bending stresses in the flanges of a member will be

lower when the web is composed of a lacing-system than when it is composed of a solid plate, "shear" deflexion contributing a larger share. The rotation of the web-members should, therefore, with normal design, increase the transverse stiffness of the webs and should decrease the intensity of the erection stresses.

(*d*) *Spans Partially Covered by Load.*

Web-members carry their maximum primary and deformation stresses under partial loads. The rotation at the end of a web-member then exceeds the full-load allowance, which rotation also causes local deformation stresses in the chords. If a web-member is pre-stressed for partial-loading conditions (that is, so that it is straight under a partial load), it will not be straight under full-load conditions. This lack of straightness will cause an undesirable stress in the chord owing to a rotation of the joints as a whole, and at the time when the chord is carrying its maximum primary stress. While generally it would be preferable, in designing, to give preference to the chords (that is, for full-load conditions), the relative merits of either method could be quickly ascertained for any specific case by applying partial loading to a model and measuring the primary deformation stresses involved at the joint under investigation.

(*e*) *Hangers.*

Hangers, if they occur in a truss, present a certain difficulty. If it is required that the bottom chord should be straight under full-load conditions, then it is clear that the full-load extension of the hangers U_1L_1 (Fig. 24, Plate 3), plus the vertical drop of point U_1, should be equal to the versine of the arc $L_0L_1L_2$ when the truss is on camber-jacks.

If U_1L_1 is designed for a normal working-stress the full-load strain in it will be much greater than the required versine in the length of chord $L_0L_1L_2$. It is therefore necessary to lift L_1 up above its normal cambered position, causing an undesirable kink and heavy deformation stresses in the chord under all but fully-loaded conditions. When all the panel-points except L_1 are fully loaded, the chord is subjected to approximately its maximum primary stress plus the deformation stress due to the kink at L_1. This undesirable feature could be eliminated by increasing the cross-section of U_1L_1, thereby reducing its full-load strain, but this process could not prove economical and hangers are better avoided.

Sub-panels generally involve sub-hangers which cause difficulties of the same kind. Such members of necessity are of light section,

and the comparatively heavy drifting which is necessary to deform the heavy chord-sections is prone to tear the holes and to defeat the object of the pre-stressing.

It is possible that satisfactory results could be obtained if special steps were taken—such as the application of jacks or turnbuckles — to enable such members to be drifted into place without risk of damage, but it is submitted that such measures are undesirable and likely to be neglected, and it is preferable to design a frame which can be satisfactorily erected without special means and with an assurance that the required results will be obtained.

The employment of sub-panels only becomes necessary when the panel-length, which is related to the depth of the truss, becomes so large that the rail-bearers become inconveniently deep and heavy. The difficulty may, however, be overcome by changing from simple **N**-trusses to **K**-trusses when the panel-length reaches about 25 feet.

(*f*) *Camber.*

It is evident that, if a model is made to the theoretical no-camber profile and then loaded upwards with the scale full load, the resulting cambered shape will depend only upon the proportions of the members, and that, unless the camber is adhered to in the design, the chords will not be straight under full load. The no-load shape cannot therefore be chosen as a smooth curve to suit requirements of track-laying, but can only be determined after the section of each member has been decided upon, and can only then be modified by changing the section of appropriate members. In those bridges in which the track is laid on cross-sleepers supported directly on the rail-bearers, the initial cambered shape may be a matter of some importance; for instance, the lifting of the L_1 points above their normal position to accommodate the large strain of the hip-hangers in an **N**-truss becomes particularly inconvenient and affords a further argument for avoiding this form of construction. As an example, the L_1 points in a span of, say, 200 feet carrying no load may well be 1 inch above the L_0 points and possibly only 25 feet away from them. The track at this point would, therefore, run into a pronounced trough, or if this is packed level under no load, then over a pronounced hump under full load.

(*g*) *Welded Joints.*

It is clear that, whilst the fixity of welded joints might be considered to be more complete than that of riveted joints, a necessary factor will be the complete control of distortion. Whether welded joints are employed in a pre-stressed truss or otherwise, an undesirable

joint-distortion due to contraction will give rise to the same harmful stresses, and this aspect of welded frame-construction requires the most careful examination.

(*h*) *Floor-Systems.*

A critical examination of the usual floor-details leads to the conclusion that many really undesirable features have crept into modern practice which are wasteful of material and induce high and incalculable stresses.

In India the practice is to use open floors on all large spans, with timber cross-sleepers supported on stringers, carried in turn by cross-girders at the panel-points. The resulting track therefore varies in stiffness from cross-girder to cross-girder and from sleeper to sleeper. In order to minimize the impact effects of such variations, floor-members have been increased in depth and stiffness and consequently in cost, but the intrinsic faults in the make-up remain. The result of this has been to increase the weight of steel in the floor until the cross-sectional area of the stringers approximates in spans of moderate length to that of the chords themselves. But the metal in the stringers, owing to accepted details of construction, cannot be used to relieve the chords, since any attempt to do so must result in the introduction of secondary stresses into the chords of greater magnitude than the relief afforded. On the other hand, a partial and temporary solution has been found in the provision of expansion joints in the stringers at intervals along the span, as noted previously. The desire to reduce local deflexions in the floor has further led to the introduction of short sub-panels which produce in the main trusses deformation stresses which are uneconomical, and difficult to eliminate.

It may be mentioned also that the introduction of timber sleepers into a bridge constructed otherwise wholly of steel is in itself undesirable, and gives rise to a recurring revenue charge for maintenance. The running-rails should be treated as an integral, although a wearing, part of the steel structure.

The sole function of a railway-bridge floor is to carry rails and nothing else, and the design should therefore be considered working backwards from the rails, and not in the reverse direction from the truss. The ideal support for a rail is a continuous support, and the introduction of cross-sleepers is in itself a retrograde step. The necessity, however, of providing a support for derailed wheels cannot be overlooked, and for this reason only the support of the rail at a series of points on cross-members can be countenanced.

If, however, these cross-members transferred the load direct to the

main chords the chords would then combine the functions of the stringers and chords, with considerable simplification in detailing, whilst a closer approximation to a track of uniform stiffness would be obtained. Further, should these cross-members take the form of a continuous troughing of suitable section an adequate lateral system of bracing would be obtained, whilst eliminating secondary stresses both in the chords and in the web-verticals.

In this connexion a further point should be mentioned. The direct effect of using a transverse-trough floor is to transfer metal from the stringers to the chords supporting the troughing. This is a matter of considerable importance when considering single-track spans of 200 feet and upwards, since the concentration of metal in the chords increases the lateral stiffness of such spans. The development of "relief of stress" by the use of ordinary stringers would have the opposite effect and would introduce certain limiting conditions.

Rails attached direct to this troughing would lose the resilience afforded by timber sleepers, but the conditions of support of the troughing, namely, at the centres of the main chords, would afford an equal or greater relief from "harshness." The alternative arrangement of carrying the track on a ballasted floor, as frequently done in England, has the disadvantage that considerable dead load is added to the span, whilst the difficulty of avoiding corrosion is considerable.

Acknowledgements.

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The Paper is accompanied by twenty-three sheets of drawings and four photographs, from some of which Plates 1, 2 and 3, the Figures in the text, and the half-tone page-plate have been prepared, and by the following Appendix.

APPENDIX.

Design of Scale Model.

It is necessary to determine the proportions of the members of the model in order to ensure that the joint-movements, both of rotation and translation, in the model will each be exaggerated the same number of times—say M times. If the model is an exact replica of the full-size structure and the linear scale of the model is $\frac{1}{N}$, then the actual magnitude of the joint-movements will be $\frac{M}{N}$ full size—say R times full size. It is convenient that R should be made 1 or 2, so that model joint-movements are either full size or twice full size.

The similarity of behaviour between the model and the full-size structure will be obtained provided the following condition is fulfilled for all members:—

$$\frac{K}{K_m} = N.$$

The notation used is as follows:—

	Structure.	Model.
Let the radius of gyration of any member be denoted by	K	K_m
,, length of any member be denoted by . . .	L	l_m
,, sectional area of any member be denoted by .	A	a_m
,, moment of inertia of any member be denoted by	I	i_m
,, modulus of elasticity of material be denoted by .	E	E_m
,, load applied to any point	P	P_m

Let the linear scale $= \frac{L}{l_m}$ be denoted by N

,, magnification of the model be denoted by M

,, ratio of movement of any point in a model-member to movement of corresponding point in structure $= \frac{M}{N}$ be denoted by R

,, ratio (any member) $\frac{I}{i_m}$ be denoted by Q

Young's modulus, E, for a built-up member is taken as 92 per cent. of E on a small specimen for first loading and 98 per cent. for subsequent loadings.

Theory.

Consider a framework, abc (*Fig. 25*), to carry a load P at b and rigidly held at a and c. On applying the load P, suppose point b to move to b′; the vertical deflexion, Δ, of b in the direction of the load will depend on

(i) the stiffness of ab and bc against bending, and

(ii) the longitudinal strains of ab and bc.

The stiffness of ab and bc against bending $= \phi_1\left(\frac{I}{L^3}\right)$, and the longitudinal stiffness of ab and bc $= \phi_2\left(\frac{A}{L}\right)$.

The proportion of the load carried by bending to the load carried by direct stress will therefore depend on the ratio

$$\frac{\phi_1\left(\frac{I}{L^3}\right)}{\phi_2\left(\frac{A}{L}\right)}, \quad \text{or } \phi_3\left(\frac{I}{L^2A}\right),$$

$$= \phi_3\left(\frac{AK^2}{L^2A}\right)$$

$$= \phi_3\left(\frac{K^2}{L^2}\right), \text{ where } \phi_1, \phi_2 \text{ and } \phi_3 \text{ are functions.}$$

The same ratio must obtain in the case of the model, whence

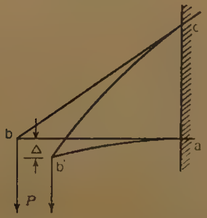
$$\frac{K^2}{L^2} = \frac{K^2_m}{l^2_m}$$

or

$$\frac{K^2}{K^2_m} = \frac{L^2}{l^2_m} = N^2 \quad \ldots\ldots\ldots \quad (1)$$

If, however, the model is cut from a sheet of material of uniform thickness, such as celluloid, the above relationship has no significance, since the forms of

Fig. 25.



the cross-sections of the model-members will in all cases differ from those of the prototype. Provided, however, that the above relationship between the transverse and longitudinal stiffness is maintained, the width of the model-members as cut out can be made any convenient dimension, and provided that this width d bears a fixed relationship to $\sqrt[3]{I}$, $\frac{I}{i_m}$ will be constant throughout. It has been found that convenient dimensions are obtained if $d = \frac{\sqrt[3]{I}}{110}$.

These proportions would give a correct relationship between the lateral stiffness of the members of a frame, but two important requirements would be lacking :—

(i) the cross-sectional areas of the members of the model would bear no relationship to those of the original, and

(ii) the elastic movements under applied loads would be too small to be detected easily, and no magnification would be obtained.

Both these requirements can be met by introducing into each member a magnifying device, the scale of which is related to the ratio $\frac{L}{A}$ of the corresponding original member. The effect of this device is to permit axial strains

in the model members far greater in magnitude than those which could occur normally, reproducing the whole axial strain throughout the length of the member at one point. (It should be noted that the axial strain in the material of the model is small.)

The error in the worst cases of box-members would rarely exceed 9 per cent. This will be referred to later. Thus, for any member

$$\frac{I}{i_m} = Q.$$

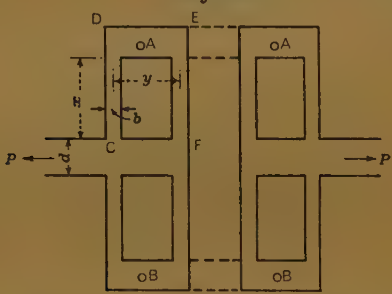
Then it is necessary that

$$\frac{\frac{A}{L}}{\frac{a_m}{l_m}} = \frac{\frac{I}{L^3}}{\frac{i_m}{l^3_m}} = \frac{I}{i_m} \times \frac{1}{N^3} = \frac{Q}{N^3}$$

Then $$a_m = \frac{AN^3}{Q} \times \frac{l_m}{L} = \frac{AN^2}{Q}. \qquad (2)$$

A convenient device which gives the magnification while preserving the stiffness is constructed as follows. Each member is cut between two inter-

Fig. 26.



section points and each cut end is provided with four identical "fixed-end" beams as indicated in *Fig. 26.* The ends are then overlapped and rigidly connected by screws through holes AA and BB. When an axial load P is applied to the member, the eight beams, such as CD and EF, will deflect and will permit a relatively large movement to occur. The portion DE is made relatively very heavy so that the ends D and E may without sensible error be considered fixed.

Let x denote the length of each of the eight beams,
,, b ,, ,, "depth" or width,
and ,, t ,, ,, thickness of the material from which the model is cut.

Then for a load P the deflexion of each half, considering points A and B as fixed, will be

$$\tfrac{1}{4}\left(\frac{Px^3}{12 \times E_m I_0}\right) = \frac{Px^3}{48 E_m I_0}, \text{ where } I_0 = \tfrac{1}{12} b^3 t,$$

and the total extension of the member with two such devices fixed at A and B will be :—

$$= \frac{2(Px^3)}{48 E_m I_0}$$

$$= \frac{Px^3}{24 E_m I_0}.$$

This must equal the theoretical extension of the member itself of area a_m, equation (2).

Then
$$\frac{Px^3}{24E_mI_0} = \frac{Pl_m}{a_mE_m},$$
or
$$x^3 = \frac{24I_0l_m}{a_m}$$
$$= \frac{24I_0l_mQ}{AN^2}.$$

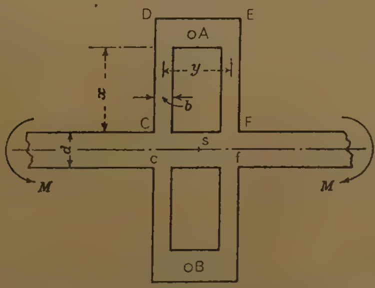
Now Q is constant for all members and
$$= \frac{AK^2}{\frac{1}{12}td^3},$$
and if d is made equal to $\frac{\sqrt[3]{I}}{110}$

then
$$Q = \frac{I \,.\, 110^3}{\frac{1}{12}tI}$$
$$= \frac{12 \times 110^3}{t},$$
and
$$x^3 = \frac{24I_0l_m \times 12 \times 110^3}{AN^2t}$$
$$= \frac{24 \times 12 \times 110^3}{N^2} \times \frac{I_0l_m}{tA}.$$

For any one model values of N, I_0 and t will be constant, and $x^3 = B\left(\frac{L}{A}\right)$, where B is a constant.

If, then, I_0 is fixed beforehand, as is most convenient, the length of the beams, x, can be at once determined.

Fig. 27

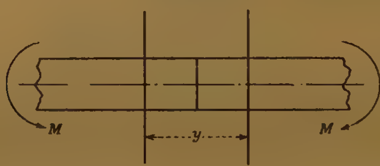


When using celluloid 0·06 inch thick, a convenient value of b was found to be 0·125 inch, though in some cases for very light members half this value is convenient. The distance apart of beams DC and EF (y in *Fig. 27*) requires to be fixed with reference to the dimension d and to the stiffness of the member.

It is convenient to maintain a standard dimension for y for all members throughout the model.

Since the two halves of the members are rigidly attached at A and B, a moment M tending to bend the member will cause no movement in DE. The moment in each half will, therefore, be resisted by DC and EF, acting as a strut and a tie, at a distance apart y. A moment M (*Fig. 28*) acting on a member of width d and of thickness t would produce a deflexion angle θ equal to $\frac{My}{2E_m i_m}$ when the length of the member under consideration is y.

Fig. 28.



In *Fig. 27*, the slope of the tangent at c distant $y/2$ from point s would therefore be $\theta/2$, and the movement of c in the direction DC would be

$$\frac{\theta y}{4},$$

$$\text{or } \frac{My^2}{8Ei_m} \text{ where } i_m = \frac{1}{12}td^3,$$

$$= \frac{3My^2}{2td^3E}.$$

Let the load in DC $\quad = \frac{1}{2} \cdot \frac{M}{y};$

then the strain is $\quad \frac{1}{2} \cdot \frac{M}{y} \times \frac{x}{E_m bt}.$

Then $\quad \frac{M}{2y} \cdot \frac{x}{E_m bt} \text{ must be } \ngtr \frac{3My^2}{2td^3E},$

or $\quad \frac{x}{yb} \ngtr \frac{3y^2}{d^3}$

or $\quad y \nless d\sqrt[3]{\frac{x}{3b}}.$

The ratio $\frac{x}{b}$ in practice will be usually about from 12 to 16,

whence $y \nless$ from $1{\cdot}6d$ to $1{\cdot}75d$, or say $y = 2{\cdot}0d$.

This reasoning will produce a joint in which there will be little or no alteration in curvature on account of the magnifying device, but it is strictly true only when the joint is in its unstrained position, that is, when CD and FE are straight. The error involved, however, when the member is under an axial load and CD and FE are bent is extremely small, and is well within the degree of accuracy demanded in evaluating deformation stresses.

If the linear scale of the model is made $\frac{1}{50}$ (that is, $N = 50$) and the magnifications of the joint movements required is 50 (that is $M = 50$), then all move-

ments in the model will be $\frac{M}{N}$ times the actual movement in the full-size structure. This scale has been found convenient for measurement, and is sufficiently large to be easily visible, so that the interaction of members can be viewed as a whole in a manner that is both striking and instructive.

Calibration of Model.

The relationship between the sectional area of a member of the full-size structure and the corresponding member of the model is (as has been shown already) given by the expression

$$a_m = A\left(\frac{N^2}{Q}\right).$$

A load P_m acting on a model-member will produce a strain $\frac{P_m l_m}{a_m E_m}$, which is required to be R times that of the original;

that is,
$$\frac{P_m l_m}{a_m E_m} = \frac{RPL}{AE}.$$

Hence
$$\frac{P}{P_m} = \frac{1}{R} \times \frac{A}{a_m} \times \frac{E}{E_m} \times \frac{l_m}{L}$$
$$= \frac{1}{R} \times \frac{Q}{N^2} \times \frac{E}{E_m} \times \frac{1}{N}$$
$$= \frac{Q}{RN^3} \cdot \frac{E}{E_m}.$$

Substituting typical values, such as $\left\{\begin{array}{l} N = 50 \\ R = 1 \\ \dfrac{E}{E_m} = 60 \end{array}\right.$

the result is $\frac{P}{P_m} = 76{,}660$,

or 1 lb. on the model is equivalent to 34·22 tons on the prototype.

Before proportioning a model it is necessary to consider in detail the make-up of each member and the type of end connexion, and to estimate approximate values for the moment of inertia, I, of each and the effective length, L. Lacing-bars on the flanges operate to increase the moment of inertia, whilst splice-plates may produce considerable local increases. Provision for such items can be made when calculating d, given by $\frac{\sqrt[3]{I}}{110}$.

It is also necessary that the length of the beams $x = 0{\cdot}794\frac{\sqrt{L}}{A}$ should be calculated on the effective lengths L, after a reduction has been made to allow for end gussets, splice-plates and lacing-bars. A further reduction of about 5 per cent. should be made generally to compensate for the longitudinal extension of the material in the model-members (see later).

The statement which follows (Table VI) gives the calculations to determine the main dimensions d and x for one of the trusses for the Nerbudda bridge. In cutting out the model the shape of the gussets at the intersection points may be reproduced on the linear scale to three-quarters their scale size.

TABLE VI.—CALCULATIONS FOR d AND x FOR MODEL OF NERBUDDA BRIDGE TRUSS.

Member.	L = length.		I = moment of inertia.		A = area of section.		$d = 0{\cdot}00909 \sqrt[3]{I}$ (I, effective): inch.	$x = 0{\cdot}794 \sqrt[3]{\frac{L}{A}}$ (L and A effective): inch.
	Theoretical: inches.	Effective: inches.	Theoretical: inch⁴ units	Effective: inch⁴ units	Theoretical: square inches.	Effective: square inches.		
Top chords :—								
U_2U_4 . . .	423	361	15,255·5	15,563·6	116·264	118·59	0·226	1·15
U_4U_6 . . .	423	372	17,353·9	17,701·0	144·14	172·968	0·237	1·024
U_6U_8 . . .	423	372	17,353·9	17,701·0	144·14	172·968	0·237	1·024
Bottom chords :—								
L_0L_1 . . .	211·5	162	11,340·27	11,907·27	76·26	80·06	0·206	1·00
L_1L_2 . . .	211·5	176·7	11,340·27	11,907·27	76·26	80·06	0·206	1·03
L_2L_3 . . .	211·5	186·0	11,340·27	11,907·27	76·26	80·06	0·206	1·05
L_3L_4 . . .	211·5	171·5	11,340·27	11,907·27	76·26	80·06	0·206	1·024
L_4L_5 . . .	211·5	176·7	15,436·00	16,207·80	113·76	119·45	0·230	0·90
L_5L_6 . . .	211·5	176·7	15,436·00	16,207·80	113·76	119·45	0·230	0·90
L_6L_7 . . .	211·5	178·5	16,388·00	17,207·00	140·00	147·00	0·235	0·845
L_7L_8 . . .	211·5	180·5	16,388·00	17,207·00	140·00	147·00	0·235	0·870
End rakers :—								
U_2M_1 . . .	319·89	270·5	15,255·5	15,723·1	116·264	119·75	0·228	1·04
M_1L_0 . . .	319·89	279·3	15,255·5	15,723·1	116·264	119·75	0·228	1·05
Verticals :—								
U_2L_2 . . .	480·00	410·00	558·56	614·41	33·748	37·122	0·0774	1·76
U_4L_4 . . .	480·00	419·00	4,615·00	5,076·00	50·1	55·11	0·156	1·555
U_6L_6 . . .	480·00	419·00	3,182·00	3,500·00	38·94	42·83	0·138	1·698
U_8L_8 . . .	480·00	419·00	983·00	1,081·00	23·06	25·36	0·0937	2·02
Diagonals :—								
U_2M_3 . . .	319·89	274·00	6,620·00	7,282·00	82·008	90·208	0·1765	1·15
M_3L_4 . . .	319·89	268·00	6,591·70	7,250·80	75·008	82·508	0·176	1·175
U_4M_5 . . .	319·89	274·00	3,056·60	3,362·20	57·25	62·97	0·1364	1·292
M_5L_6 . . .	319·89	268·00	3,035·20	3,038·70	52·00	57·20	0·136	1·322
U_6M_7 . . .	319·89	274·00	2,591·60	2,850·70	41·00	45·10	0·129	1·444
M_7L_8 . . .	319·89	268·00	2,591·60	2,850·70	41·00	45·10	0·129	1·438
Sub-verticals :—								
M_1L_1, M_3L_3, M_5L_5, M_7L_7	240·00	204·25	162·22	178·44	19·008	20·908	0·0512	1·698
Sub-diagonals :—								
M_1L_2, M_3L_2, M_5L_4, M_7L_6	319·89	277·00	284·76	313·23	18·00	19·8	0·0618	1·912

Effect of longitudinal strain in model members.

The beams of the expansion elements have been proportioned to represent a sectional area

$$a_m = \frac{AN^2}{Q}$$

$$= \frac{AN^2t}{12 \times (110)^3}.$$

The actual cross-sectional area of the celluloid is fixed by the dimension d and is

$$td = \frac{\sqrt[3]{I}}{110} \times t$$
$$= \frac{t}{110}\sqrt[3]{AK^2}.$$

Then the ratio

$$\frac{\text{beam stiffness}}{\text{sectional-area stiffness}} = \frac{\dfrac{AN^2t}{12 \times (110)^3}}{\dfrac{t}{110}\sqrt[3]{AK^2}}$$
$$= \frac{N^2}{12 \times (110)^2}\left(\frac{\sqrt[3]{A^3}}{\sqrt[3]{AK^2}}\right)$$
$$= \frac{2{,}500}{12 \times (110)^2}\left(\sqrt[3]{\frac{A^2}{K^2}}\right)$$
$$= 0{\cdot}0172\left(\sqrt[3]{\frac{A^2}{K^2}}\right).$$

Substituting values for typical members, the following results were obtained for the trusses of the Nerbudda bridge.

Top chord U_6U_8.
$A = 140$ square inches.
$K = 11$ inches.
Ratio $= 9{\cdot}4$ per cent.

End raker L_0M_1
$A = 115$ square inches.
$K = 13{\cdot}5$ inches.
Ratio $= 7{\cdot}2$ per cent.

Vertical U_4L_4.
$A = 49$ square inches.
$K = 9{\cdot}62$ inches.
Ratio $= 5{\cdot}1$ per cent.

The effect, therefore, of the longitudinal extension of the celluloid in the member as compared with the extension of the beams is small, and may be sufficiently adjusted by reducing the effective lengths of all members by 5 per cent.

It should be noted that the importance of the effect varies directly as N^2, and inversely as the square of the arbitrary factor 110, which controls the width d of the members (that is, as d^2).

Discussion.

The Chairman. The CHAIRMAN remarked that it was not often that an engineer had an opportunity of making a full-scale analysis of the magnitude of that referred to in the Paper, and the Author was to be congratulated on the use which he had made of that opportunity. An enormous amount of work had been carried out, both in the taking of the forty thousand readings and in their analysis, which most employers and most contractors would not have been willing to undertake. For that reason, The Institution welcomed the opportunity of publishing the Paper; the investigations which it recorded were undoubtedly in many respects unique.

Mr. Gribble. Mr. CONRAD GRIBBLE said that he had not gathered from the Paper itself to what extent the experiments and the method of erection had been adopted throughout the structure, which was one of fifteen spans; he now understood from the Author that all the spans were erected in precisely the same manner but that stress-readings were taken on only one truss. Was it reasonable to suppose that the pre-stressing had given similar results on the others? As the Author had pointed out, the fact that there were fifteen spans in the bridge had enabled the method to be used without undue expense, as the extra cost of manufacturing the bridge by jig-drilling (which was essential for the method) was small when spread over the whole structure. In England, on the other hand, single-span bridges were usual, and it might be that the extra cost of erection in the manner described, to say nothing of the cost of the records which were taken for a check, would far outweigh any economy in the design of the structure which might be obtained owing to the more accurate knowledge of the stresses.

In the early days of lattice-girders it had been the general practice to use pin-jointed trusses, because engineers in those days had been of opinion that they were more easily understood; but on account of cost and to obtain greater rigidity their use had diminished, and for bridges of the type in question they had been abandoned altogether. Engineers had had an uneasy feeling, therefore, that their ordinary stress-calculations gave only a very rough idea of the actual stresses in a truss. He had heard it said by those whose opinions were worthy of respect that in a particular member the value of the extreme stress on one corner of the member was of less importance than the average stress on the member. Although that view might

be fairly sound for tension members, he felt that it was a little optimistic in regard to struts, because if an initial bending stress were put into a strut its power of resistance would certainly be diminished. Mr. Gribble.

Secondary stresses had for many years been a bugbear to engineers designing bridges, and some most laborious calculations had been made. Ten or twelve years ago he had seen a paper, published in America, the author of which had made a colossal calculation of the secondary stresses in a truss, using about seven different methods, and some of the simultaneous equations were most remarkable, because one term might have a coefficient in which there were five figures to the left of the decimal point while the next term would have five noughts before arriving at a significant figure. With simultaneous equations of that order the prospect of arriving at accurate results seemed to be remote. The author in question, however, had obtained results, but the calculation was so lengthy a nature that such methods could never be adopted for general use. Because the matter was so difficult, engineers had been inclined to put it aside as not being vitally important, and they had been confirmed in their view by the fact that, after all, the many thousands of truss bridges which had been erected without any particular calculation of secondary stresses had served their purpose without any sign of failure, and many of them were actually bearing loads far heavier than those for which they had been designed.

The Author's experiment had been made with the object not so much of ascertaining the secondary stresses as of avoiding them, and had of necessity to be somewhat limited. The Author had had to concentrate on deformations in the plane of the truss, and explained that there was no way of eliminating the secondary stresses due to floor and top-lateral systems, suggesting that that part of the problem should be dealt with by omitting the usual type of floor altogether and using a transverse-trough floor, which would not produce any bending moments in the truss-members. That suggestion was presumably made because the Author recognized that it was impossible, in dealing with a bridge of the kind in question, to eliminate the secondary stresses in a plane at right angles to the truss.

On p. 91 the Author mentioned that the Indian authorities had agreed to the working stress in the girders being raised from 8 to 9 tons per square inch provided that steps were taken to limit, if not to eliminate, the deformation-stresses. Since that date, a British Standard Specification had been issued which permitted a stress of 9 tons per square inch without any such proviso. If trusses could safely be built with that working stress without any special provision in regard to secondary stresses, it appeared that a still higher working stress might be used if with confidence it was possible to build them

Mr. Gribble. applying the Author's methods in such a manner as to minimize those stresses.

On p. 117 the Author, in describing a method of pre-stressing web members, suggested that "the riveting of the lacing-systems could be left until a full design load had been applied to the span." That seemed to be quite feasible for tension members, but it was difficult to see how the riveting-up of a strut member could be deferred until the full load was on it; the unriveted strut would appear to be in a very unsatisfactory state to resist that full load. It had occurred to Mr. Gribble that if the method in question could be applied to spans much larger than those under consideration, it might be quite sufficient to pre-stress them for dead loads only and to ignore the live loads, as for longer spans the live load would become very much less in proportion to the dead load.

The Author had described his most interesting and valuable experiment without giving any very definite lead as to how far he considered that the procedure could economically be applied to bridge-work in general, or to what kinds of spans it might best be applied. It would be of interest if the Author would give his views on that point, showing what economy had been attained, and how far the saving on account of the increase of stress permitted had been offset by what must have been a very big increase in the cost of erection of the structure.

Mr. Fereday. Mr. H. J. Fereday observed that the first recorded measurements of the actual stresses in trusses had been made by Professor Frankel [1] in 1883, a few years after the publication of H. Manderla's celebrated thesis on secondary stresses.[2] In 1905 W. Gehler had conducted a series of tests and measurements on a Pratt-truss skew railway-bridge. Dr. D. B. Steinman, commenting [3] on these results, said: "The results, on the whole, afforded a valuable proof of the remarkably close agreement attainable between theoretical computation and actual conditions in a bridge structure." In 1917, Dr. Steinman had written a paper [4] entitled "Stress Measurements on the Hell Gate Arch Bridge." That huge structure had been used as an instrument of scientific research, the cost of which had been personally defrayed by the late Dr. Gustav Lindenthal as a contribution to engineering science. Notwithstanding those outstanding contributions, there was a paucity of experiment aiding the mathematical analysis of secondary stresses.

Whatever merit might be attributed to pre-stressing, Mr. Fereday

[1] *Der Civilingenieur*, 1883.
[2] *Allgemeine Bauzeitung*, 1880.
[3] Trans. Am. Soc. C.E., vol. lxxxii (1918), p. 1043.
[4] Trans. Am. Soc. C.E., vol. lxxxii (1918), p. 1040.

Mr. Fereday.

imself had been personally responsible for the introduction of its pplication to bridges in India. The subject of the present disussion should be the application, as distinct from the principle, of he pre-stressing of framed structures, as the principle was well nderstood. The Author had nowhere in his Paper given a definition f pre-stressing for framed structures. Stated broadly, it comprised he imposition of secondary stresses during erection equal and pposite in sign to those which would be induced in the structure y the loading. It should be mentioned that, for pre-stressing, the econdary stresses were not calculated. They were imposed nechanically during erection and eliminated by the loading for which he structure was pre-stressed. If success were to be achieved in he application of the principle, the fabrication, erection, drifting nd riveting would have to be carefully done. If the drifting were lone in such a way that rivet-holes were distorted, much of the value f pre-stressing would be lost. If the degree of success fell below he expected standard, that could only be due to non-compliance vith the necessary conditions, since no fault could be attributed to he principle itself.

The success of the erection of the Nerbudda bridge could have been ested by comparing readings on corresponding members of the two russes forming one span. It seemed a pity that, out of the 40,000 eadings taken on the bridge, none, apparently, had been taken on he opposite truss. The Author attributed the failure to realize a igher degree of secondary-stress elimination than that attained artly to the form of the truss and its necessary sub-members. Nevertheless, any form of truss, whether statically determinate or not, ould be effectively pre-stressed, and that statement was fully subtantiated by information published regarding the Sciotoville bridge n America.[1] Not only had that structure the same form of truss, ut it was a continuous structure which added to the complexity of he problem. Again, pre-stressing had been adopted on the Willinglon bridge [2] under the direction of Mr. L. H. Swain, M. Inst. C.E., nd other examples of its successful application could be cited.

Dealing with specific points in the Paper, on p. 91 it was implied hat the first proposal to raise the working stress in mild steel to tons per square inch, on condition that steps were taken to liminate or reduce secondary stresses, had been made by the Consulting Engineers in respect of the Nerbudda bridge in 1931; ctually, in 1925 the Consulting Engineers had suggested to the Railway Board that those conditions should be allowed.

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[1] Trans. Am. Soc. C.E., vol. lxxxv (1922), p. 910.

[2] Minutes of Proceedings Inst. C.E., vol. 235 (1932–33, Part I), p. 36.

Mr. Fereday.

On p. 94, it was stated: "It was specified that . . . the top chord was to be erected . . . each panel-length being cambered in turn as soon as the splices were made." It was in fact specified in the contract documents for the Nerbudda bridge that each top chord was to be erected horizontally, with the joints drifted and bolted, and then allowed to fall into its proper cambered shape. Steel brackets were provided to support those chords, and an examination of the top-chord details revealed that sufficient riveting could have been done to pre-stress those members properly. Incidentally, *Fig. 3*, p. 95, showed the top-chord distortions incorrectly.

There were many important statements in the Paper which were not clear to Mr. Fereday. For example, on p. 100, it was stated that "the zero differences were checked by three completely independent sets of readings made by different observers." That was a necessary precaution, but it did not appear to be stated whether the stress-readings on which the Paper was largely based had also been taken by independent observers, nor, if so, how the mean results had been derived.

Reference was made by the Author to the use of the Fereday-Palmer stress-recorder. Mr. Fereday did not suggest that it was an ideal instrument for the investigation in question, but he felt bound to say that the manner in which it was used in punched holes could not be expected to give satisfaction and would result in exasperation and tiredness for the operators. Accessories had now been designed and extensively used to enable the instrument to be used with confidence for the type of stress-recording described, which was of a difficult and exacting nature. He was afraid that to avoid their use on account of time or cost could result only in uncertainty and perhaps unfair criticism of the instrument, and in such circumstances some other type of instrument should be adopted.

On p. 102 it was stated that camber was lost by oscillating the structure. The loss of camber was partly explained by reference to p. 120, where it was stated that rivet-holes were damaged.

On pp. 103 and 104 it was stated: "These results pointed to a substantial permanent set in all members except the top chord . . ." That would means, from the usually understood meaning of "permanent set," that the stresses in all those members had exceeded the yield-stress, whereas elsewhere in the Paper it was stated, as would be expected, that the maximum stresses were everywhere well below that value and within the permissible limits.

Various values were given to the elastic modulus of the steel. The elastic modulus, however, was a definite mechanical property which was independent of the geometry of the frame.

On p. 112 the Author stated that "the pre-stressing was also

Mr. Fereday.

argely neutralized by the application of the dead load." If that statement meant that the secondary stresses were largely neutralized by the dead load, then the application of the live load, which was ıpproximately twice the dead load, should have produced, if the iveting were good, secondary stresses about twice the magnitude of hose originally induced by pre-stressing, and of opposite sign. Possibly either the wording of the Paper, or else the readings or heir interpretation, was at fault.

On p. 113 the following statement was made: "In the case of he member symmetrical in section, the same stresses, but of opposite ign, apply to the other flanges." That statement was contradicted on p. 123, where the Author remarked that "even in the simplest ections, the distribution of stress is far from uniform." Mr. Fereday assumed that "simplest" meant symmetrical, and that "uniform" meant linear. On p. 113, however, the Author assumed a linear relation for stress in the end raker, which was not a symmetrical section.

In conclusion, the failure to realize the desired amount of pre-stressing in the structure was not due to the type of truss but to he state of affairs mentioned on p. 120, where it was remarked that he imposition of stresses "damaged the rivet-holes and destroyed he accuracy of the riveted attachments." It was pertinent to ask whether the erection-staff were given, and made to follow, full instructions for the procedure to be observed. Unless he had been nstructed, the first impression that a riveter would get on seeing a eries of partially-matched rivet-holes would be that the fabrication vas at fault, but that somehow he had to get the rivets in position; ie would then proceed accordingly. It appeared that the rivet-ioles were damaged because the drifting had not been done with ufficient care. If the drifting and riveting were faulty, much of the oss of pre-stressing in the Nerbudda bridge could be attributed to hose causes. The specification provided for drifts to the extent f at least 40 per cent. of the number of field rivets to be driven.

Mr. Durant.

Mr. N. J. Durant remarked that the Author, having had difficulty n reconciling his calculated secondary stresses with those derived rom his experiments, appeared to consider that the only satisfactory vay to determine those stresses was either to experiment on the full-cale structure or to use a model constructed to satisfy the law of imilitude. The first method was costly and the second method, ıvolving similitude, was not very generally understood. If the uthor's contentions were correct, the determination of secondary tresses would be a rather hopeless task. Properly applied, however, he analytical method was capable of yielding accurate results with reater rapidity than either method recommended by the Author.

Mr. Durant. The analytical method given by Johnson, Bryan, and Turneaure, and referred to by the Author,[1] was suitable for didactic purposes, but it needed to be used with circumspection. Those interested in secondary-stress analyses might be referred to two excellent recent papers, one by Professor Hardy Cross [2] and the other by Professor R. V. Southwell.[3] Both those methods were independent of simultaneous equations.

The Author implied on p. 95 that secondary stresses could be eliminated for all conditions of loading, but it could hardly be believed that that was his intended meaning, as it was impossible.

On p. 125 the Author, referring to redundant frames, said: "Primary stresses cannot be obtained with any certainty from rigid-body statics." "Statics" would appear to be a better term. There would be general agreement with the literal interpretation of that statement, but if the Author meant, as the context seemed to show, that redundant frames could not be correctly analysed, Mr. Durant did not agree. Every rigid airship was a space frame whose order of redundancy was far in excess of that of any bridge structure. The conditions of loading presented a problem of analysis more difficult than that likely to confront the bridge engineer, and yet the factor of safety required was much lower than that demanded in bridge design specifications. Professor Southwell had evolved an elegant method of analysing such structures. Probably one of the finest examples of bridge engineering was the design of the towers of the George Washington bridge. Those towers were necessarily highly redundant frames, but Mr. L. S. Moisseiff had designed them so that the stresses in the legs were equal, notwithstanding the fact that the two applied loads of 112,000 tons were each eccentric by many feet. The designer's success might be gauged from the published description of the work.[4] Incidentally, Professor A. J. S. Pippard had given a method for the direct design of redundant frames.[5]

The Author's construction and use of a model were open to criticism. The Author recommended the use of models apparently for two purposes: firstly, to modify the prototype of a proposed design

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[1] Footnote 1, p. 92.

[2] Prof. Hardy Cross, "Analysis of Continuous Frames by Distributing Fixed-End Moments." Trans. Am. Soc. C.E., vol. 96 (1932), p. 1.

[3] Prof. R. V. Southwell, "Stress Calculation in Frameworks by the Method of Systematic Relaxation of Constraints." Proc. Roy. Soc. (A), vols. 151 and 153 (1935, 1936), pp. 56, 41.

[4] L. S. Moisseiff, "George Washington Bridge: Design of the Towers." Trans. Am. Soc. C.E., vol. 97 (1933), p. 164.

[5] Reports and Memoranda No. 793. Aeronautical Research Committee, London.

Mr. Durant.

and secondly, to evaluate the secondary stresses in the full-scale structure. The first recommendation was perhaps a good one, and, under the right conditions and with the right technique, a model might lead to results of considerable value when applied to a complicated structure. In a simple structure such as the Nerbudda bridge truss the use of a model did not appear to be necessary. The second recommendation could also be made effective, but with much greater difficulty, because in order to evaluate secondary stresses in that way the model had to satisfy the law of similitude more exactly than for the previous, or indeed any other, purpose. No single method was known by which all the factors influencing the problem could be simulated, and considerable judgment had to be exercised to give proper weight to the different similarity requirements.

Valuable results, however, could be and were obtained not by insistence on theoretical rigour but by the acceptance of a procedure that was partly based on similitude, justified by the fact that it led to the desired results. Probably the best-known method was that due to Professor G. E. Beggs. The Author, like Professor Beggs, made the width of a member in the model proportional to the cube root of the moment of inertia of the corresponding member of the prototype. That violated the law of similitude, but as absolute obedience to the law was not to be expected, it was necessary to violate some of the similarity-requirements in order to satisfy others having a greater bearing on the results. Professor Beggs used the model in quite a different way; briefly, his object was to find the effect at a point on the structure due to a measured displacement at a second point.

In the Appendix to the Paper there were certain relationships among various quantities, but, as the analysis proceeded, those values were arbitrarily changed. Again, as the Author was aware, the magnifying device was a source of error. The value given by Professors Coker and Filon for the elastic modulus of celluloid was 300,000 lbs. per square inch, whereas the Author's value was 500,000 lbs. per square inch, an increase of 66 per cent. Assuming the Author's analysis to be correct and his value of the elastic modulus to be in excess by the amount stated, then 1 lb. on the model was equivalent to 23 tons, and not 34 tons as stated. Professor J. F. Baker, in his researches for the Steel Structures Research Committee, had discarded celluloid because of its liability to creep.

As so many of the Author's conclusions were based on the results of experiments on his model, Mr. Durant had felt constrained to make the foregoing observations. In studying the Paper, he had tried to attain some sympathetic understanding of the Author's difficulties,

Mr. Durant. and had confined his remarks to what appeared to him to be major issues. It appeared to him that the greatest value of the Paper was that it directed attention to the necessity of a more general acceptance of modern research. The fault was not in the designer's command of analysis or of practical requirements but in the assumptions that he was often compelled to make. The need for a specification more general in character had been suggested by the Author.

The series of tests for static loading was one of the most comprehensive on record, and the engineering profession was indebted to the Author for carrying them out and to the Bombay, Baroda, and Central India Railway for making them possible.

Mr. Bateson. Mr. Ernest Bateson remarked that in *Fig. 3* the Author gave a diagram of the distorted structure. That diagram was of course exaggerated, but Mr. Bateson could not quite understand the severe kinks which occurred at the top-chord panel-points. He would have thought that the upper chord would be the most easily bent, as the top-flange covers could first be placed in position; then, as the chord was in compression with butt-joints, it could be allowed to fall into place, and the kinks shown in *Fig. 3* would have been avoided.

On pp. 96–100 the Author described his difficulties with regard to the determination of the zero readings on the various members. Mr. Bateson could well appreciate those difficulties; in his own experience in connexion with temperature-stress measurements on a structure it had been found that even when a drilling jig was used there were appreciable variations between different sets of plugs inserted by means of the same jig. Those variations, however, did not matter, so long as they were within the range of the instrument, but to obtain consistent results it had been found necessary to use properly-designed plugs with properly-shaped holes and to see that they were perfectly clean when the instrument was placed in position. He would imagine that it would be extremely difficult to obtain accurate readings with punch-marks such as the Author referred to, but at the same time he realized that the expense of inserting special points for a large number of positions would have been heavy.

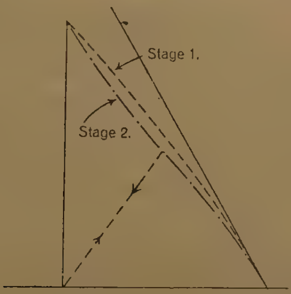
On p. 103 the Author referred to relief of stress due to the upper lateral system. As, in the truss in question, the upper lateral bracing was a K-system, it was difficult to see how such relief could occur except in the two middle bays to which the Author referred, where the bracing became a cross-brace system. At the bottom of the same page, the Author referred to a permanent increase in length of the members. He could not understand that statement, as the stresses in the members were certainly below the elastic limit of the

Mr. Bateson.

material, and, as the stress-recorders were placed outside the connexions, the question of slip did not arise. He would be glad if the Author could give some more information on that point.

The Author on p. 113 mentioned the difficulty of avoiding damage due to drifting in the fairing of holes. It seemed hardly reasonable to expect to be able to fair holes in such a case as that described without the use of some kind of straining device. In the case shown in the sketch (*Fig. 29*), it was arranged that two members did not come together at the top panel-point. It would therefore be necessary to deflect one member to bring the ends together (stage 1), but that would only result in fairing-up a few of the rivet-holes, and it would then be necessary further to deflect the member (stage 2) to fair up the remaining holes. That could be done by some very simple form of straining tackle.

Fig. 29.



On p. 114 the Author, referring to a member 40 feet long, stated hat an error of 0·01 inch in its length would cause an undesired axial tress of 0·28 ton per square inch. That would be true if each end f the member were rigidly attached to some immovable point, but n actual fact the whole of the structure was elastic; further some of he members might be an equivalent amount longer and some an quivalent amount shorter than their designed lengths, so that the tress set up would probably not be so great as that referred to.

On p. 117, discussing the action of lacing-bars, the Author suggested hat the members should be turned through 90 degrees to bring the cing-bars into the plane of the truss instead of at right angles to . In many practical cases, however, lacing-bars were attached by ne rivet only, and it would be very difficult indeed to distort the ember accurately by a single-riveted connexion.

With regard to the Author's remark on p. 125 that "Pre-stressing ould provide for dead plus live plus impact loads," Mr. Bateson lt that possibly a certain amount of impact should be provided for,

Mr. Bateson. but the present formula seemed to make an excessive allowance. Moreover, impact was a vibratory stress, the stresses in the members oscillating about a mean value due to dead plus live load and the amplitude of oscillation on each side of that value being the amount due to impact. Before accepting the Author's statement as it stood, it would be desirable to have more information about the effects of impact upon secondary stresses.

With regard to the possible use of welded joints, referred to by the Author on p. 127, Mr. Bateson imagined that it would be extremely difficult to pre-stress a welded structure which was erected piecemeal in the same way as a bridge, and which could not be completely welded in the shops or on the ground. Further, he could not see the possible advantage of welding in the case in question; the Author found that there was no rotation of the riveted joints, so that presumably they gave the required degree of fixity.

The suggestion that the ordinary type of bridge floor should be eliminated and that a transverse-trough floor should be used would seem to involve many practical difficulties, apart from the question of design. As a suggestion for eliminating secondary stresses in chords it seemed to be unsatisfactory, as the chord itself would have to function both as a chord and a stringer, and would be subjected to considerable bending stresses while carrying a live load. A practical difficulty would be the question of camber of the truss as affecting the track.

With regard to the cost of erection, Mr. Bateson did not think that the method of pre-stressing outlined by the Author need add very much to the cost of the bridge, but he thought that it would be essential to develop a technique in erection. It would not be good enough simply to send the steelwork along to the job and leave it to the ordinary kind of bridge-erector, who, if he saw holes which did not fair up, would simply blame the works and proceed to drift them into position by brute force. With the method of manufacture in question, the holes had to be drawn into place by proper methods.

Mr. Greet. Mr. E. H. Greet said, with regard to the relative merits of eliminating deformation-stresses by pre-stressing or of calculating them and including them in the total stresses, that there was one point which stood out clearly. The British Standard Specification for girder bridges had always, rightly or wrongly, specified that for spans below 200 feet and of normal proportions deformation-stresses might be ignored. Fortunately for the average bridge-designer, by far the greater number of bridges produced fell within that limit, so that a designer might go on for years in blissful indifference to, or ignorance of, the significance of deformation-stresses; but, when once he had been compelled to calculate them, even approximately, he was

nclined for ever after to shun them like the plague. To say that Mr. Greet. hey trebled the amount of work was to state the case very mildly, nd, in spite of the exhilaration which often accompanied the cademic solution of a different problem, engineers would welcome ny simpler alternative if they were convinced of its effectiveness. The present Paper showed an effort to provide such an alternative, vith the added advantage of a reduction in the weight of steel; and, n the main, the evidence of the experiments vindicated the theoretical nvestigation.

There were, however, a number of disturbing factors to which he vould like briefly to call attention. In the first place, in spite of the bviously careful supervision and the somewhat stringent tolerances ermitted in the positioning and fairing-up of the holes, as shown in Table III, the net result had been that rather less than half of the ery high deformation-stresses had finally been eliminated by pre-tressing. That was shown in Table V; the deformation-stresses emaining in various members ranged from 11 to 39 per cent. of the rimary stress, and in the sub-diagonal the residual stress was stated o be as high as 153 per cent. of the primary stress.

On p. 95, the Author said: " The contract for the manufacture of he steelwork was in due course let to a firm of contractors in India vho have specialized in the use of hard steel bushed jigs and who have leveloped this method of interchangeable manufacture to a high tate of excellence. Since there was every assurance that the ccuracy of manufacture would represent the highest standard that vas likely to be obtained at present. . . ." It was obvious that he value of pre-stressing would be governed very largely by the skill nd workmanship shown, and that the work would have to be most arefully inspected and supervised. Therefore, without deprecating he skill of contractors, he was inclined to wonder whether the method id not require a higher standard than that to which most contractors vere able to conform. It had also to be borne in mind that the igher the degree of accuracy demanded the fewer became the firms which the engineer could go for tenders for supply, and the less ompetitive became the tendering, with consequent increase of cost. dding that to the increased cost of the extra work, he imagined that ne saving of 9 per cent. of material indicated on p. 92 might be more han counterbalanced by the extra price per ton.

Could the Author state whether any bridges had been erected by ne pre-stressing method without stress-readings being taken, and, so, whether he had the same faith in those bridges, or whether he ad any knowledge of the magnitude of the residual deformation-resses? Before reading the Paper Mr. Greet had been puzzled to now how it was possible to avoid damage to rivet-holes consequent

Mr. Greet. on forcing members to bend by driving drifts into the compact groups of holes at the ends. That was made clear on pp. 117 to 120, but it seemed that damage did actually occur, in the sub-verticals and sub-diagonals at least, owing to the light members being compelled to conform to the requirements of the heavy members. The Author's suggestion that web-members should be of box section with the plane of the lacing parallel to the longitudinal axis of the girder seemed useful, though hardly adding to the appearance of the bridge.

In view of the very high deformation-stresses of the Nerbudda bridge, it would be of interest if the Author would say whether it was his opinion that a sub-divided truss of similar proportions but a much smaller span, say 150 feet, designed to the same working stress, would develop deformations of the same magnitude as the 282-foot span. It was a disturbing thought that the British Standard Specification might be at fault in permitting deformation-stresses to be ignored in short spans. If the magnitude of the deformation-stresses depended only on the type of truss and not at all on its length, the whole problem would require thorough investigation. Those designers who were not prepared to accept pre-stressing would then have two alternatives only; either the deformation-stresses would have to be calculated for each bridge, or a series of computations would have to be made for a number of types of truss to assess an all-round allowance for deformation-stresses for each one.

Mr. Needham. Mr. E. S. Needham said that the subject of the so-called secondary stresses had received much attention from the American Society of Civil Engineers, and some extracts from its publications were pertinent to the present discussion. Mr. O. H. Ammann had stated [1] that it remained an open question whether rigid gussets, producing large secondary stresses, were not a source of strength rather than of weakness, and had inclined to the view that they were a source of strength. Dr. D. B. Steinman had stated [2] that the actual secondary stresses would generally be lower than the calculated values, because there was an automatic readjustment of strains within a structure in such directions as to relieve the secondary stresses.

In a more recent paper Messrs. J. I. Parcel and E. B. Murer had stated,[3] from tests closely simulating the conditions for bridge-members, that "the ultimate strength is practically unaffected, even by high secondary stresses," and that "when the extreme fiber stresses reach the neighbourhood of the yield point, a radical

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[1] Trans. Am. Soc. C.E., vol. 89 (1926), p. 151.
[2] *Ibid.*, vol. 82 (1918), p. 1071.
[3] *Ibid.*, vol. 101 (1936), pp. 289, 323.

readjustment takes place, greatly relieving the flexural stresses." Mr. Needham.
As a result of the readjustment in stress—strain relations, they had drawn the following inferences :—(*a*) Any tension-member tended, as the load increased, to a condition of uniformity of stress over the cross-section, except for a local over-strain in a short section near each end. (*b*) Compression-members bent in single curvature might be seriously affected with regard to failure by buckling if the deflection due to secondary bending became large. That could occur, however, only in the case of large secondary moments and flexible members, a combination which was not ordinarily realizable. It had been noted that for values of $L/r \leqq 70$, the effect of secondary action in inducing failure by buckling was small. In such a case the rigid-joint action which gave rise to secondary stress acted as a brake on the deflection of the long column before the point of ultimate column-strength was reached. (*c*) For a column bent in double curvature, the secondary action, by forcing the curvature into two "waves," might actually have a beneficial effect with regard to failure by buckling. (*d*) For "stocky" columns, with values of $L/r \leqq$ about 40, which were ordinarily the only members that developed high secondary stresses, failure was nearly always due to local overstrain. Until the average stress approached the yield-point the transverse deflection was negligible, and column-failure in the ordinary sense could not occur. For such columns the secondary stresses, whether resulting in single or double curvature (the latter would almost invariably be the case for high secondary stresses), would merely result in high stresses on the compressive face, which were rapidly relieved by plastic flow of the material as the yield-point was approached. In conclusion, Messrs. Parcel and Murer made the following statements :—(i) The ultimate practical utilizable strength of a tension-member was not affected by any secondary-stress action within reasonable limits. (ii) Compression-members sufficiently flexible to develop failure by buckling before the yield-point was reached would usually exhibit too low a value for the secondary bending to reduce the ultimate carrying-capacity of the member materially. (iii) The rigid type of built column, in which alone high secondary stresses were to be expected, almost invariably failed by local over-stress, usually at a point at which local buckling could readily take place. High secondary stresses had no effect in hastening such local failure, and therefore had no effect in reducing the actual ultimate strength of such compression-members, provided that the proportions of the section were governed by the limits of present standard specifications.

Professor Hardy Cross wrote: [1] "At present secondary stresses are

[1] Trans. Am. Soc. C.E., vol. 101 (1936), p. 1365.

Mr. Needham. accepted as a necessary evil. Try to keep them within a reasonable figure, and otherwise forget about them. The idea of discounting secondary stresses is not new in America, but accurate data as to the amount that may be discounted are much needed."

Reverting to the Paper under discussion, which supplied the data needed, Table V was very instructive, but left some doubt as to whether pre-stressing was worth while, especially if the view were taken that the factor of safety on the utilizable strength of the bridge was not greatly improved thereby. Mr. Needham was fully in agreement with the necessity of investigating a design for probable deformation-stresses, especially to find out those compression-members which were distorted in single curvature. Compression-members bent in double curvature were affected only at the ends, where the area of section was usually sufficient, because of the low working stress used for the member as a column.

The Author stated that the displacement of the neutral axis in the top and bottom chords by the U-shaped section in both cases operated to reduce the effective deformation-stress. It was not clear why that was so. Referring to *Fig. 3*, p. 95, most of the top-chord members were bent in single curvature, and it would appear from the direction of bending that the bottom flange would suffer, being farthest from the neutral axis. A symmetrical chord-section without cover-plate would appear to be an improvement in the design, not only in regard to deformation-stresses but also, from his own experience, because of the difficulty of getting good rivets in the splices of the cover-plate.

One of the most interesting features of the Paper was the description of the model, which deserved a Paper to itself. Mechanical analysis was becoming more and more important as a time-saver in solving stresses due to joint-rotations, and was to be recommended when the technique was available. The analytical determination of deformation-stresses, however, could be made in a practical form by the method of moment-distribution in conjunction with the Williot diagram.

Mr. Moon. Mr. Ramsay Moon observed that on p. 127 the Author referred to the question of the use of welding in association with pre-stressing, or, perhaps it would be more correct to say, to the use of pre-stressing in a welded structure, suggesting that there might be some difficulty in combining the two techniques. He had discussed the matter at some length with the Author, and his remarks would be confined to that question. So far, there had been no attempt to combine the two techniques, and it was therefore not possible to discuss the matter on the basis of experience, but the known behaviour of welds and of welded joints suggested not only that the use of welding was possible but also that it might provide a means by which the technique

of pre-stressing might be used with a precision and certainty as to the distribution of stress which the use of riveting did not seem to afford. Mr. Moon.

In view of the fact that both welding and pre-stressing were still in the development stage and presented some degree of indeterminacy, it might be correct to suggest that to combine them at present would result in a second degree of indeterminacy which would be undesirable. Thus, until they were both developed more completely, it might be as well to regard the two techniques as rivals rather than as complementary. As the aim in each case was to effect reduction in weight, it might be suggested that more advantage was likely to accrue from the use of welding rather than of pre-stressing, since for pre-stressing a claim for a saving of about 10 per cent. was made, whereas a saving of from 15 to 20 per cent. might have been expected had the truss been designed for welded construction.

When a satisfactory arrangement was evolved for the joints of the welded truss, the application of pre-stressing would present very interesting possibilities. As pointed out in the Paper, there were two inherent difficulties in the riveting technique. Firstly, owing to the slip of the rivets which took place on the first application of the load, the distribution of the initial stresses was altered in a way which was variable and which could not be calculated. Secondly, it was difficult to put the stress into the members, particularly if they were stiff, though the Author made a useful suggestion in that respect. There was some danger of damaging the metal in the neighbourhood of the hole, and always considerable uncertainty as to the magnitude and the distribution of the stresses induced. The use of welding might provide a solution of both those difficulties. Firstly, a welded joint was essentially rigid, and no appreciable slip or change of stress-distribution was likely to take place on the first loading, so that once the stress was put in it remained. Secondly, the shrinkage of the weld-metal on cooling, to which the Author referred as a possible source of difficulty, might in fact provide the means for introducing stresses of any required magnitude into the members. Engineers who had had trouble with shrinkage and distortion of welded members might be inclined to laugh at that suggestion, but the technique of welded construction was gradually approaching a stage where accurate control would become possible. If the joints were of the type which was normal in riveted construction, with plates lapped, he saw no way in which welding could be satisfactorily applied or in which stresses could be effectively induced thereby; but it was certain that when trusses of the size and type in question were built by welding they would be built with butt-joints. Experiments had shown that the shrinkage across a weld was proportional to the sectional area of the weld. Thus the stress induced in a particular flange of a member could be controlled by

Mr. Moon. the design of the weld. If it were shown that too much stress or too little stress had been induced by a particular weld, the stress could be adjusted by adding additional runs of metal or by removing a certain amount of metal and re-welding. He would suggest, therefore, that when welding technique had been developed sufficiently, which was not the case at present, it might prove to be a valuable means of applying pre-stressing.

Mr. Thornton. Mr. D. LAUGHARNE THORNTON remarked that the Paper touched an important aspect of structural engineering, particularly in relation to the construction and design of large suspension-bridges with unstiffened trusses. Moreover, the process of pre-stressing described by the Author afforded a measure of control over the natural frequency of vibration of a structure, and was therefore of interest in connexion with bridges designed to withstand the consequences of prescribed hammer-blows produced by locomotives, since one effect of a partial constraint was that of raising the gravest frequency in particular.

In various parts of the Paper reference was made to apparent variations in the value for the direct modulus of elasticity concerned, which were enumerated in Table II. Those observations lent support to the view that the variations in question might advantageously be regarded as a measure of the "mechanical efficiency" of the bridge considered as a machine which was describing small displacements about a mean position. In view of the different sizes and types of joints associated with the structural system, the magnitude of that efficiency would be different for different members, and for certain parts of the bridge the value of the modulus was as likely to be slightly greater than 100 per cent. of E as it was to be less than that figure, since for structural members connected to a common joint the relative motion was probably executed in a series of jerks. That might also partly account for the observed loss of camber.

With reference to dynamic loading alone, the transverse beams mentioned on p. 108 and indicated in *Fig. 15* would act as "condensers" with respect to the stress-waves associated with the resulting oscillations of the model. Consequently, were it possible to take autographic records of the stresses acting on various parts of the model, the graphs would in general not be of the same shape or form as those taken on corresponding members of the actual bridge, which did not include such transverse beams. For corresponding parts of the model and bridge the difference in shape of the two graphs would in some respects resemble the effect of the phenomenon of interference in the case of complicated structures, in so far as some of the harmonic components of the stresses would on that account be "masked," and therefore extremely difficult to evaluate from records taken with the aid of the model.

The process of pre-stressing might involve the reversal of some or

all of the forces acting on the members of the bridge compared with the direction of the forces imposed on similar systems which were not constructed in accordance with the pre-stressing procedure. In those circumstances due consideration should be given to the disturbed motion of systems with reversed forces of a dynamical character. That arose from the fact that, in any dynamical system acted on by constraints independent of the time t and by forces that depended only on the configuration of the system, the integrals of the equations of motion remained real if $\sqrt{-1}t$ were substituted for the original symbol t and $-\sqrt{-1}v_1$, $-\sqrt{-1}v_2$, . . . $-\sqrt{-1}v_n$ in turn substituted for the initial velocities v_1, v_2, . . . v_n of the n points on the system under examination. The expressions thus obtained represented the motion which the same system would have if, with the same initial conditions, it were acted on by the same forces reversed in direction. It was therefore in general advisable to utilize a complex variable in the process of comparing the disturbed motions of two similar systems, one of which was acted on by reversed forces. Mr. Thornton.

The AUTHOR, in reply, wished to express his deep appreciation of the encouraging reception accorded to the Paper and of the very interesting and instructive discussion which it had provoked, even though its subject was not a "popular" one. There seemed to be little doubt that a fuller knowledge of the internal stresses in the members of a truss when carrying its design load would make possible economies of materials which could not be ignored. The Author.

The elimination of secondary and deformation stresses, as Mr. Gribble had suggested, would undoubtedly in time lead to the general acceptance of higher working-stresses for design purposes, but before that could come about it would be necessary that a method of design and erection should become general that would ensure, even under ordinary conditions of moderately careful erection, that the desired results would always be attained. The method put forward, namely, that of rotating the web-members to bring the lacing-bars into the plane of the truss and riveting them up when under load, was one which did ensure that effectiveness, and the operations involved could be under the complete control of supervising staff. Further, a truss erected in that way and free from serious deformation-stress could always be recognized as such, a matter of some importance in ascertaining the carrying-capacity of girders after they had been in use for some years. In the case of trusses composed of light members the mere rotation of the members without leaving the lacing-bars loose should be all that was necessary to ensure effective pre-stressing. With such members a double advantage was obtained, in that the members were made more flexible in the plane of the truss and easier to deform by drifting at the ends, and also that, should

The Author. the resulting pre-stressing be in defect, subsequent distortion in th same plane would produce smaller stresses than would occur if th webs were solid. Mr. Bateson's objection to dealing with single riveted lacing-bars would not arise in such cases.

Mr. Greet had asked whether there was any justification in dis tingishing between spans of less than or more than 200 feet. I was clear that, provided the same working-stresses were used, joint rotations of the same magnitude would occur in spans of simila geometrical outline regardless of the span. The intensity of deforma tion-stresses would then be directly dependent on the cross-sectiona dimensions of the members. In short spans those dimensions woul normally be less than in longer spans, but the intensities of loadin for which they were designed might counteract that effect. Thos factors, together with numberless variations in details, would mak any attempt to standardize allowances for particular types of trus little more than guess-work.

Mr. Bateson's question with regard to impact would appear t be answered at the top of p. 107. If the span were to be pre-stresse to eliminate deformation-stresses for a load covering the whole spa then the object sought would be to reduce to zero the deformatio stresses when the span was deflected by a load equivalent to the tot of the dead, live, and impact loads.

The important feature of the proposed method of eliminatin deformation-stresses outlined in the Paper was the fact that b loading the span it would be ensured that all members were bein effectively dealt with; as observed by Mr. Greet, that could not b guaranteed by methods based on the use of drifts or on strainin devices, and Mr. Fereday's statement that "other examples of th successful application of pre-stressing could be cited" could no carry great conviction in the absence of test results. So far as th Author was aware, no strain-measurements had been carried o on the bridges referred to by Mr. Fereday, with the exception of th Willingdon bridge, in which case, owing to special circumstanc obtaining at the site, a series of readings at one cross-section on of a bottom chord had been obtained. Further, he understo that on the Willingdon bridge the chords only had been dea with, by being provided with square butt-joints and cambered aft riveting.

The cambering of the top chords required some consideratio since if the splices occurred at the panel-points the chords cou not be fully riveted in the "straight" position, as it was necessa to drop each splice on to the top of the post before all rivet-hol could be filled. That point was mentioned on p. 125, and he cou assure Mr. Fereday that nothing short of a completely-riveted spli was good enough before cambering, if stress were not to be lo

The Author.

The specification as quoted on p. 94 was worked to in the erection, for the reason given above.

The shape of the top chord in its distorted shape, which had been queried by Mr. Fereday and others, was correctly shown in *Fig. 3*, p. 95. That diagram, with the conventional exaggerations, showed the top chord when correctly pre-stressed to be in tension throughout on the top flange except near the hip joint, and that distribution was confirmed by calculation, by actual measurement, and by the model.

Mr. Needham had asked why the U-shape of the top and bottom chords operated to reduce the effective deformation-stresses. The reason was that the neutral axis was nearer the bottom flange in the case of the tension (bottom) chord, and deflexion causing a sag at the centre of the span would cause a reduced tension in the bottom flange but a larger compression in the top flange. The compressive stress was, however, of little importance, since it was neutralized by the primary tensile stress. The reverse occurred in the case of the top (compression) chord, where the U-section was inverted.

He appreciated Mr. Fereday's remarks as to the value of taking stress-readings on more than one truss, but had the time, money, and energy been available for more readings he would have been more inclined to double the number of readings on the test-truss, before taking spot readings on other trusses, since in his opinion more valuable information would thereby have been obtained.

All stress-readings and all work connected with the tests had been carried out by the Author's staff and not, as Sir Clement Hindley had suggested, by the Contractors' staff. Extra work required of the Contractor, such as the erection of a truss in the prone position, had been paid for as an extra.

He did not contend, as Mr. Fereday appeared to believe he did, that statically-indeterminate structures could not be effectively pre-stressed. The Paper pointed out certain difficulties which had been met with in practice on the Nerbudda trusses: on p. 125 proposals were put forward to assist in overcoming those difficulties in the case of redundant frames. Again, Mr. Fereday took exception to the statement made on p. 112 that "the pre-stressing was also largely neutralized by the application of the dead load." That statement was intended to be taken literally, and an examination of Figs. 5 to 10 (Plate 1) would substantiate it; Mr. Greet, in his remarks, appeared to have interpreted it correctly.

The statement on p. 123 should also be taken literally, and not necessarily as interpreted by Mr. Fereday. By a "simple" section the Author meant a section composed of few scantlings and not necessarily symmetrical; and by "uniform" the Author meant a uniform distribution across a section of a member subjected to an axial load.

The Author. Mr. Gribble raised the interesting question of eliminating secondary stresses resulting from the floor-system as usually designed. There was no doubt that substantial relief could be obtained from lateral distortions of the bottom chords (in a through truss) and cross girders by the expedient of putting a full design load on a span before finally riveting the floor-members and bottom lateral bracing. Relief to the web verticals might be obtained as stated on p. 123 by inclining slightly the ends of the cross girders, but if those steps were to be put into effect there would still remain a duplication of metal in the stringers and bottom chords as referred to on p. 128. A more satisfactory and more economical solution would evidently be to combine the duties of stringer and bottom chord in the same member; Mr. Bateson's objection to doing so could not easily be understood. It had to be remembered that, whatever undesirable stresses might remain after making that change, under conditions of partial or concentrated loading, the same stresses would also be present though probably to a larger degree in a truss as designed at present. The Author was satisfied that if Mr. Bateson cared to examine the camber of a truss as a specific case he would find that the shape of the bottom chord was improved rather than otherwise. The Indian authorities had already accepted a cambered track on an unloaded truss as satisfactory, provided the camber did not exceed 1 inch per 100 feet of span.

With regard to the top-chord lateral system, if a K-system were used the secondary stresses induced (as noted on pp. 105 and 121) should always be small, except perhaps at the centre of the truss, where continuity over the two centre panels would occur unless the Ks pointed away from the centre.

He was at a loss to understand how Mr. Durant could have so misread the statements made on p. 95 as to have deduced that it was the Author's contention that a span could be free from secondary stresses under all conditions of loading. The further reference to this question made on p. 126 discussed the difficulties to be dealt with and the alternatives available. It would appear to the Author that the methods of obtaining "automatic" pre-stressing put forward in the Paper could with advantage be applied to all types and dimensions of truss spans, and, in fact, wherever pre-stressing was contemplated. Even where no pre-stressing was desired, the suggested rotation of web-members would itself be of assistance in bridge spans of all types, including open-spandrel arches, since that rotation would produce extra stiffness in the transverse plane where it was usually very desirable.

Although the day was probably rather distant when an all-welded pre-stressed truss would be produced, yet the possibility of a reduction in weight and cost made the prospect an attractive one, and Mr.

Moon's remarks as to possible directions of development were most interesting. It appeared that if Mr. Moon's forecast became a reality it would be possible to induce deformation-stress in a member by adding repeated runs of weld-metal until strain-meters indicated that exactly the required stress had been obtained. The Author.

There appeared to be a widespread opinion that the introduction of the use of hard steel jigs increased the cost of manufacture of steelwork unless quite a large number of similar spans were to be fabricated, and that view appeared to be held by many British manufacturers. The Author, however, was convinced that a works thoroughly organized for the use of jigs would be able enormously to increase its output, with savings in both labour and plant. Such reorganization had been carried out in the case of at least one firm of bridge manufacturers, involving some initial outlay, but he understood from them that their capacity had thereby been greatly increased and their working expenses so far reduced that even for twin or single girders the use of steel-bushed jigs was more economical than the usual method of manufacture. It was interesting to record that for reasons of economy steel-bushed jigs were being used for every joint in the Howrah cantilever bridge, which was in fact a single span. Even in that case there would be four or eight members of each type to manufacture.

The extra cost feared by Mr. Gribble of erecting spans in the manner proposed in the Paper would be confined to the cost of the extra field riveting—a small item, as noted by Mr. Bateson. The certainty of removing deformation-stresses should be sufficient to permit the use of higher working stresses, as Mr. Gribble had remarked, perhaps 10 tons per square inch, and that point merited serious consideration. Even with such a very moderate increase in working stress, an overall economy in cost of from 5 to 10 per cent. could be anticipated, apart from the considerable saving which in India would result from the elimination of timber sleepers.

Mr. Gribble had questioned the possibility of putting compression-members under full load before the lacing-bars (or batten-plates) were riveted to them. That point, however, offered no difficulty when the details of the unriveted member were considered more closely. The lacing-bars would in any case be attached by small bolts allowing a movement of $\frac{1}{16}$ inch to $\frac{1}{8}$ inch, which would be sufficient to permit the deflexions required by the pre-stressing method but quite insufficient seriously to affect the capacity of the member as a strut. In fact, many erecting-crane jibs worked constantly with bolted connexions and lacing-bars, and were virtually under the conditions visualized for pre-stressing compression-members.

With regard to the permanent set which appeared to have occurred,

The Author. particularly in tension-members (pp. 103 and 104), stresses measured on the 10-inch bases were in general well below the yield-point of the metal (though at three or four points stresses up to about 12·5 tons per square inch were measured), but it was possible that members might have behaved much as a wire rope did under its first loading. Minor wrinkles were bound to exist in rolled sections and plates, and a tensile load well below the yield-point of the metal could operate to draw them out and pack the scantlings more closely together, the resulting strain resembling a yield in the members as a whole and reducing the effective modulus of elasticity. Mr. Thornton's theory to account for departures both above and below the recognized value of E was most interesting, and the Author would be glad to hear more about it.

The model had been primarily intended to be used as a more convenient and rapid method of analysing complicated frames, as noted by Mr. Needham, but the Author was not aware that he had implied, as stated by Mr. Durant, that more cumbersome mathematical methods were impossible. He was also not aware of any departure in the Appendix from a true-scale relationship, as Mr. Durant seemed to fear. The material used in the construction of the model, though sold commercially as celluloid, was in fact some other synthetic composition, and its elastic modulus was not the same as that for celluloid quoted by Mr. Durant. He was of the opinion that it was easier to represent the many structural details of a truss in a model, with some approach to accuracy, than to make proper provision for them in a purely mathematical calculation. Gusset details at a joint were a typical éxample.

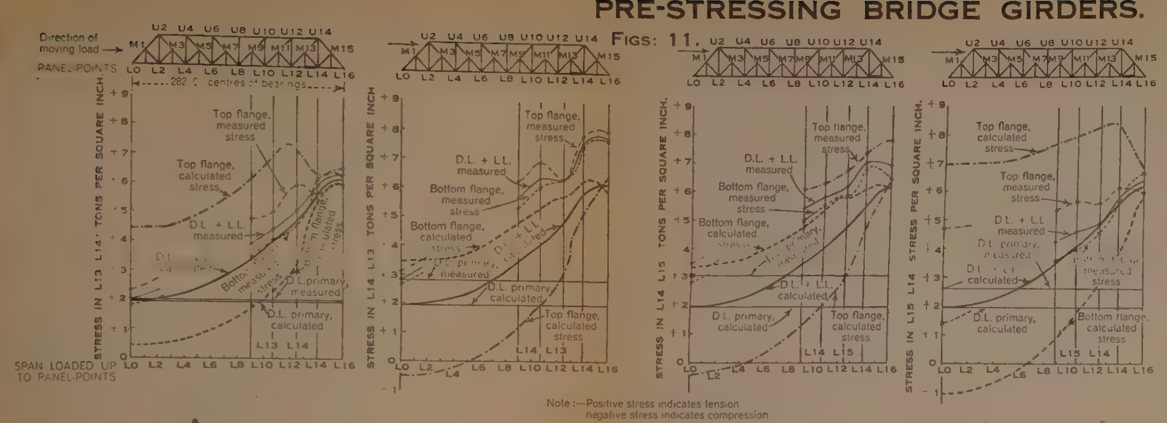
The condenser effect under dynamic loading of the magnifying beams used on the model could be understood, but it should be emphasized that the model had not been intended in any way to examine the effects of dynamic loads.

Mr. Needham's précis of current American opinion on secondary stresses was of great interest, and indicated that the question, which had been largely neglected in Britain, had received careful study in America. It should be remembered, however, that Amercan bridge practice differed considerably from British practice. The effective depth of American trusses was considerably greater than that of those built in Britain, so that the deflexions and distortions were smaller, and, further, the members were in general more slender and suffered less from a given distortion. Moreover, a dead-load plus live-load plus secondary stress might be just below the yield-point and be perfectly safe for a single application but disastrous when occurring as a repeated stress.

⁂ The Correspondence on the foregoing Paper will be published in the Institution Journal for October, 1937.—Sec. Inst. C.E.

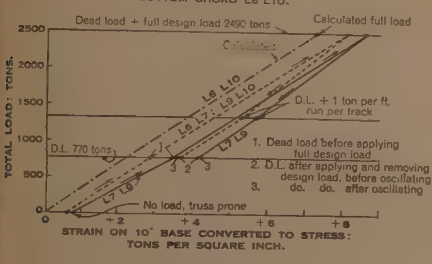
PRE-STRESSING BRIDGE GIRDERS.

Figs: 11.

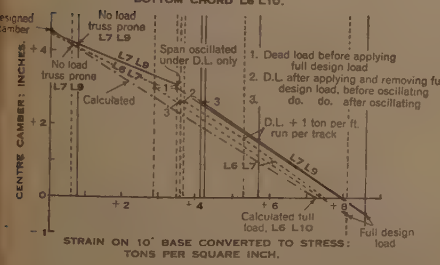


Note:—Positive stress indicates tension
negative stress indicates compression

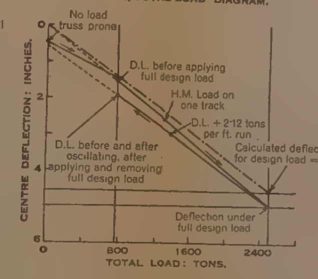
Figs: 16.



A
TOTAL-LOAD/STRESS DIAGRAM.
BOTTOM CHORD L8 L10.

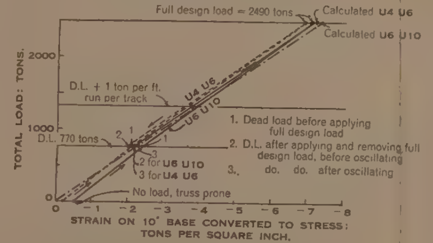


B
CAMBER/STRESS DIAGRAM.
BOTTOM CHORD L8 L10.

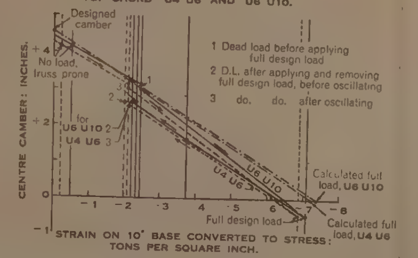


E
DEFLECTION/TOTAL-LOAD DIAGRAM.

Note:—Positive stress indicates tension.
Calculated values based on E = 13500 tons per square inch.

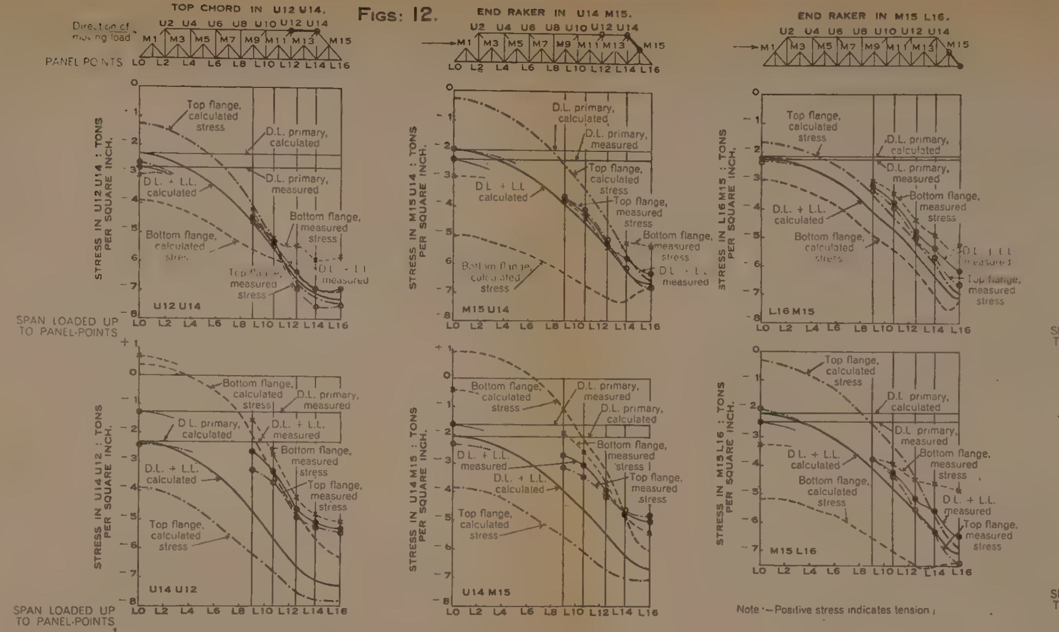


C
TOTAL-LOAD/STRESS DIAGRAM.
TOP CHORD U4 U6 AND U8 U10.



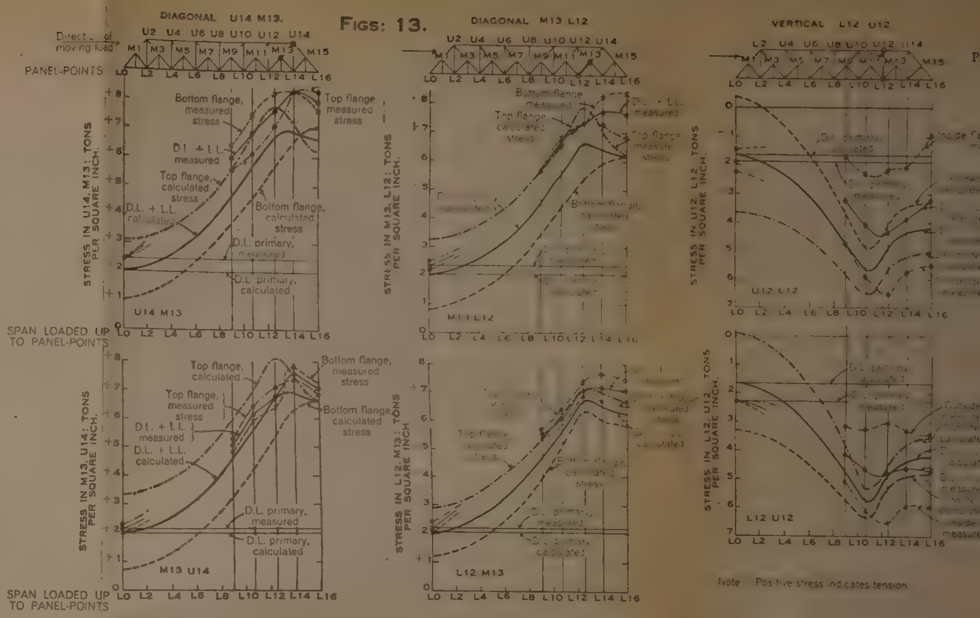
D
CAMBER/STRESS DIAGRAM.
TOP CHORD U4 U6 AND U8 U10.

Figs: 12.



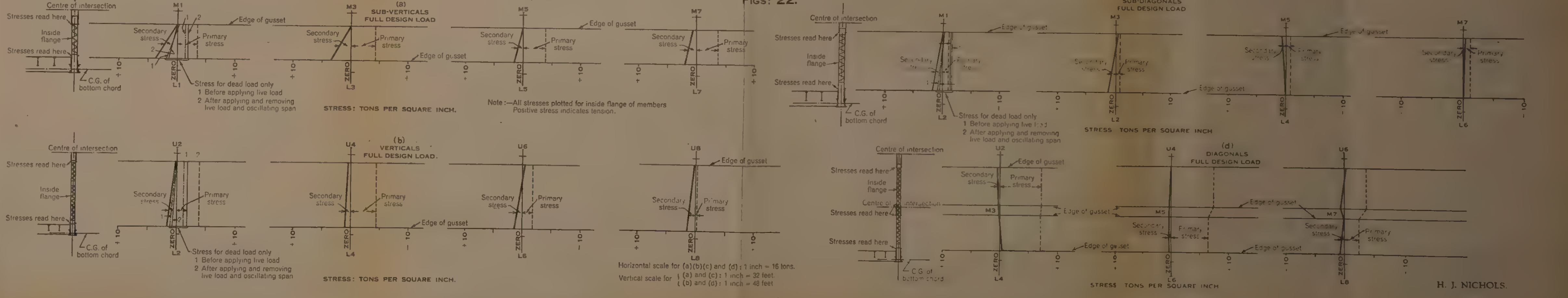
Note:—Positive stress indicates tension

Figs: 13.



Note:—Positive stress indicates tension.

Figs: 22.



(a) SUB-VERTICALS FULL DESIGN LOAD

(b) VERTICALS FULL DESIGN LOAD.

(c) SUB-DIAGONALS FULL DESIGN LOAD

(d) DIAGONALS FULL DESIGN LOAD

Note:—All stresses plotted for inside flange of members
Positive stress indicates tension.

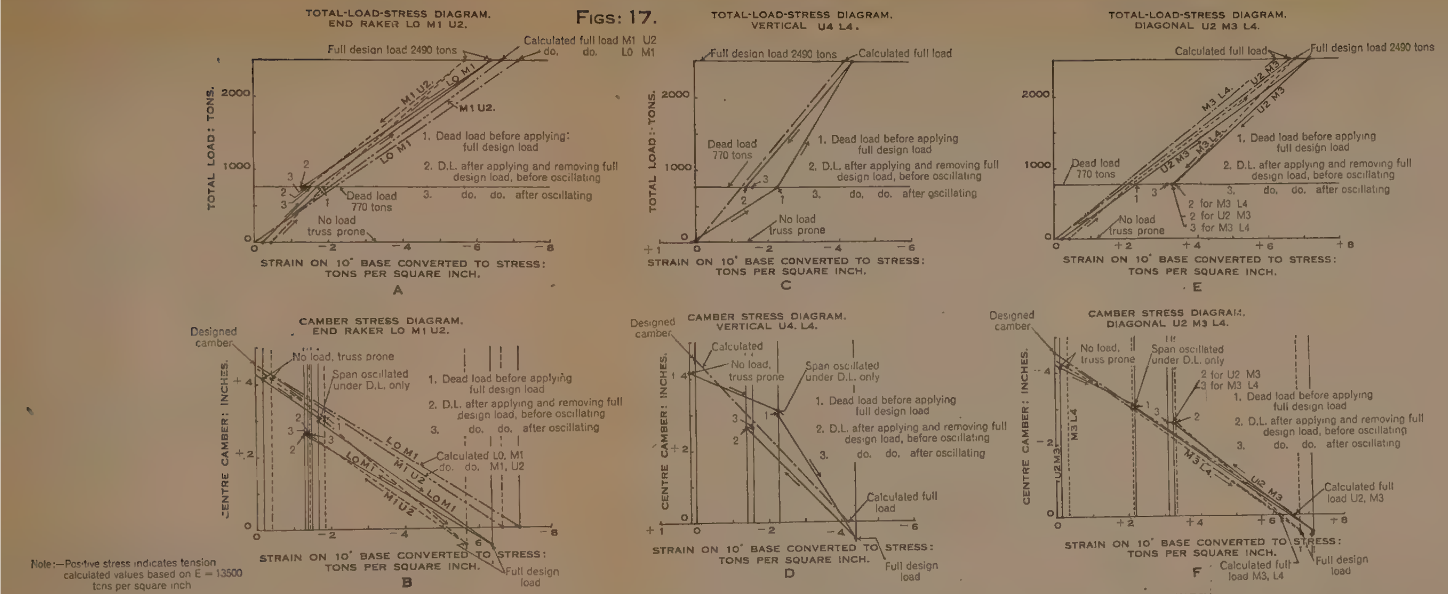
Horizontal scale for (a)(b)(c) and (d): 1 inch = 16 tons.
Vertical scale for (a) and (c): 1 inch = 32 feet.
(b) and (d): 1 inch = 48 feet.

H. J. NICHOLS

e A

PRE-STRESSING BRIDGE GIRDERS.

FIGS: 17.



FIGS: 21

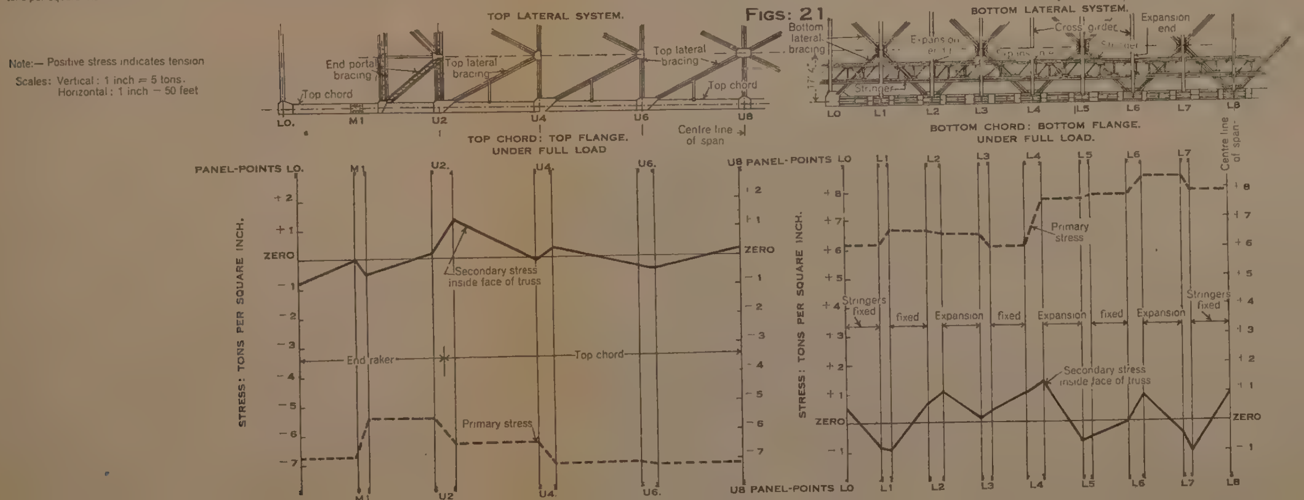
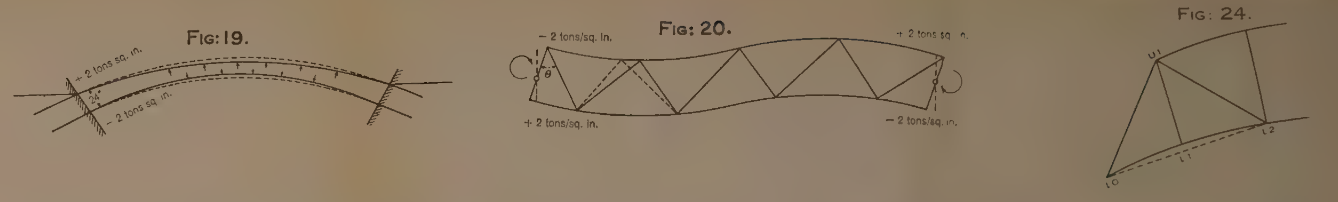


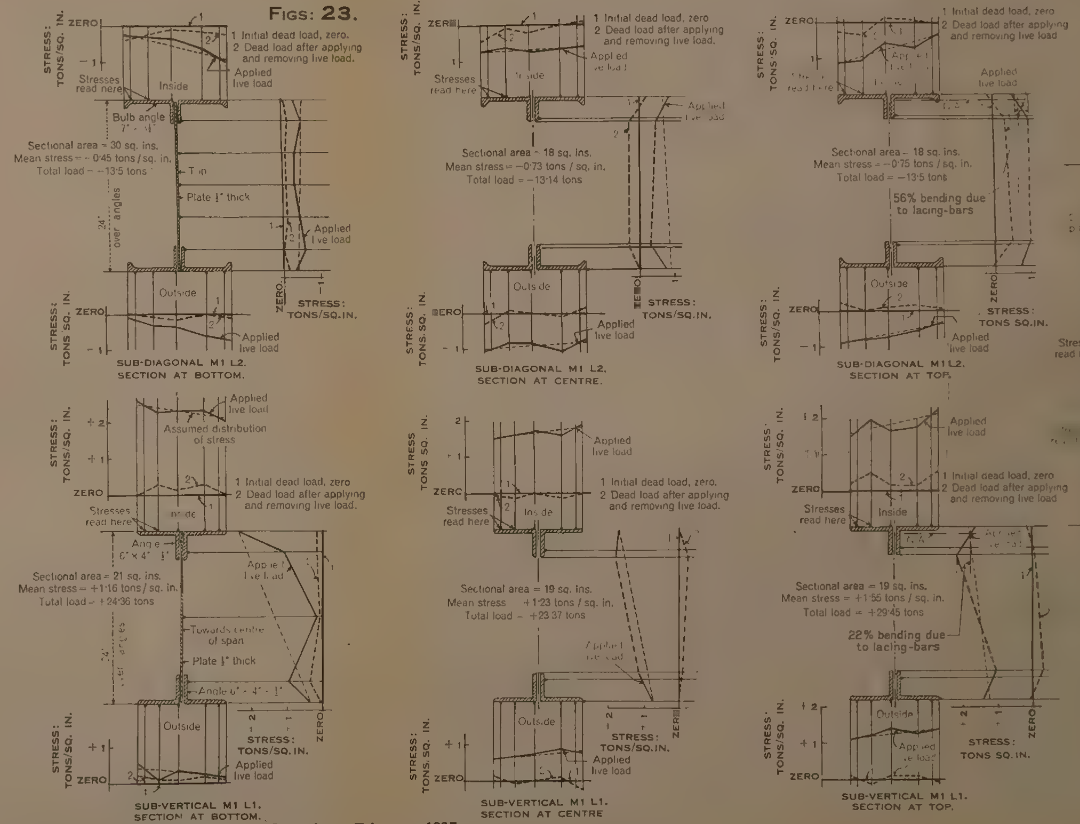
FIG: 19.

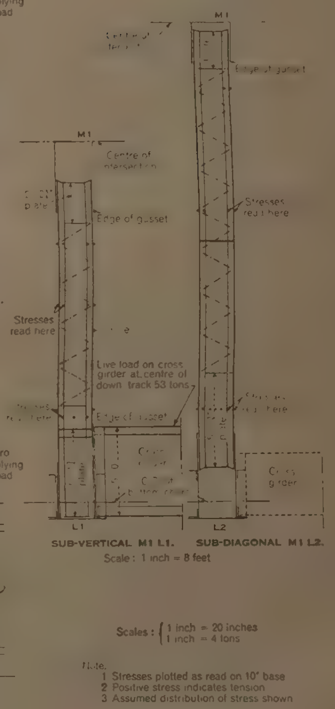
FIG: 20.

FIG: 24.



FIGS: 23.





H. J. NICHOLS.

A

Paper No. 5070.

"Fluctuating Loads in Sleeve Bearings."

By Professor HERBERT WALKER SWIFT, M.A., D.Sc. (Eng.).

(*Ordered by the Council to be published with written discussion.*) [1]

TABLE OF CONTENTS.

INTRODUCTION.

WHEN a journal bearing is subjected to a steady load the shaft normally takes up a fixed position relative to the bearing. This position depends upon the geometrical arrangement of the bearing and upon the value of the dimensionless product $\frac{P}{\lambda U}$, where P denotes the load on the bearing per unit of axial length, λ the viscosity of the lubricant and U the peripheral rubbing speed. Any alteration in either the magnitude or direction of the load gives a different equilibrium-position of the journal, and therefore causes a movement of the journal-centre. A change in the pressure-distribution occurs due not only to the altered position of the centre but also to the fact that the centre is in motion. In other words, the distribution of pressure at any instant, and so the resultant pressure, depends not only on the position but also on the translational velocity of the journal-centre in the bearing.

In the present Paper it is proposed to examine the paths followed by the journal-centre under certain simple conditions of loading, and for the most part in sleeve bearings completely surrounding the shaft. In this investigation use will be made of the theory developed by Dr. A. Sommerfeld [2] and Mr. W. J. Harrison [3] independently and

[1] Correspondence on this Paper can be accepted until the 15th June, 1937, and will be published in the Institution Journal for October, 1937.—SEC. INST. C.E.

[2] *Zeitschrift für Mathematik und Physik*, 1904, p. 97.

[3] Trans. Camb. Phil. Soc., vol. xxii (1912–23), p. 39.

applied by the latter to certain unsteady conditions.[1] This theory, in common with all other journal-bearing theories, neglects side leakage and local changes in viscosity. Prominent in it also is the assumption that there is continuity of the pressure-film around the whole bearing arc.

In the case of partial bearings this assumption has proved untenable [2] on grounds of stability and also in the light of experiment, and in the sleeve bearing running steadily under constant load there is experimental evidence that the pressure-film only exists over a portion of the bearing arc, and a smaller portion as the working eccentricity increases. When, however, the journal-centre has a cyclic motion of its own apart from the rotation, so that the points of maximum and minimum pressure are rapidly moving round the bearing, a systematic periodic sequence of rupture and re-formation of the film at successive points is not easy to visualize, and for descriptive and comparative purposes it is doubtful whether any more reasonable assumption could be found than the simple one of film-continuity. This assumption is made more plausible by the fact that in the theory of partial bearings under steady load the particular assumption made as regards the rupture or continuity of the film is found to make comparatively little difference to the results, at any rate over the more important range of eccentricities. For the most part, therefore, it will be assumed in the present treatment that the oil film is continuous around the bearing.

The path followed by the journal-centre under specified variations in load is most simply investigated if the effect of this movement is considered independently of rotation in the first instance.

No Rotation of Shaft; Radial Movement of Centre.

A journal will be considered (*Fig. 1*), of radius R and centre J, mounted in a clearance-bearing of radius $R + r$ and centre O. The journal is assumed to have no rotation, but to move in the clearance space with a radial velocity V. If instantaneously OJ $= \epsilon r$, then $V = r\dfrac{d\epsilon}{dt}$, the film-thickness at any point defined by the angle θ measured from the line of centres JO is $h = r(1 + \epsilon \cos \theta)$ and $\dfrac{\partial h}{\partial t} = V \cos \theta$.

The film-thickness being small compared with the radius of the journal, the effect of the tangential component of motion may be

[1] Trans. Camb. Phil. Soc., vol. xxii (1912–23), p. 373.

[2] H. W. Swift, "The Stability of Lubricating Films in Journal Bearings." Minutes of Proceedings Inst. C.E., vol. 233 (1931–32, Part I), p. 267.

neglected[1] and the peripheral velocity of the lubricant at any point in the film distant y from the surface of the journal will be

$$u = \frac{1}{2\lambda}\frac{\partial p}{\partial x}y(y-h),$$

where $\frac{\partial p}{\partial x}$ denotes the peripheral pressure-gradient at the section and λ the viscosity of the lubricant.

Fig. 1.

The equation of continuity of flow for an incompressible fluid becomes in this case:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) = 12\lambda\frac{\partial h}{\partial t},$$

or

$$\frac{\partial}{\partial \theta}\left(h^3\frac{\partial p}{\partial \theta}\right) = 12\lambda V R^2 \cos\theta.$$

Hence

$$\frac{\partial p}{\partial \theta} = \frac{12\lambda V R^2}{r^3}\cdot\frac{\sin\theta - \sin\psi}{(1+\epsilon\cos\theta)^3},$$

where ψ denotes the value of θ at which $\frac{\partial p}{\partial \theta} = 0$, and is at present undetermined.

If Dr. Sommerfeld's notation is adopted in the form

$$i_1 = \frac{1}{\alpha+\cos\theta} \qquad j_1 = \int\frac{d\theta}{\alpha+\cos\theta}$$

$$i_2 = \frac{1}{(\alpha+\cos\theta)^2} \qquad j_2 = \int\frac{d\theta}{(\alpha+\cos\theta)^2}$$

$$i_3 = \frac{1}{(\alpha+\cos\theta)^3} \qquad j_3 = \int\frac{d\theta}{(\alpha+\cos\theta)^3} \qquad \left(\text{where } \alpha = \frac{1}{\epsilon}\right),$$

[1] Prof. David Robertson ("Whirling of a Journal in a Sleeve Bearing," *Phil. Mag.*, vol. xv (1933), p. 113) criticizes Mr. Harrison for the omission of this component, but it proves negligible later in his own analysis, and both arrive at the same results.

then with datum pressure at both ends of the film ($\theta_1 = \pi - \beta_1$, $\theta_2 = \pi + \beta_2$) the pressure at any section is

$$p = \frac{12\lambda VR^2}{r^3\epsilon^3}\left[\frac{1}{2}i_2 - j_3 \sin\psi\right]_{\theta_1}^{\theta},$$

where $$[p]_{\theta_1} = 0.$$

This determines the angle ψ, as $\sin\psi = \dfrac{1}{2}\dfrac{\left[i_2\right]_{\theta_1}^{\theta_2}}{\left[j_3\right]_{\theta_1}^{\theta_2}} = \dfrac{I_2}{2J_3}$, say. The components of resultant pressure on the journal per unit of axial width are

$$P\cos\phi = -\int_{\theta_1}^{\theta_2} p\cos\theta \,.\, Rd\theta = R\int_{\theta_1}^{\theta_2}\frac{\partial p}{\partial\theta}\sin\theta d\theta$$

$$P\sin\phi = -\int_{\theta_1}^{\theta_2} p\sin\theta \,.\, Rd\theta = -R\int_{\theta_1}^{\theta_2}\frac{\partial p}{\partial\theta}\cos\theta d\theta,$$

which can be evaluated in the form

$$P\cos\phi = \frac{12\lambda VR^3}{r^3\epsilon^3}\{-J_1 + 2\alpha J_2 + (1-\alpha^2)J_3 - \tfrac{1}{2}I_2\sin\psi\}$$

$$P\sin\phi = \frac{12\lambda VR^3}{r^3\epsilon^3}\{-I_1 + 2\alpha I_2 + J_2\sin\psi - \alpha J_3\sin\psi\}.$$

When $\beta_1 = \beta_2 = \frac{1}{2}\beta$, then $\sin\psi = 0$, $\phi = 0$, and

$$[p]_{\theta_1}^{\theta} = \frac{6\lambda VR^2}{r^3}\left[\frac{\alpha^3}{(\alpha+\cos\theta)^2}\right]_{\theta_1}^{\theta},$$

$$P = \frac{24\lambda VR^3}{r^2(1-\epsilon^2)^{\frac{3}{2}}}\left(\frac{\pi}{2} - \frac{\gamma}{2} + \tfrac{1}{4}\sin 2\gamma\right), \text{ where } \cos\gamma = \frac{1-\alpha\cos\dfrac{\beta}{2}}{\alpha-\cos\dfrac{\beta}{2}}.$$

In two cases of special interest this result becomes :—

(*a*) for a complete 360-degree bearing,

$$P = \frac{12\pi\lambda VR^3}{r^3}\cdot\frac{1}{(1-\epsilon^2)^{\frac{3}{2}}} \quad \dots\dots\dots\dots \quad (1)$$

(*b*) for a half bearing,

$$P = \frac{12\pi\lambda VR^3}{r^3}\cdot\frac{\pi - \cos^{-1}\epsilon + \epsilon\sqrt{1-\epsilon^2}}{(1-\epsilon^2)^{\frac{3}{2}}} \quad \dots \quad (2)$$

This problem is of some practical interest in connexion with bearings of the knuckle type which are subjected to reciprocating

loads with little or no rotation. If the effects of rotation or oscillation are neglected and the bearing load per unit of axial width can be expressed in the form $P = P_0 \sin \omega t$, then, provided that inertia forces are small compared with viscous forces,[1] the movement of the journal in a complete sleeve bearing will satisfy the equation

$$P_0 \sin \omega t = \frac{12\pi\lambda R^3}{r^2} \cdot \frac{1}{(1-\epsilon^2)^{\frac{3}{2}}} \frac{d\epsilon}{dt},$$

which, by integration, gives

$$\left[\frac{P_0 r^2 \cos \omega t}{12\pi\lambda\omega R^3}\right]_{t_0}^{t} = \left[\frac{\epsilon}{\sqrt{1-\epsilon^2}}\right]_{\epsilon_0}^{\epsilon},$$

and corresponds to an oscillatory motion in the line of the alternating load. The range of this oscillation depends on the initial conditions, being in general from ϵ_0 to ϵ_1 where

$$\tan (\sin^{-1}\epsilon_1) = \tan (\sin^{-1}\epsilon_0) - \frac{P_0 r^2}{6\pi\lambda\omega R^3}.$$

The range for symmetrical oscillations about the central position is $\pm\dfrac{r\Delta_0}{\sqrt{1+\Delta_0{}^2}}$, where $\Delta_0 = \dfrac{P_0 r^2}{12\pi\lambda\omega R^3}$.

If the maximum eccentricity $\dfrac{\Delta_0}{\sqrt{1+\Delta_0{}^2}} = \epsilon_0$ and the minimum

---

[1] The relative importance of inertia forces is another point of difference between Prof. Robertson and Mr. Harrison. Some idea of their effect under practical conditions may be obtained from a simplified equation of motion of the type $P_0 \sin \omega t - \mu\dfrac{dx}{dt} = \dfrac{W}{g}\dfrac{d^2x}{dt^2}$, where $x = \epsilon r$, μ is a representative value of $\dfrac{12\pi\lambda R^3}{r^3(1-\epsilon^2)^{\frac{3}{2}}}$, and $\dfrac{W}{g}$ denotes the oscillating mass.

The forced oscillation represented by this equation has periodicity $\dfrac{\omega}{2\pi}$ and amplitude $X = \dfrac{P_0}{\mu}\dfrac{\cos \alpha}{\cos 2\alpha}$, where $\tan \alpha = -\dfrac{W\omega}{\mu g}$.

When $\dfrac{W\omega}{\mu g}$ is small, $X = X_0\left(1 + \dfrac{3}{2}\alpha^2\right)$, $X_0 = \dfrac{P_0}{\mu}$ being the amplitude when inertia is neglected. The proportionate error $\dfrac{3}{2}\alpha^2 = \dfrac{3}{2}\left(\dfrac{W}{P_0} \cdot \dfrac{\omega X_0}{g}\right)^2$. In a case where the inertia corresponds to the maximum load, so that $W = P_0$, and where $\omega = 100\pi$ and $X_0 = 0{\cdot}01$ in., corresponding to full movement at 3,000 alternations per minute in a diametral clearance of 0·020 inch, the error is about 1 in 10,000.

Inertia is apparently quite negligible except in such a case as a vertical shaft with no nominal load.

film-thickness $= t$, then

$$t^2 = r^2(1-\epsilon_0)^2 = \frac{12\pi\lambda\omega R^3}{P_0} \cdot \frac{\epsilon_0(1-\epsilon_0)^2}{\sqrt{1-\epsilon_0{}^2}}.$$

It will be found that t has a maximum value when $\epsilon_0 = \dfrac{\sqrt{3}-1}{2}$ $= 0{\cdot}366$, which corresponds to a value of $\Delta_0 = 0{\cdot}393$. Hence the minimum film-thickness will be greatest when the radial clearance r is such that

$$\frac{P_0 r^2}{\lambda\omega R^3} = 14{\cdot}8\,;$$

or, in terms of the pressure-criterion,

$$\frac{p_0}{\lambda\omega} \cdot \frac{r^2}{R^2} = 7{\cdot}4, \qquad (3)$$

where p_0 denotes the pressure per unit of projected area corresponding to the maximum load P_0.

The minimum film-thickness in this case is given by

$$\left(\frac{t}{R}\right)^2 = 2{\cdot}97\frac{\lambda\omega}{p_0}.$$

The work done in overcoming viscous resistance in the sleeve bearing during a complete cycle is

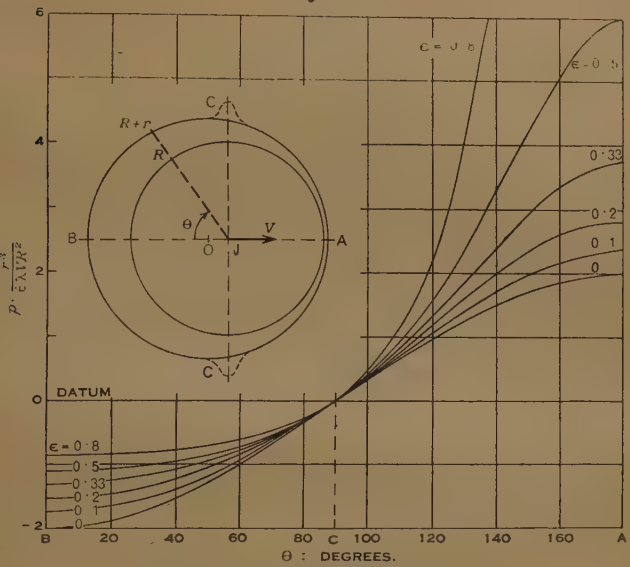
$$4\int_0^{\epsilon_0} P + d\epsilon = 4\sqrt{12\pi\lambda\omega R^3 P_0}\int_0^{\theta_0}\sqrt{\frac{\tan^2\theta_0 - \tan^2\theta}{\tan\theta_0}}\cos\theta\,.\,d\theta.$$

This leads to an elliptic integral, but it will be found that its value increases continuously with increase of θ_0, within the relevant range, showing that the energy-loss increases with $\dfrac{p_0}{\lambda\omega}\cdot\dfrac{r^2}{R^2}$. When the clearance is so arranged that $\epsilon_0 = 0{\cdot}366$, the energy-loss per cycle is found to be $4{\cdot}49\sqrt{\lambda\omega R^3 P_0}$, and since in this case $\dfrac{p_0}{\lambda\omega}\cdot\dfrac{t^2}{R^2} = 2{\cdot}97$, the energy-loss can be written in the more convenient form $1{\cdot}84 P_0 t$.

The form of the dimensionless product $\dfrac{p_0}{\lambda\omega}\cdot\dfrac{r^2}{R^2}$ given above is closely analogous to that which controls the operation of a rotating bearing under constant load, ω in that case being the angular velocity of the shaft and p_0 the applied pressure per unit area. It is to be noted that the higher the rate of alternation and the smaller the bearing-clearance, the smaller will be the fluctuations in eccentricity and *a fortiori* the smaller the linear range of oscillation $\pm\epsilon_0 r$.

During these oscillations it is worthy of note that the pressure always has its greatest value at the point towards which motion is

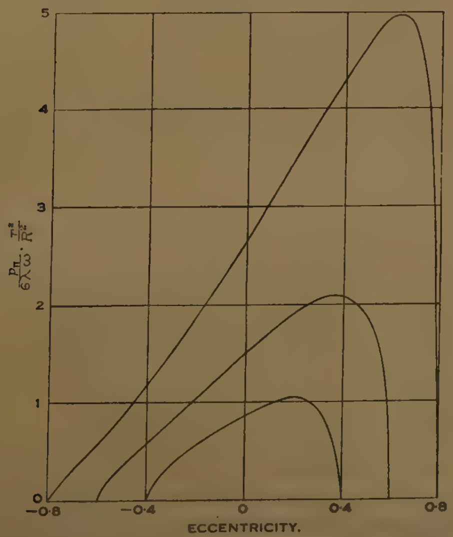
Fig. 2.



Radial Movement, No Rotation.

instantaneously taking place and its least value at the opposite point. Typical pressure-curves are shown in *Figs*. *2* and *3*, *Fig*. *2*

Fig. 3.



Pressure-Changes in Load-Line: Harmonic Load, No Rotation.

showing the pressures at different points around the bearing for different values of the instantaneous eccentricity, and *Fig. 3* showing the changes in maximum pressure (p_π) due to a simple harmonic load, for three different ranges of eccentricity.

It will be noticed that the absolute value of the pressure does not enter into the problem, and so long as side-leakage is prevented and the interspace is initially filled with lubricant cavitation cannot theoretically occur. In bearings of ordinary axial widths side-leakage is bound to occur, and cavitation can only be avoided if fresh oil is supplied at a sufficient rate to balance the leakage. This replenishment is made difficult by the fact that the low-pressure region—where supply is most conveniently arranged—becomes the high-pressure region on reversal of the load, and replenishment at points C,C in *Fig. 2*, unless under pressure, gives no real assurance against low pressures and cavitation at the "suction" end of the load-line.

In cases where the applied load P is subject to other than simple harmonic variations, the changes in eccentricity will follow the condition

$$\int P dt = \frac{24\lambda R^3}{r^2}\int_{\epsilon_0}^{\epsilon}\frac{\frac{\pi}{2}-\frac{\gamma}{2}+\frac{1}{4}\sin 2\gamma}{(1-\epsilon^2)^{\frac{3}{2}}}d\epsilon,$$

which for the complete sleeve bearing becomes

$$\frac{r^2}{12\pi\lambda R^3}\int P dt = \frac{\epsilon}{\sqrt{1-\epsilon^2}} - \frac{\epsilon_0}{\sqrt{1-\epsilon_0{}^2}},$$

and which can easily be traced when the law of variation of P is known.

It will be noticed that if the load P has any constant or unsymmetrical component, however small, the value of the integral above will eventually become infinite, the eccentricity will become unity and the film of lubricant will break down. Hence a fluctuating load can only maintain the conditions of film lubrication in a non-rotating bearing if the load is strictly alternating and symmetrical. In practice this condition is rarely if ever fulfilled, and it follows that in general film lubrication is just as dependent on rotation of the shaft with fluctuating as with constant loading in a single plane. From the practical standpoint, therefore, the theory of bearings under alternating loads without rotation must be regarded as mainly introductory to the case where rotation and load-variation are combined, it being anticipated that conditions which tend to improve the operation of a bearing under these ideal conditions are likely to be applicable in a descriptive way at least to the more complex problem which approximates to practical conditions.

For this reason it may not be unprofitable to compare the conditions in a perfectly cylindrical sleeve bearing with those in split bearings of types not infrequently employed in practice for reciprocating loads and indicated in *Figs. 4*. *Fig. 4* (*a*) shows a split bush which has been bored to a clearance with shims inserted (of thickness $O_A O_B$) and subsequently bolted together with the shims removed, so as to provide an initial eccentricity in each half of the bush. *Fig. 4* (*b*) shows a bush in which both halves have been bedded to the shaft over an angle 2β and separated by shims of thickness $O_A O_B$ so as to introduce some working clearance.

For the purpose of this particular comparison it is proposed to set aside the assumption of film-continuity around the whole of the bearings, and to assume instead that each half of the brass is "single-acting," providing a pressure-film during the "compression-stroke"

Fig. 4 (*a*).

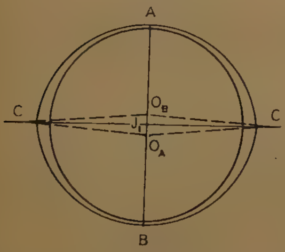
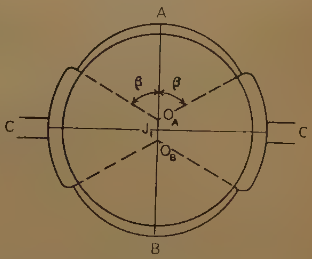


Fig. 4 (*b*).



and being replenished with oil from some external source (as is necessary in practice to overcome leakage) during the "out-stroke" in such a way as not to contribute materially to the resultant pressure during that period. This is probably a truer statement of conditions in split bearings, and does not appear unfair from the comparative point of view.

Clearance-Bearings.

When this assumption is made, a simple harmonic load in a cylindrical sleeve bearing will cause oscillations of the type

$$-\frac{P_0 r^2}{12\lambda\omega R^3}[\cos\omega t] = \int \frac{\pi - \cos^{-1}\epsilon + \epsilon\sqrt{1-\epsilon^2}}{(1-\epsilon^2)^{\frac{3}{2}}}\, d\epsilon = [(\pi - \theta)\cot\theta],$$

where $\cos\theta = \epsilon$.

For symmetrical oscillations the limits of eccentricity are $\pm\epsilon_0$, where

$$\frac{P_0 r^2}{6\lambda\omega R^3} = \pi\cot\theta_0 = \frac{\pi\epsilon_0,}{\sqrt{1-\epsilon_0{}^2}},$$

and the minimum film-thickness $t = r(1 - \epsilon_0)$ is given by

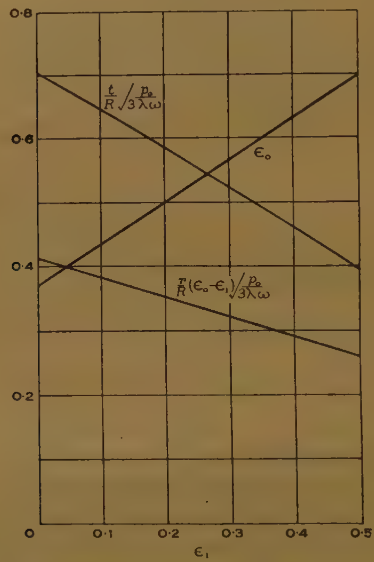
$$\frac{P_0 t^2}{6\pi\lambda\omega R^3} = \frac{\epsilon_0(1-\epsilon_0)^2}{\sqrt{1-\epsilon_0^2}}.$$

This, as before, has a maximum value when $\epsilon_0 = 0{\cdot}366$, in which case

$$\frac{p_0}{\lambda\omega} \cdot \frac{t^2}{R^2} = 1{\cdot}48 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The range of oscillation $2r\epsilon_0$ increases continuously with the clearance.

Fig. 5.



Alternating Load Without Rotation: Optimum Film-Conditions.

A clearance-bearing of the type shown in *Fig. 4* (*a*), drawn up to the extent necessary to provide an initial eccentricity ϵ_1 in each half when the shaft is central, will next be considered. In *Fig. 4* (*a*) $J_1O_A = J_1O_B = \epsilon_1 r$. If under an alternating load the maximum eccentricity in each half alternately is ϵ_0, then the minimum film-thickness will be $t = r(1 - \epsilon_0)$, and this can be evaluated for any prescribed conditions by the methods described above.

For any assumed value of ϵ_1 it will be found that the film-thickness-criterion $\frac{t}{R}\sqrt{\frac{p_0}{\lambda\omega}}$ has a maximum value for a certain value of ϵ_0, values of which will be found in *Fig. 5* plotted against various initial eccentricities ϵ_1. In the same diagram are also plotted

corresponding values of the film-thickness $\left(\frac{t}{R}\sqrt{\frac{p_0}{\lambda\omega}}\right)$ and amplitude $\left(\frac{r}{R}(\epsilon_0-\epsilon_1)\sqrt{\frac{p_0}{\lambda\omega}}\right)$ criteria, treating each half of the bearing as effective only during "compression," that is, while the eccentricity changes from $2\epsilon_1 - \epsilon_0$ to ϵ_0. It will be seen that the bearing capacity for a specified value of t is greatest when there is no initial eccentricity, while the movement of the shaft is least when the clearance and fitting are arranged to give high initial eccentricity. As compared with the cylindrical sleeve bearing this type of construction clearly has the effect of reducing the amplitude of the oscillations, but at the expense of film-thickness.

Bedded Bearings.

In the case of bedded bearings (*Fig. 4* (*b*)) the analytical procedure is modified to some extent. Considering a bedded brass of total angle 2β, with the load applied centrally, if θ is measured from the central radial plane of the brass, the film-thickness at any point will be $h = c\cos\theta$, where c denotes the clearance in the central radial plane.

The pressure-distribution is determined by the equation

$$\frac{\partial p}{\partial\theta} = -\frac{12\lambda VR^2}{c^3}\cdot\frac{\sin\theta}{\cos^3\theta},$$

where $V = -\frac{dc}{dt}$ is the speed at which the surfaces are approaching one another in the central plane.

The viscous resistance to movement per unit of axial width is

$$P = \frac{24\lambda VR^3}{c^3}\cdot\frac{1}{2}\{\tan\beta\sec\beta - \log(\tan\beta+\sec\beta)\},$$

which will be written as $\frac{12\lambda VR^3}{c^3}$. B.

If the applied loading is simple harmonic ($P = P_0\sin\omega t$) and the brasses are assumed to act alternately as before, then the semi-amplitude of the oscillations will be given by

$$\frac{P_0}{12\lambda\omega R^3}\Big[\cos\omega t\Big]_\pi^0 = \frac{P_0}{6\lambda\omega R^3} = \frac{B}{2}\left\{\frac{1}{(c_0-r)^2} - \frac{1}{(c_0+r)^2}\right\},$$

where c_0 denotes the mean central clearance.

Hence $$\frac{P_0t^2}{12\lambda\omega R^3} = B.\frac{\frac{r}{t}\left(1+\frac{r}{t}\right)}{\left(1+\frac{2r}{t}\right)^2}, \text{ which will be written as } B.C, \quad . \quad . \quad (5)$$

$t(=c_0-r)$ denoting the minimum film-thickness in the central plane, namely, the plane of loading.

Since the value of C in this expression increases continuously from 0 to $\frac{1}{4}$ as r/t increases, the residual central film-thickness for any value of β can only be increased by increasing the amplitude of oscillation, but it will be found that C increases very slowly when r/t becomes large. For example, a value of $r/t = 0{\cdot}6$ agrees reasonably with the optimum conditions found in the case of clearance brasses. If this value is adopted it will be found that $C = 0{\cdot}20$ nearly, whereas the limiting value when r/t is large is only 0·25.

The value of B in the above expression increases continuously with increasing values of β, and becomes infinite as β approaches $\frac{\pi}{2}$, corresponding to a 180-degree bedded brass. It must be remembered however, that in brasses with large bearing angles the clearance at the "horns" becomes small compared with that in the load-line, and although film-breakdown and wear at these points are less important the choice of a suitable value for the angle β might well be made in the light of this consideration.

The minimum film-thickness at the "horns" will be $t_1 = (c_0 - r)\cos\beta$, and the expression for the thickness-criterion becomes

$$\frac{P_0 t_1^2}{12\lambda\omega R^3} = \{\sin\beta - \cos^2\beta \log(\tan\beta + \sec\beta)\}\frac{\frac{r}{t_1}\cos\beta\left(1 + \frac{r}{t_1}\cos\beta\right)}{\left(1 + 2\frac{r}{t_1}\cos\beta\right)^2}.$$

For any assigned value of r/t_1 this attains a maximum with a certain value of β, this value increasing from 63 degrees when r/t_1 is small to 90 degrees when r/t_1 is large. For a value of $r/t_1 = 0{\cdot}6$ the maximum occurs when $\beta = 70$ degrees approximately, and it will then be found that $B = 6{\cdot}30$ and $\frac{p_0}{\lambda\omega}\cdot\frac{t_1^2}{R^2} = 0{\cdot}55$, while the criterion $\frac{p_0}{\lambda\omega}\cdot\frac{t^2}{R^2} = 7{\cdot}5$ under the same conditions.

It was found that under optimum conditions in a non-offset clearance-bearing the value of this criterion was 1·48. Hence it should be possible to increase the loading-criterion $\frac{p_0}{\lambda\omega}$ to five times its value in a clearance-bearing if a bedded bearing consisting of two 140-degree arcs is employed, spaced in such a way that $\frac{p_0}{\lambda\omega}\cdot\frac{c_0^2}{R^2} = 19{\cdot}2$. Alternatively, if the same loading-criterion is employed in the two cases, the amplitude of oscillations in the bedded bearing would only be that corresponding to $C = 0{\cdot}039$, which gives $r/t = 0{\cdot}045$

approximately, less than one-thirteenth of the amplitude for the optimum clearance-bearing.

The work absorbed per cycle is $2\int_{c_0-r}^{c_0+r} P_0 \sin \omega t dc$,

where $$\cos \omega t = 1 - \frac{6\lambda\omega R^3}{P_0} B\left\{\frac{1}{c_0} - \frac{1}{(c_0-r)^2}\right\}.$$

This gives an elliptic integral, but graphical integration in the particular case when $2\beta = 140$ degrees and $r/t = 0{\cdot}6$ shows that the work per cycle is $1{\cdot}67P_0t$, whereas in the case of the optimum clearance-bearing this work is $1{\cdot}84P_0t$.

It seems reasonable, therefore, to predict that a suitable form of bearing for alternating loads with relatively little rotation would consist of two bedded brasses, each embracing an angle of about 140 degrees, and spaced so as to give a total clearance in the line of loading of $2c_0$, where $\frac{c_0}{R} = 4{\cdot}4\sqrt{\frac{\lambda\omega}{p_0}}$ approximately.

No Rotation of Shaft; Rotary Movement of Centre.

Considering a journal of radius R whose centre J is moving with velocity V_1 perpendicular to the line OJ joining it to the centre of a sleeve bearing of radius $R + r$. Then if angles are measured from the radius JO produced, as in *Fig. 1*, $h = r(1 + \epsilon \cos \theta)$, in the same notation as before, and $\frac{\partial h}{\partial t} = V_1 \sin \theta$.

Hence $$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 12\lambda V_1 \sin \theta,$$

which gives $$\frac{\partial p}{\partial \theta} = -\frac{12\lambda V_1 R^2}{r^3\epsilon^3} \cdot \frac{\cos \theta - \cos \psi}{(\alpha + \cos \theta)^3},$$

where ψ corresponds to the point of maximum pressure.

Continuity of pressure requires

$$\Big[p\Big]_0^{2\pi} = -\frac{12\lambda V_1 R^2}{r^3\epsilon^3}\Big[j_2 - (\alpha + \cos \psi) j_3\Big]_0^{2\pi} = 0,$$

which gives $\cos \psi = \frac{-3\alpha}{2\alpha^2 + 1}$.

The components of resultant force per unit of axial width are

$$P \cos \phi = -\int_0^{2\pi} p \cos \theta \, . \, Rd\theta = R\int_0^{2\pi} \frac{\partial p}{\partial \theta} \sin \theta \, . \, d\theta = 0,$$

$$P \sin \phi = -\int_0^{2\pi} p \sin \theta \, . \, Rd\theta = -R\int_0^{2\pi} \frac{\partial p}{\partial \theta} \cos \theta \, . \, d\theta$$

$$= \frac{12\pi\lambda V_1 R^3}{r^3} \cdot \frac{2}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}.$$

If the motion is continuous, so that the shaft-centre has an angular velocity ω_1 about O, then $V_1 = \epsilon r \omega_1$, and

$$P = \frac{24\pi\lambda\omega_1 R^3}{r^2} \cdot \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}, \quad . \quad . \quad . \quad (6)$$

acting in a direction directly opposed to the motion V_1. This expression bears a marked similarity to that for the steady-pressure resultant produced on the same shaft when rotating in the same sleeve bearing at the same eccentricity, $Q = \frac{12\pi\lambda\omega R^3}{r^2} \cdot \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}$ perpendicular to and in advance of the line of centres OJ.

The above analysis shows that, so long as inertia-effects are negligible, a steady rotational movement of the shaft-centre requires a load of constant magnitude rotating at the same angular speed, the instantaneous direction of motion being in the line of the applied load and the eccentricity of the circular locus determined by equation (6) above. The greater the speed of rotation ω_1 of the load and the smaller the radial clearance r of the bearing, the smaller will be the eccentricity-ratio ϵ.

An expression for the minimum film-thickness $t = r(1-\epsilon)$ can be obtained in a form independent of the radial clearance r.

$$t^2 = \frac{24\pi\lambda\omega R^3}{P} \cdot \frac{\epsilon(1-\epsilon)^2}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}.$$

For fixed values of the other conditions this has a maximum value $t^2 = 5{\cdot}56\frac{\lambda\omega R^3}{P}$ when $\epsilon = 0{\cdot}366$ as before.

Hence the greatest film-thickness under otherwise specified conditions will be obtained when the radial clearance r is so chosen as to give this eccentricity. This condition requires

$$\frac{p}{\lambda\omega_1} \cdot \frac{t^2}{R^2} = 2{\cdot}78, \text{ and hence : } \frac{r^2}{R^2} = 6{\cdot}9\frac{\lambda\omega_1}{p},$$

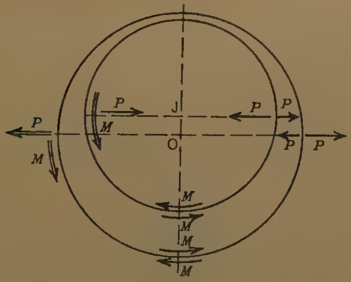
where p denotes the applied pressure per unit of projected area $= \dfrac{P}{2R}$.

Since the shaft has no appreciable peripheral velocity, the frictional traction at any point is simply $f = \dfrac{h}{2R} \cdot \dfrac{\partial p}{\partial \theta}$, and the resultant frictional moment M, assisting the rotation of J about O, is found to be

$$M = \frac{12\pi\lambda V_1 R^3}{r^2} \cdot \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}} = \tfrac{1}{2}P\epsilon r.$$

The sense of the moment is worthy of comment. Since there is no rotation about J the frictional traction due to the lubricant has the same value and acts in the same sense on both shaft and bearing. Hence the forces acting in the system are as shown in *Fig. 6*, each

Fig. 6.



moment and force acting in the direction indicated and on the particular member on which it is shown. The external applied forces are indicated by double lines. In each case the forces will be seen to balance when account is taken of the fact that $M = \frac{1}{2}P\epsilon r$. When account is taken of the moment of P about O there is found to be a resultant moment resisting the rotation of J about O of value $P\epsilon r - M = \frac{1}{2}P\epsilon r$, and the work absorbed by the bearing per revolution of J is $\pi P\epsilon r$, which agrees with the work absorbed by viscous friction.

Since the running eccentricity is related to the clearance r, the cyclic work can be expressed in a form independent of r, thus:

$$\sqrt{P\lambda\omega_1 R^3} \cdot \frac{4\pi\epsilon\sqrt{6\pi}}{(2+\epsilon^2)^{\frac{1}{2}}(1-\epsilon^2)^{\frac{1}{4}}}.$$

For fixed values of the quantities under the root sign this increases continuously with the eccentricity, that is, as the clearance r is made

greater. In the particular case when r is chosen as above so that $\epsilon = 0{\cdot}366$ the cyclic work becomes $14{\cdot}2\sqrt{P\lambda\omega_1 R^3} = 6Pt$, where t denotes the least film-thickness.

Shaft and Load Rotating.

It will now be supposed that the shaft in a sleeve bearing rotates with angular velocity ω while the load rotates with angular velocity ω_1. These conditions will be satisfied if the journal-centre J rotates in a circular path about the bearing centre O, with the same angular velocity ω_1 as the applied load and at such an eccentricity that the resultant pressure Q (90 degrees in advance of OJ, owing to the rotation ω of the shaft) and P (90 degrees behind OJ, owing to the movement ω_1 of the shaft centre) together equilibrate the applied load W per unit of axial width. This condition requires that

$$W = P - Q = \frac{12\pi\lambda R^3}{r^2} \cdot \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}(2\omega_1 - \omega) \quad . \quad . \quad (7)$$

So long as $\omega < 2\omega_1$, the point of nearest approach, being always in the line OJ, will lag 90 degrees behind the load-vector W, whilst if $\omega > 2\omega_1$ the point of nearest approach will be 90 degrees in advance of this load-vector.

A case of special concern arises when $\omega_1 = \frac{1}{2}\omega$. In this case no load can be carried by the bearing with any eccentricity other than unity. Hence, in theory at any rate, a load rotating at half the shaft-speed neutralizes the effects of pressure-film lubrication. In the commonest case, where $\omega_1 = \omega$, OJ will rotate at the same angular speed as the shaft, lagging 90 degrees behind the load-line, and the eccentricity will be precisely the same as if the given load W were applied in a fixed direction.

The right-angular relationship between the load-line and the line OJ is an essential condition for the circular locus, and the shaft-centre J will not follow a path of this simple type if the load-vector is bound to the centre-line OJ in any other way. This limitation is of special significance in connexion with centrifugal force, since a perfectly balanced rotor would have its centre of gravity at J and a circular locus would then introduce a centrifugal load in the line OJ. Under these ideal conditions, P and Q being both perpendicular to OJ, it would seem that the motion of the journal-centre must always have an outward radial component which must ultimately bring the shaft into contact with the bearing-surface and break the lubricating film. Professor David Robertson, who has drawn attention to this matter, regards film-breakdown as inevitable whether or not the bearing is under a directly-applied load. On the other hand, there is

sufficient experimental evidence of steady running in sleeve bearings to demand an explanation.

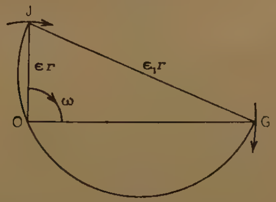
For one explanation it is not necessary to go beyond the present theory, but merely to concede that in practice no rotor is perfectly balanced. In that case, if the centre of gravity G lies at a distance $\epsilon_1 r$ from J a state of equilibrium is attained when the centres lie as indicated in *Fig. 7*, O being at such a point on the semicircle with JG as diameter that

$$\frac{W}{g}\omega^2\sqrt{\epsilon_1^2-\epsilon^2}=\frac{12\pi\lambda\omega R^3}{r^2}\cdot\frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}.$$

This equation is always satisfied by some positive value of $\epsilon<\epsilon_1$.

A less academic explanation arises from the established fact that the resultant pressure due to rotation of the shaft in actual bearings is not perpendicular to OJ but has an inward radial component along JO which increases with the working eccentricity. On this account

Fig. 7.



there is always a restoring force available to overcome an outward radial load in the line OJ, and at some eccentricity stable conditions are reached. When centrifugal force is superimposed on a steady load the journal-centre follows a small closed path round its basic position of equilibrium, instead of a small circular path round the centre O.

The work absorbed per unit of time when a rotating shaft carries a rotating load is the sum of the losses due to both rotations, and may be written as

$$\frac{4\pi\lambda R^3}{r}\cdot\frac{(1+2\epsilon^2)\omega^2+3\epsilon^2\omega_1^2}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}.$$

When expressed in terms of the applied load W this becomes

$$Wr\,.\,\frac{(1+2\epsilon^2)\omega^2+3\epsilon^2\omega_1^2}{3\epsilon(2\omega_1-\omega)},$$

and if $\omega_1=\omega$ it corresponds to an effective coefficient of friction, measured at the journal-surface, $\mu=\frac{r}{R}\cdot\frac{1+5\epsilon^2}{3\epsilon}$, which for a given

film-thickness t has a minimum value of $2{\cdot}3\ t/R$ when $\epsilon = 0{\cdot}29$. With the same steady load the coefficient of friction would be

$$\mu \doteq \frac{r}{R} \cdot \frac{1 + 2\epsilon^2}{3\epsilon},$$

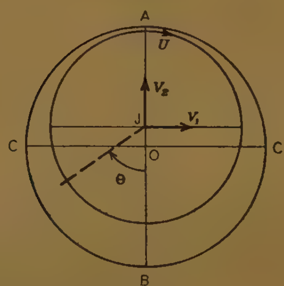
which has a minimum value of $1{\cdot}82\ t/R$ when $\epsilon = 0{\cdot}366$.

The General Case of Journal-Movement.

Let it be supposed that a journal is rotating with surface-velocity U in a clearance sleeve-bearing (*Fig. 8*) and that at any moment its centre J is moving relative to the bearing-centre O with a velocity whose components are V_2 in the direction OJ and V_1 perpendicular to this direction. Then the velocity at any point in the film distant y from the journal-surface will be

$$u = \frac{1}{2\gamma} \frac{\partial p}{\partial x} y(y - h) + U \cdot \frac{h - y}{h}.$$

Fig. 8.



The volumetric-continuity condition is

$$\frac{\partial}{\partial x} \int_0^h u dy + \frac{\partial h}{\partial t} = 0,$$

and, since $\dfrac{\partial h}{\partial t} = V_2 \cos \theta + V_1 \sin \theta$, this takes the form

$$\frac{1}{12\lambda R^2} \cdot \frac{\partial p}{\partial \theta} = \frac{U}{2R} \cdot \frac{1}{h^2} + \frac{V_2 \sin \theta - V_1 \cos \theta}{h^3} + \frac{C}{h^3}, \quad . \quad . \quad . \quad (8)$$

where C is a constant of integration which must be chosen to satisfy the pressure-continuity condition

$$\Big[p\Big]_0^{2\pi} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If p_0 denotes the pressure at any point due to journal-rotation alone,

and p_1, p_2 the pressures due to V_1 and V_2 respectively acting alone, then these pressures vary according to the equations

$$\frac{1}{12\lambda R^2}\cdot\frac{\partial p_0}{\partial\theta}=\frac{U}{2R}\cdot\frac{1}{h^2}-\frac{U}{2R}\cdot\frac{h_1}{h^3},$$

$$\frac{1}{12\lambda R^2}\cdot\frac{\partial p_1}{\partial\theta}=-\frac{V_1\cos\theta-V_1\cos\theta_1}{h^3},$$

$$\frac{1}{12\lambda R^2}\cdot\frac{\partial p_2}{\partial\theta}=\frac{V_2\sin\theta-V_2\sin\theta_2}{h^3},$$

where h_1, θ_1, and θ_2 are each separately determined by the continuity-conditions $\Big[p_0\Big]_0^{2\pi}=\Big[p_1\Big]_0^{2\pi}=\Big[p_2\Big]_0^{2\pi}=0$.

The equations can therefore be combined in the form

$$\frac{1}{12\lambda R^2}\cdot\frac{\partial}{\partial\theta}(p_0+p_1+p_2)=\frac{U}{2R}\cdot\frac{h-h_1}{h^3}$$
$$+\frac{V_2(\sin\theta-\sin\theta_2)-V_1(\cos\theta-\cos\theta_1)}{h^3},$$

where $$\Big[p_0+p_1+p_2\Big]_0^{2\pi}=0.$$

Hence $p=p_0+p_1+p_2$ satisfies the equation (8) and the continuity-condition (9) above, and it follows that the principle of superposition may be applied to determine the pressure-distribution or resultant due to a combined system of shaft-displacements when that due to each of the component displacements is known. It does not, however, follow, and indeed is not true, that the displacements due to a combined system of forces acting on the shaft may be obtained by superposition in the same way.

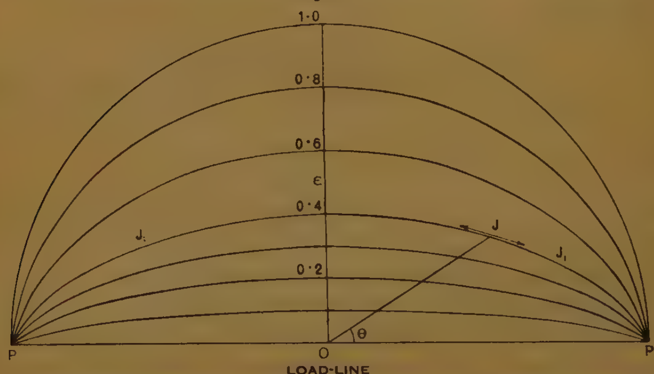
As regards the frictional resistance of the bearing, since the local traction is $f=\frac{\lambda V}{h}+\frac{h}{2R}\cdot\frac{\partial p}{\partial\theta}$, it is clear that the resultant surface-traction may be found by superposition. It is important to note, however, that this resultant does not represent the total resistance which has to be overcome by the shaft. To obtain this it is necessary in addition to take into account the direct resistance to the translational motion of the shaft-centre. The time-rate of energy-absorption, when a shaft is rotating with angular velocity ω at an eccentricity ϵ and its centre has tangential and radial velocity-components $V_1=\epsilon r\omega_1$ and $V_2=r\frac{d\epsilon}{dt}$, will be compounded of :—

(*a*) due to rotation,

	Tractive.	Direct.
(*a*)	$\frac{4\pi\lambda\omega^2 R^3}{r} \cdot \frac{1+2\epsilon^2}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}$	0
(*b*) due to tangential motion,	$\frac{-12\pi\lambda\omega\omega_1 R^3}{r} \cdot \frac{\epsilon^2}{(2+2^2)\sqrt{1-\epsilon^2}}$	$\frac{24\pi\lambda\omega_1{}^2 R^3}{r} \cdot \frac{\epsilon^2}{(2+\epsilon^2)\sqrt{1-\epsilon^2}}$
(*c*) due to radial motion,	0	$\frac{12\pi\lambda R^3}{r} \cdot \frac{\left(\frac{d\epsilon}{dt}\right)^2}{(1-\epsilon^2)^{\frac{3}{2}}}$.

The total effective resisting moment on the shaft is therefore obtained by dividing the sum of these terms by the rotational speed ω. An expression embodying the tractive components only of this resistance will be found in Mr. W. J. Harrison's Paper.[1]

Fig. 9.



OFFSET CENTRE-PATHS: ALTERNATING LOAD, NO ROTATION.

A relatively simple case to which the principle of superposition may be applied is that of a journal which is not rotating but whose centre is moving with velocity V at an angle α to the line of centres OJ. It will be found that the resultant fluid pressure on the journal is then inclined at an angle ϕ to the line JO,

where $$\tan\phi = \frac{2(1-\epsilon^2)}{2+\epsilon^2}\tan\alpha.$$

The direction of movement α of the journal is therefore not simply related to the direction ϕ of the applied load, but depends also on the eccentricity. An inspection of this relationship shows that a load acting in a fixed direction will only produce a rectilinear displace-

[1] Trans. Camb. Phil. Soc., vol. xxii (1912–23), p. 374.

ment of J if the angle $\phi = 0$, in which case the line OJ is coincident with the load-line. The effect of a load in a fixed direction OP (*Fig. 9*), when for any reason the shaft-centre J is offset from OP, can be determined as follows.

Considering the journal-centre in the position J and moving along its assumed locus J_1J_2, the components of velocity along and perpendicular to OJ are

$$V_1 = \epsilon r \frac{d\theta}{dt} \quad \text{and} \quad V_2 = r\frac{d\epsilon}{dt} \text{ respectively,}$$

from which the pressure-components in these directions may be written down. If the line of the impressed load corresponds to $\theta = 0$, then whatever the law of variation of this load its component perpendicular to this line must vanish. Hence arises the condition

$$P_1 \cos\theta + P_2 \sin\theta = 0,$$

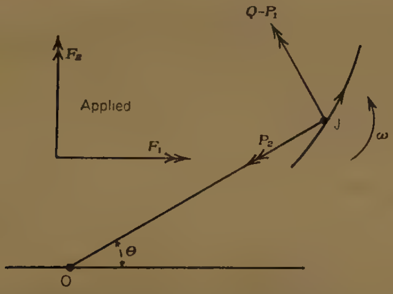
which takes the form:

$$\frac{2\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}} \cdot \cos\theta \frac{d\theta}{dt} + \frac{1}{(1-\epsilon^2)^{\frac{3}{2}}} \frac{d\epsilon}{dt} \cdot \sin\theta = 0$$

and gives, on integration, $\sin^2\theta = \dfrac{\epsilon_0^2}{(1-\epsilon_0^2)^{\frac{3}{2}}} \cdot \dfrac{(1-\epsilon^2)^{\frac{3}{2}}}{\epsilon^2}$,

where ϵ_0 is the value of ϵ when $\theta = \dfrac{\pi}{2}$.

Fig. 10.



This is the general polar equation to centre-loci for loads which do not alter in direction. *Fig. 9* shows a number of these loci, calculated by substituting various values of ϵ_0 in the above equation. The centre-locus in any particular case will be that member of the family on which the point J happens to lie when the load is first applied, and the actual time-history and range of oscillation along this locus will depend on the law of variation of the load P.

Considering next a journal rotating with angular velocity ω whose centre J is at the same time moving, as indicated in *Fig. 10*, then by

virtue of the rotation and the position of J a resultant pressure

$$\frac{Q = 12\pi\lambda\omega R^3}{r^2} \cdot \frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}} \quad . \quad . \quad . \quad (10)$$

will be produced per unit of axial width of bearing.

Hence, so long as inertia is neglible, the conditions of equilibrium under an applied load whose components are F_1 and F_2 as shown will be

$(Q-P_1)\sin\theta + P_2\cos\theta = F_1$ and $(Q-P_1)\cos\theta - P_2\sin\theta = -F_2$,

which give

$$\left(\omega - 2\frac{d\theta}{dt}\right)\frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}} \cdot \sin\theta + \frac{\frac{d\epsilon}{dt}}{(1-\epsilon^2)^{\frac{3}{2}}} \cdot \cos\theta = \frac{F_1 r^2}{12\pi\lambda R^3} \quad . \quad (11)$$

and an analogous equation.

Provided that the laws of variation of F_1 and F_2 and the values of the other impressed conditions are known, these two equations suffice in theory to determine θ in terms of ϵ and both in terms of t.

Case I.

If $F_1 = F_2 = 0$, then it follows that

$$\omega - 2\frac{d\theta}{dt} = 0 \quad \text{and} \quad \frac{d\epsilon}{dt} = 0.$$

The journal-centre will therefore, as shown independently above, follow a circular orbit, which may be at any eccentricity whatever, with angular velocity $\frac{\omega}{2}$.

Case II.

If $F_1 = \text{constant} = F_0$ say, and $F_2 = 0$, the equations may then be written in the form

$$\left(\omega - 2\frac{d\theta}{dt}\right)\cot\theta = \frac{d\epsilon}{dt} \cdot \frac{2+\epsilon^2}{\epsilon(1-\epsilon^2)}, \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{F_0 r^2}{12\pi\lambda R^3} \cdot \cos\theta = \frac{d\epsilon}{dt} \cdot \frac{1}{(1-\epsilon^2)^{\frac{3}{2}}} \quad . \quad . \quad . \quad . \quad (13)$$

Hence it will be found that

$$\cos\theta \cdot \frac{d\theta}{dt} + \sin\theta \cdot \frac{2+\epsilon^2}{2\epsilon(1-\epsilon^2)} - \frac{1}{2\Delta_0(1-\epsilon^2)^{\frac{3}{2}}} = 0, \quad . \quad (14)$$

$$1 - 2\frac{d\theta}{d\phi} = \Delta_0 \sin\theta \cdot \frac{(2+\epsilon^2)\sqrt{1-\epsilon^2}}{\epsilon}, \quad . \quad . \quad (15)$$

where $\Delta_0 = \dfrac{F_0 r^2}{12\pi\lambda\omega R^3} = \dfrac{\epsilon_0}{(2+\epsilon_0^2)\sqrt{1-\epsilon_0^2}}$ and $\phi = \omega t$.

Equation (14) is a linear equation of the first order, which leads to the solution

$$\sin\theta = \frac{1}{5\Delta_0\epsilon\sqrt{1-\epsilon^2}} + c\frac{(1-\epsilon^2)^{\frac{3}{4}}}{\epsilon},$$

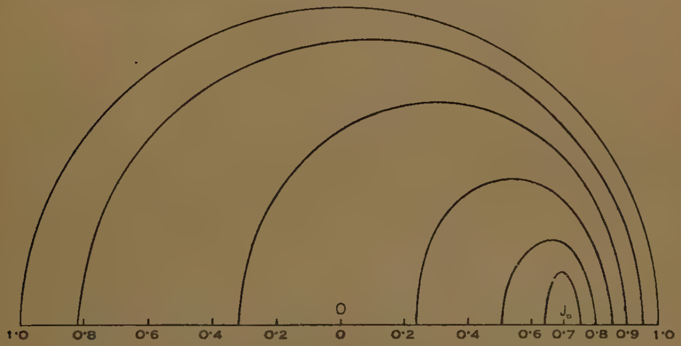
where c is determined by initial conditions. For purposes of computation it is convenient to assume some value $\epsilon = \epsilon_1$ for the eccentricity when $\theta = \frac{\pi}{2}$. It will then be found that

$$\sin\theta = \frac{1}{A\Delta_0} + \left(1 - \frac{1}{A_1\Delta_0}\right)\frac{B}{B_1}, \quad . \quad . \quad . \quad . \quad (16)$$

where $A = 5\epsilon\sqrt{1-\epsilon^2}$ and $B = \dfrac{(1-\epsilon^2)^{\frac{3}{4}}}{\epsilon}$.

By applying this expression with various values of ϵ_1 it is a simple matter to plot polar curves showing the family of paths for any assumed basic eccentricity ϵ_0. A family of curves for $\epsilon_0 = 0{\cdot}4$ will be found in *Fig. 7* of Mr. W. J. Harrison's Paper,[1] and for $\epsilon_0 = 0{\cdot}7$ in

Fig. 11.



FREE OSCILLATIONS OF SHAFT WITH CONSTANT LOAD: $\epsilon_0 = 0{\cdot}7$.

Fig. 11 of this Paper, for comparison and ready reference.

The periodicity of the oscillations represented by these curves may in theory be determined from the relation :

$$\frac{d\phi}{d\theta} = \frac{2}{1 - \dfrac{\Delta_0}{\Delta}\sin\theta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Δ being evaluated from the established relationship between ϵ

[1] *Ibid.*, p. 379.

and sin θ. General solution of this equation is not easy, but a graphical method enables the average value of $\frac{d\phi}{d\theta}$ over a complete cycle to be obtained in all cases where the orbit of J encloses the bearing centre O, since in these cases θ increases continuously from 0 to 2π. An examination of curves in which $\frac{d\phi}{d\theta}$ is plotted against θ shows that the average value of $\frac{d\phi}{d\theta}$ is always 2·0 when the orbit occupies the whole of the clearance-space in the bearing, and steadily increases as the orbit becomes smaller within the range of application of the graphical method. In cases where the shaft-orbit is not large enough to include the bearing-centre O the Author has been unable to pursue the examination, except in the limiting case where the movement is so small that approximations are admissible. This limiting case has been examined by Mr. W. J. Harrison [1] but there appears to be an algebraic error in his final expression for the periodic time, which affects the subsequent numerical Table of results.

If ψ is written for $\frac{\pi}{2} - \theta$, then for small orbits ψ will be small, as will also be values of $\epsilon - \epsilon_0 = \xi$. The equations of motion may then be written with sufficient accuracy as

$$\psi\left(\omega + 2\frac{d\psi}{dt}\right) = \frac{d\xi}{dt} \cdot \frac{2 + \epsilon_0{}^2}{\epsilon_0(1 - \epsilon_0{}^2)}$$

and

$$\psi \, . \, \Delta_0\omega = \frac{d\xi}{dt} \cdot \frac{1}{(1 - \epsilon_0)^2)^{\frac{3}{4}}},$$

which, on elimination of ξ, gives

$$\frac{d^2\psi}{d\phi^2} + \frac{1 - \frac{1}{2}\epsilon^2 + \epsilon^4}{(2 + \epsilon^2)^2}\psi = 0 \text{ (where } \phi = \omega t \text{, as before).}$$

Hence, if ω_2 denotes the mean angular velocity of the journal-centre in its orbit,

$$\frac{\omega_2}{\omega} = \frac{\sqrt{1 - \frac{1}{2}\epsilon^2 + \epsilon^4}}{2 + \epsilon^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Values of $\frac{\omega_2}{\omega}$ for various eccentricities are as follows :—

ϵ	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$\frac{\omega_2}{\omega}$	0·50	0·496	0·486	0·470	0·450	0·431	0·412	0·401	0·395	0·398	0·408

[1] *Ibid.*, p. 380.

Considering this Table in conjunction with the results of the graphical analysis, it seems clear that the periodicity of free oscillations bears a ratio to the speed of rotation which does not vary greatly over the whole range of eccentricities and amplitudes. This would explain not only the evidences of instability at speeds about twice the critical speed of the rotor, but also the remarkable immunity from vibration at ordinary speeds which makes possible the experimental exploration of the centre-locus under running conditions.

Case III.

In this case $F_1 = F \cos \omega_1 t$ and $F_2 = F \sin \omega_1 t$, representing a load F rotating with angular velocity ω_1 in the same sense as the journal. The circular orbit (equation (7), p. 176), with angular velocity ω_1 and eccentricity such that

$$\frac{Fr^2}{12\pi\lambda R^3} = (\omega - 2\omega_1)\frac{\epsilon}{(2+\epsilon^2)\sqrt{1-\epsilon^2}},$$

is a particular solution representing the " forced " motion, but the general solution will be found to lie in the equations

$$\frac{Fr_2}{12\pi\lambda R^3} \cdot \cos(\theta - \omega_1 t) = \frac{d\epsilon}{dt} \cdot \frac{1}{(1-\epsilon^2)^{\frac{3}{2}}},$$

$$\left\{(\omega - 2\omega_1) - 2\frac{d}{dt}(\theta - \omega_1 t)\right\}\cos(\theta - \omega_1 t) = \frac{d\epsilon}{dt} \cdot \frac{2+\epsilon^2}{\epsilon(1-\epsilon^2)}.$$

A comparison of these equations with equations (13) and (12) will show that the free oscillations of the shaft-centre are generally similar to those which would occur under a steady load $P = \dfrac{F\omega}{\omega - 2\omega_1}$, but relative to an axis revolving about O with angular velocity ω_1. In particular, if $\omega_1 = \omega$, then $P = F$ numerically and the periodicity of oscillations is precisely the same as if the load were steady. It is worthy of note, however, that unless the periodicity of the free oscillations bears some integral relationship to that of the forced circular movement on which they are superimposed the resultant track of the journal-centre will not form a closed curve.

Case IV.

The conditions next considered are that $F_1 = F \sin \theta$, $F_2 = 0$. The important case of an alternating load in a single plane could best be treated in principle by assuming some relationship on a time-basis such as $F_1 = F \sin \omega t$, but this substitution is found to lead to troublesome equations. If, however, the load is expressed as a

simple harmonic function of the angular displacement θ of the line of centres OJ, equations are obtained which are more easily solved. This substitution is from the physical standpoint frankly a makeshift, since the implied fluctuations of F_1 on a time-basis do not follow any simple law, nor are they similar in form with various periodicities and intensities of load. On the other hand, the substitution provides useful comparative and descriptive results, and it will be found in the outcome that by the choice of suitable limits cases can be covered not only of alternating loads but also of loads which fluctuate without change of sign.

When the above substitutions are made in equations (12) and (13) (p. 182) it will be found after manipulation that these can be reduced to the forms

$$\frac{d}{dt}\sin^2\theta - \sin^2\theta \cdot \frac{2+\epsilon^2}{\epsilon(1-\epsilon^2)} = \frac{1}{\Delta_0} \cdot \frac{1}{(1-\epsilon^2)^{\frac{3}{2}}} \quad . \quad . \quad (19)$$

and

$$1 - 2\frac{d\theta}{d\phi} = \Delta_0 \sin^2\theta \cdot \frac{(2+\epsilon^2)\sqrt{1-\epsilon^2}}{\epsilon}, \quad . \quad . \quad . \quad (20)$$

where $\Delta_0 = \dfrac{Fr^2}{12\pi\lambda\omega R^3}$ and $\phi = \omega t$.

Of these the former is a linear equation in $\sin^2\theta$, which gives on integration

$$8\Delta_0 \sin^2\theta = \frac{(1-\epsilon^2)^{\frac{3}{2}}}{\epsilon^2}\left\{\frac{\epsilon(1+\epsilon^2)}{(1-\epsilon^2)^2} - \tfrac{1}{2}\log_e\frac{1+\epsilon}{1-\epsilon} + C\right\} \quad . \quad (21)$$

where C is a constant of integration. This defines the polar form of the path of the journal-centre J. Equation (20) may then be regarded as determining the periodicity of the motion, ϵ having been found in terms of θ.

Considering first the case when equation (21) gives some real value of ϵ, say ϵ_1, when $\theta = 0$, this corresponds to a truly alternating load, the change in sign occurring when $\theta = 0, \pi$. Under these conditions equation (21) becomes

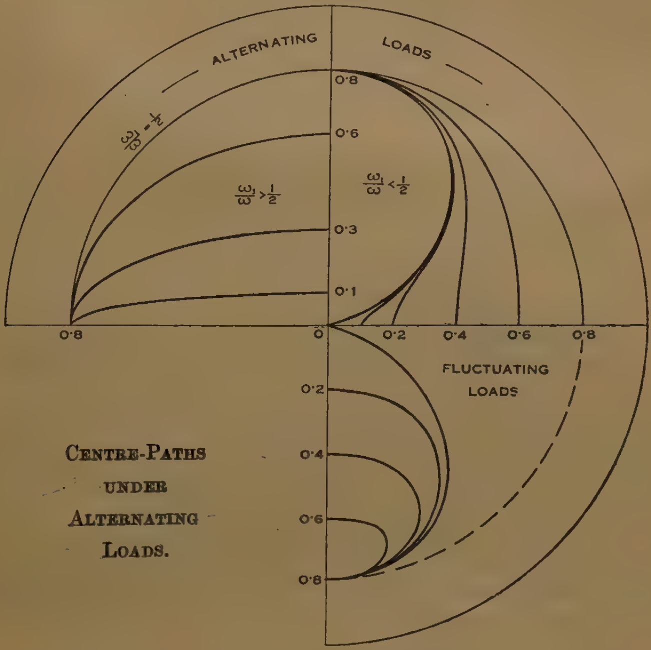
$$8\Delta_0 \sin^2\theta = \frac{A - A_1}{B}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where $A = \dfrac{\epsilon(1+\epsilon^2)}{(1-\epsilon^2)^2} - \tfrac{1}{2}\log\dfrac{1+\epsilon}{1-\epsilon}$, $B = \dfrac{\epsilon^2}{(1-\epsilon^2)^{\frac{3}{2}}}$, and A_1 is the value of A when $\epsilon = \epsilon_1$. If $\epsilon = \epsilon_2$ when $\theta = \dfrac{\pi}{2}$, then the appropriate value of Δ_0 is fixed, as $\Delta_0 = \tfrac{1}{8} \cdot \dfrac{A_2 - A_1}{B_2}$, and the relationship between ϵ

and θ is $\sin^2 \theta = \dfrac{B_2(A - A_1)}{B(A_2 - A_1)}$, which is now in a suitable form for computation.

From this equation polar curves can be plotted showing the path followed by the journal-centre for any assumed values of ϵ_1 and ϵ_2. Typical curves derived in this way are plotted in *Fig. 12*. It will be noticed that they reveal two distinct modes of precession according as $\epsilon_1 \gtrless \epsilon_2$.

Fig. 12.



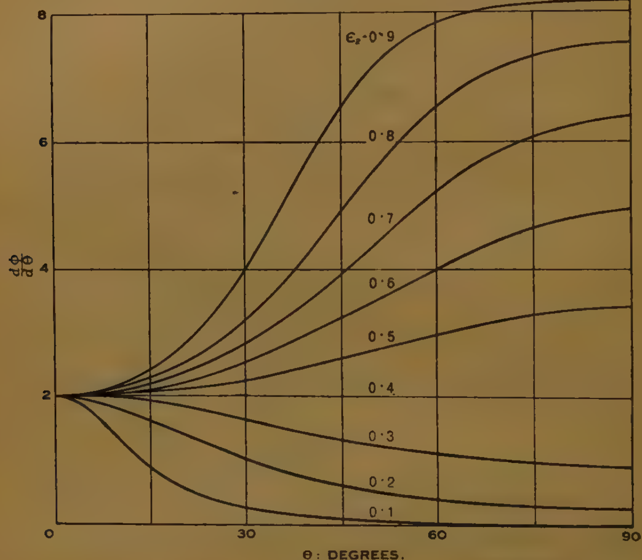
The time-history and periodicity of these oscillations can be determined from equation (20), which for purposes of computation is conveniently written in the form:

$$\frac{d\phi}{d\theta} = \frac{2}{1 - D(A - A_1)}, \quad \text{where } D = \frac{(2+\epsilon^2)(1-\epsilon^2)^2}{8\epsilon^3}.$$

From this equation values of $\dfrac{d\phi}{d\theta}$ may be calculated for assumed values of ϵ and ϵ_1, and then referred to the appropriate values of θ found from equation (22). Typical curves showing $\dfrac{d\phi}{d\theta}$ in terms of θ

obtained in this way are given in *Fig. 13*. From any such curve the appropriate relationship between ϕ and θ may be obtained by graphical integration; in particular, the mean ordinate of the curve is a measure of the ratio $\frac{\omega}{\omega_1}$ of the periodicity of rotation to that of load-alternation.

Fig. 13.



RATES OF PRECESSION WITH ALTERNATING LOADS: $\epsilon_1 = 0{\cdot}4$.

By the procedure described above it is possible to determine, for assumed values of ϵ_1 and ϵ_2,

(*a*) the value of $\frac{Fr^2}{12\pi\lambda\omega R^3} = \frac{1}{8}\frac{A_2 - A_1}{B_2}$,

(*b*) the rate of alternation as a fraction of the angular speed ω,

(*c*) the time-history and wave-form of the load F_1, since $\frac{F}{F_1} = \sin\theta$, which can be found in terms of $\phi = \omega t$ from curves of the type shown in *Fig. 13*.

The range of greatest practical interest is found to be that in which $\epsilon_1 > \epsilon_2$, giving the mode of oscillation illustrated in the left-hand portion of *Fig. 12*. Within this range the value of $\frac{d\phi}{d\theta}$ falls con-

tinuously from its initial value of 2 as θ increases from 0 to $\frac{\pi}{2}$, the fall being more pronounced the smaller the value of ϵ_2 for any assumed value of ϵ_1. Hence the range $\epsilon_1 > \epsilon_2$ corresponds to cases in which $\frac{\omega_1}{\omega} > \frac{1}{2}$, the flatter curves in *Fig. 12* representing the higher rates of load-alternation. In the limit when $\frac{\omega_1}{\omega}$ is very large the centre-locus coincides with the axis $\theta = 0$ between the limits $\pm \epsilon_1$.

Conditions within this range may be investigated in the following way. A value of ϵ_1 is assumed, curves of $\frac{d\phi}{d\theta}$ are drawn, and values of $\frac{\omega_1}{\omega}$ deduced for various values of ϵ_2. These values are plotted in terms of ϵ_2, as also are the corresponding values of $\frac{Fr^2}{\lambda\omega R^3} = \Delta_1$, say. From these curves the appropriate value of ϵ_2 for any particular ratio $\frac{\omega_1}{\omega}$ is read off, together with the value of the load-criterion Δ_1; the latter may also be found by direct calculation. A distinctive general characteristic of the curves is the rapid rise in the value of the load-criterion for any particular eccentricity ϵ_1 as the rate of load-alternation increases.

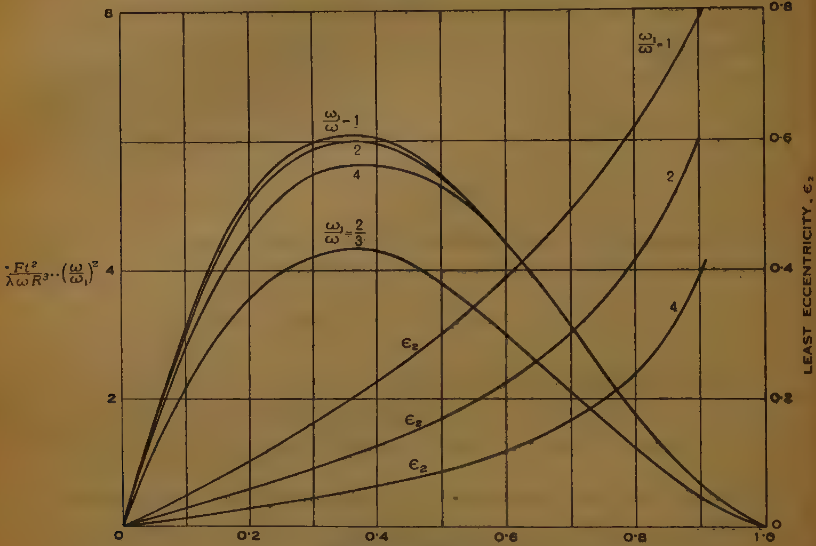
From the standpoint of design an important criterion is the minimum film-thickness $t = r(1 - \epsilon_1)$ under specified conditions $F, \omega, \omega_1, \lambda, R$. When these are known the working range of eccentricities is controlled by the choice of clearance r, and the minimum film-thickness for a bearing designed in this way to give any chosen value of ϵ_1 can be found from the relation

$$t^2 \cdot \frac{F}{\lambda\omega R^3} = \Delta_1(1 - \epsilon_1)^2.$$

Hence the product $\Delta_1(1 - \epsilon_1)^2$ is a criterion of film-thickness and consequently of safety of operation.

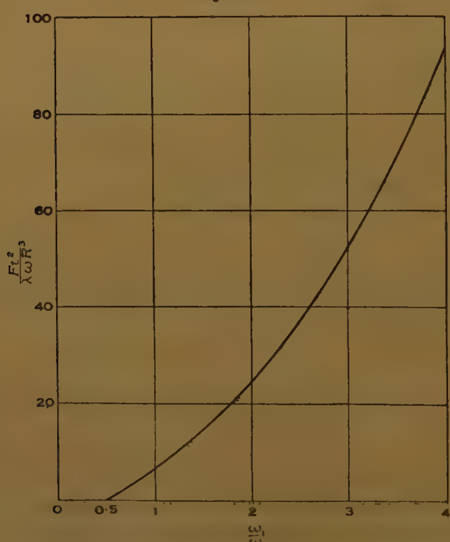
Values of this product for different values of the maximum eccentricity ϵ_1 and for certain chosen periodicity-ratios $\frac{\omega_1}{\omega}$ are plotted in *Fig. 14* (p. 190). These curves show that in each case optimum conditions of film-thickness are obtained when the clearance r is so chosen that $\epsilon_1 = 0{\cdot}35$ to $0{\cdot}40$, either greater or smaller values of r giving a thinner film. This optimum condition is notably similar to those found in the cases of a rotating bearing with steady or rotating load and of a non-rotating bearing under alternating load.

Fig. 14.



Eccentricity-Ranges and Film-Thicknesses: Alternating Load and Rotating Shaft.

Fig. 15.



Effect of Speed of Load-Alternation on Bearing-Capacity: $\epsilon_1 = 0{\cdot}4$.

The influence of the periodicity of the applied load on the load-criterion corresponding to this optimum eccentricity is indicated in *Fig. 15*, from which it will be seen that for values of $\frac{\omega_1}{\omega}$ between 1 and 4 the permissible load increases nearly in proportion to $\left(\frac{\omega_1}{\omega}\right)^2$. The value of the load-criterion which provides optimum conditions between these limits is about $\frac{Fr^2}{\lambda\omega R^3} = 16\left(\frac{\omega_1}{\omega}\right)^2$, or, if the maximum pressure per unit of projected area is p_0,

$$\frac{p_0}{\lambda\omega}\cdot\frac{r^2}{R^2} = 8\left(\frac{\omega_1}{\omega}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

It is worth noting that a value of $\frac{\omega_1}{\omega} = \frac{1}{2}$ appears incapable of producing a pressure-bearing film under alternating loads, in the same way as was found in the case of rotating loads. This is a matter of some concern in connexion with the main bearings of single-cylinder four-stroke internal-combustion engines, but it should be borne in mind that, although in such cases the fundamental harmonic has half the periodicity of the crankshaft, the higher harmonics are very considerable and no doubt play an important part in maintaining the oil film.

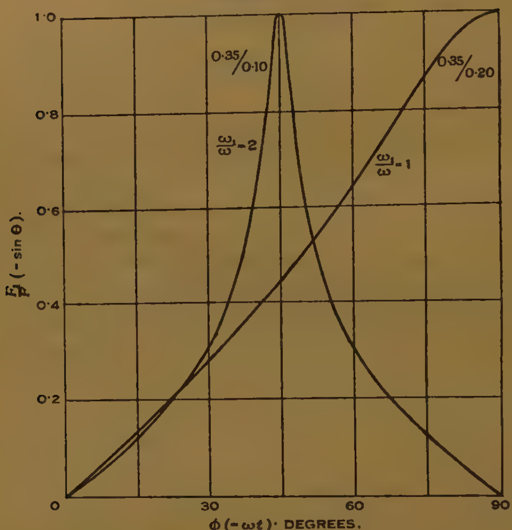
A portion of the wave-form of the load corresponding to $\epsilon_1 = 0{\cdot}35$ and $\epsilon_2 = 0{\cdot}20$, which are nearly the optimum conditions when $\omega_1 = \omega$, is shown in *Fig. 16* (p. 192), together with a portion of that when $\epsilon_1 = 0{\cdot}35$ and $\epsilon_2 = 0{\cdot}10$, which is near the optimum when $\omega_1 = 2\omega$. It will be noticed that the wave-forms are distinctly different, an inherent characteristic of the substitution $F_1 = F \sin \theta$.

It is not permissible to apply the results obtained above to wave-forms other than those involved in this substitution, but the fact that in each case the optimum eccentricity ϵ_1 appears to be nearly the same, although the corresponding wave forms are quite different, together with the constant recurrence of this optimum under such varied conditions of loading and operation, suggests that it is probably valid with sinusoidal or other ordinary types of alternating load. The pressure-criterion $\frac{p}{\lambda\omega}\cdot\frac{r^2}{R^2}$ required to give these optimum conditions is likely to be more dependent on the wave-form, but since the variations in film-thickness do not exceed 5 per cent. for values of ϵ_1 from 0·25 to 0·45, between which limits the pressure-criterion increases by nearly 100 per cent., it would appear that a bearing designed to give optimum conditions under the present assumption

should operate satisfactorily with alternating loads of any ordinary wave-form.

The range of alternating loads in which $\frac{\omega_1}{\omega} < \frac{1}{2}$ corresponds to values of $\epsilon_2 > \epsilon_1$ and to a type of centre-path indicated in the upper right-hand portion of *Fig. 12.* The smaller the value of ϵ_1 for any given value of ϵ_2, the lower is the corresponding periodicity and, over this range, the higher the load-criterion.

Fig. 16.



Wave-Forms of Load when $F_1 = F \sin \theta$.

It will be noticed from the figure that a limiting condition appears to arise in this range when $\epsilon_1 \to 0$, which corresponds in the case illustrated ($\epsilon_2 = 0{\cdot}8$) to a value of $\frac{\omega_1}{\omega} = \frac{1}{7}$ approximately. This marks a limit, not in the conditions of lubrication, but in the range of application of the assumed relation $F_1 = F \sin \theta$. A consideration of the physical conditions suggests that as the periodicity becomes smaller the centre-path takes a dumb-bell form which steadily becomes more attenuated as the periodicity falls until at very slow alternations of load it coincides with the axis $\theta = \frac{\pi}{2}$ between the appropriate values of $\pm \epsilon_2$.

The procedure followed above for values of $\frac{\omega_1}{\omega} > \frac{1}{2}$ in the examina-

tion of periodicity, load-criterion and wave-form can of course be extended to cases where $\frac{\omega_1}{\omega} < \frac{1}{2}$ within the range of application of the assumed relation $F_1 = F \sin \theta$, but as these cases are of purely academic interest it is doubtful whether a systematic study would repay the not inconsiderable labour of computation.

A certain amount of information regarding fluctuating loads of constant sign can be obtained by applying the relationship $F_1 = F \sin \theta$ in cases where there are two values ϵ_2 and ϵ_2' of the eccentricity of the same sign corresponding to $\theta = \frac{\pi}{2}$. The paths in these cases do not cross the axis $\theta = 0$ but form oval loops of the type indicated in the lower right-hand portion of *Fig. 12.* The equation to the centre-path in a form convenient for computation is

$$\sin^2 \theta = \frac{A_1 - A}{B} \cdot \frac{B_2}{A_1 - A_2} + \frac{A - A_2}{B} \cdot \frac{B_1}{A_1 - A_2},$$

where A and B have the same significance as before. No special feature arises in the plotting of the centre-path, but since the angle θ does not in this case change continuously from 0 to 2π and the value of $\frac{d\phi}{d\theta}$ becomes infinite at two points in the path, the graphical method of determining the periodicity cannot so readily be applied or the wave form obtained on a time-basis.

Some idea of the influence of speed and load may be obtained from a consideration of the relatively small orbits produced by small fluctuations of load. If the product $\frac{Fr^2}{12\pi\lambda\omega R^3} = \Delta_0 + \delta_0 \sin \omega_1 t$, where $\frac{\delta_0}{\Delta_0}$ is small, then the approximate equation for the time-changes in $\psi\left(= \frac{\pi}{2} - \theta\right)$ is found to take the form

$$\frac{d^2\psi}{d\phi^2} + \psi \Delta_0{}^2 \frac{(2 - \epsilon^2 + 2\epsilon^4)(1 - \epsilon^2)}{2\epsilon^2} = \delta_0 \frac{\omega_1}{\omega} \cdot \frac{(2 + \epsilon^2)\sqrt{1 - \epsilon^2}}{\epsilon} \cdot \cos \frac{\omega_1}{\omega}\phi,$$

where $\phi = \omega t$ as before and ϵ has a mean value corresponding to a steady-load-criterion Δ_0.

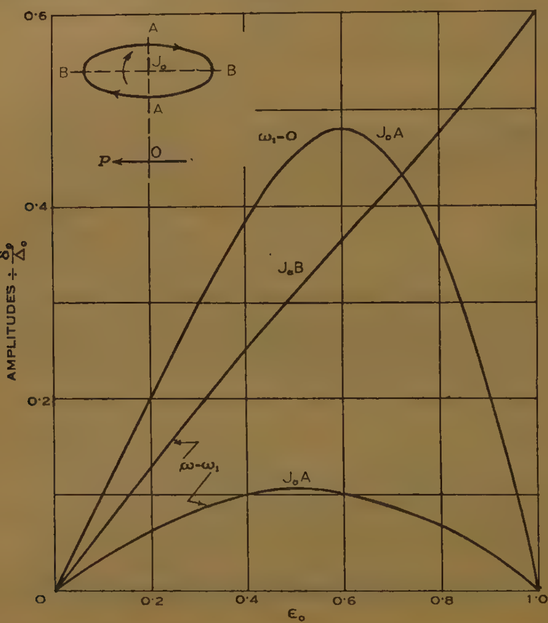
The general solution of this equation represents a system of free oscillations of undefined amplitude and of periodicity corresponding to $\frac{\omega_2}{\omega} = \frac{\sqrt{1 - \frac{1}{2}\epsilon^2 + \epsilon^4}}{2 + \epsilon^2}$, superimposed on forced oscillations of

periodicity corresponding to ω_1 and amplitudes such that:

$$\psi_{\max} = \frac{\delta_0}{\Delta_0} \cdot \frac{\frac{1}{2}\frac{\omega_1}{\omega}}{\left(\frac{\omega_1}{\omega}\right)^2 - \left(\frac{\omega_2}{\omega}\right)^2}, \qquad \xi_{\max} = \frac{\delta_0}{\Delta_0} \cdot \frac{\frac{1}{2} \cdot \frac{\epsilon(1-\epsilon^2)}{2+\epsilon^2}}{\left(\frac{\omega_1}{\omega}\right)^2 - \left(\frac{\omega_2}{\omega}\right)^2}.$$

These amplitudes decrease rapidly as the speed of alternation

Fig. 17.



Centre-Paths due to Small Cyclic Fluctuations of Load or Speed.

increases, and for values of $\frac{\omega_1}{\omega}$ greater than about 3 they become nearly

$$\psi_{\max} = \frac{1}{2} \cdot \frac{\delta_0}{\Delta_0} \cdot \frac{\omega}{\omega_1}, \qquad \xi_{\max} = \frac{1}{2} \cdot \frac{\delta_0}{\Delta_0} \cdot \left(\frac{\omega}{\omega_1}\right)^2 \cdot \frac{\epsilon(1-\epsilon^2)}{2+\epsilon^2}.$$

Some idea of the forms and dimensions of paths when the periodicity of fluctuation corresponds to the speed of rotation can be obtained from *Fig. 17*. From this it will be found, for example, that a cyclic load-fluctuation of 10 per cent. on either side of the mean, when the bearing is designed to give a mean eccentricity of 0·4, will produce variations in eccentricity from 0·39 to 0·41 only, while

the amplitude in the load-line will be about $2\frac{1}{2}$ times as great. It is worth noting that in general changes in eccentricity due to reasonable load-fluctuations of periodicity not less than that of rotation are quite small, and insufficient to require any special allowance in design until the fluctuations become large.

When the ratio $\frac{\omega_1}{\omega}$ approaches $\frac{\omega_2}{\omega}$ (whose value lies between 0·4 and 0·5 for almost all values of the eccentricity) a condition of instability arises, however small the load-fluctuation may be, but at smaller values of the ratio $\frac{\omega_1}{\omega}$ conditions become stable again, and at some value of $\frac{\omega_1}{\omega}$ less than $\frac{1}{6}$, depending on the working eccentricity, the major axis of the centre-path swings round from BB to AA in *Fig. 17*. At very low rates of fluctuation the centre-path lies entirely in the line AA, and its amplitude as compared with that when $\omega_1 = \omega$ is shown in *Fig. 17*.

It will be seen that the analysis given above can be applied to small cyclic variations of speed as well as load, the ration $\frac{\delta_0}{\Delta_0}$ then representing the proportional fluctuation of the speed above and below its mean value, and all other symbols having the same significance as before.

The Paper is accompanied by seventeen diagrams, from which the Figures in the text have been prepared.

Paper No. 5091.

"Stress-Determination for a Three-Dimensional Rigid-Jointed Framework by the Method of Systematic Relaxation of Constraints."

By JOHN CHRISTIAN RICHARDS, B.A., B.E.

(Ordered by the Council to be published with written discussion.)[1]

TABLE OF CONTENTS.

FOREWORD.[2]

THE use of "relaxation methods" to investigate stresses in frameworks was explained in two Papers recently published by the Royal Society[3]; Parts I and II dealt with pin-jointed, and Part III with rigidly-jointed, members. Numerical examples were considered in both Papers, but for brevity (and in order that attention might be focussed on essentials) the problems chosen were of simpler type than those which normally confront a designer, and for that reason they hardly serve to reveal the utility (as distinct from the validity) of the new methods. More recently several research students at Oxford have been engaged in applying relaxation methods to problems of greater complexity—some of them quite outside the range of orthodox analysis—and they have had no difficulty in arriving at solutions which are substantially correct. It thus

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[1] Correspondence on this Paper can be accepted until the 15th June, 1937, and will be published in the Institution Journal for October, 1937.—SEC. INST. C.E.

[2] Contributed by Professor R. V. Southwell, M.A., F.R.S.

[3] Professor R. V. Southwell, "Stress-Calculation in Frameworks by the Method of Systematic Relaxation of Constraints—I and II." Proc. Roy. Soc. (A), vol. 151 (1935), p. 56.

—— "Stress-Calculation in Frameworks by the Method of Systematic Relaxation of Constraints—III." Proc. Roy. Soc. (A), vol. 153 (1936), p. 41.

appears that the methods can be recommended to engineers who may have occasion to make such calculations, and the following Paper has been written with this object. The Author's problem is a rigid-jointed framework having extension in three dimensions, and loaded so that the resulting strain involves torsion of all members, as well as bending and shear. Its order of redundancy is 114—a figure which renders orthodox methods impracticable—but it has yielded to attack by relaxation methods, in which redundancy is an almost irrelevant consideration, because they do not entail the solving of simultaneous equations, but only a sequence of operations depending (in number) mainly on the number of the joints.

It is a feature of relaxation methods that they aim not at accuracy (the word has no meaning if, as is always true in practice, the loads are not exactly known) but at a margin of error which is within the range of uncertainty of the specified loading. At every stage account is kept of the "residual" forces and moments; that is, of that part of the specified loading which has not yet been accounted for. If the loading were known with certainty, these "residuals" would be a measure of the inaccuracy of the solution, but in practice the loading can be specified only within limits, and when the residuals have been brought between these limits it is not only useless, but meaningless, to attempt further refinement. Thus all that is required of the method is that it should be possible to reduce the residuals as far as may be desired: nothing is gained by bringing them actually to zero.

In the present instance *Fig. 3* (p. 203) shows the nature of the framework and the loading to which it is subjected, and Table IV (p. 216) shows the accuracy of the solution which has been obtained. The largest residual force is 1·35 lb., compared with an applied force of 100 lbs., and the largest residual moment is slightly less than 12 lbs.-feet, whereas a shearing action of only 1 lb. in one of the shortest members could entail a bending moment of 20 lbs.-feet. Evidently, in this instance, the solution has been carried quite as far as would be useful in practical work.

The Paper contains all that is necessary in order to show that a solution has been obtained. Table II (p. 212) gives the displacements and rotations of each end of every member; the distortion is therefore known, and the consequent action in every member can be calculated (Table III, pp. 214–5). All that remains is to verify that under these actions every joint is in equilibrium, within the accuracy represented by Table IV (p. 216).

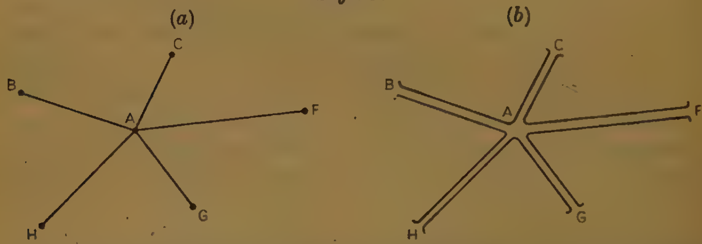
To describe the calculations in detail would be very largely to extend the length of the Paper, and this is not necessary, because the methods and essential formulas have been fully explained in

Professor Southwell's Papers published by the Royal Society.[1] A short account of relaxation methods, emphasizing principles but avoiding mathematical detail, may however be of value here.

Let a plane framework be considered, either pin-jointed as in *Figs. 1* (*a*) or having rigid joints as in *Figs. 1* (*b*). Let A be any joint in the framework, and let the members which are attached to A be AB, AC, AF, AG, AH. Suppose that the load system is specified, so that the external force at A is known.

The actions which the load-system imposes upon AB, . . . AH will be governed partly by the elastic properties of those members and by the force at A, but partly by the forces at other joints and by the elastic properties of other members. It is for this reason that an " exact " solution presents such difficulties in relation to a complicated framework: every part has influence on the stresses in every part. The relaxation method simplifies the approach by confining attention, in the first place, to the members which run

Figs. 1.



from a single joint A, and examining the effects of a displacement imposed at A while the surrounding joints B, C, F, G and H are held immovably.

Clearly the effects on any one member will depend upon the elastic properties of that member, and on nothing else; thus the " unit problem," in the relaxation method as applied to frameworks, is more simple even than that just stated. It is reduced merely to this :— One end of a specified member being held immovably, specified displacements are imposed on the other : what actions, in consequence, will be transmitted by the member to the joints which it connects ? If this question can be solved, then by mere addition an allowance can be made for any number of members connecting A with other joints ; dealing in the same way with the remaining joints of the framework, it is possible to tabulate the consequences of "operations", performed one at a time, which in combination would serve to distort the framework into any required shape.

[1] *Loc. cit.*

The "unit problem" relates to a single member and to the actions entailed at its two ends by displacements imposed on one. Suppose, for example, that the member is AB, that it forms part of a plane (two-dimensional) framework, and that the imposed displacement is a movement of the end A, in the direction Ox, through a distance u_A. The consequent actions in the member (tension or thrust, shear, etc.) will, by Hooke's law, be proportional to u_A, and in the general case (of rigid joints) they will include a force and a moment at either end. Thus the actions exerted by the member AB on the fixed joint B may be expressed in the forms

$$X_B = (x,x)_F u_A, \quad Y_B = (y,x)_F u_A, \quad N_B = (r,x)_F u_A,$$

where $(x,x)_F$, $(y,x)_F$, $(r,x)_F$ are constants depending on the elastic properties of the member, but independent of the magnitude of u_A. These are termed "influence-coefficients." Similarly, the actions exerted by AB on the joint A which is moved may be expressed in the forms

$$X_A = (x,x)_M u_A, \quad Y_A = (y,x)_M u_A, \quad N_A = (r,x)_M u_A,$$

involving influence-coefficients $(x,x)_M$, . . . etc., which in general differ from $(x,x)_F$, . . . etc.

In the Royal Society Papers cited above [1] there are given formulas whereby, in the most general case, values can be found for the influence-coefficients when the elastic properties of a member are specified. Since a member may have any direction in space, and since (in relation to a three-dimensional framework) three component displacements and three component rotations have to be contemplated as being possible to the end which is moved, rather lengthy calculations are required to cover all possibilities. The resulting formulas are, however, simple, and their forms are such that numerical values of the influence-coefficients can be computed very quickly. When these are known, particular "operations" can be investigated.

Again considering a displacement u_A, but now imagining it to be imposed on the joint A of the complete framework, it can be seen from *Figs. 1* that only the member AB will in consequence exert forces on the joint at B, but that each (in general) of the members AB, AC, AF, AG, AH will exert forces at A. Thus the resultant actions at A are to be found by addition of the influence-coefficients appropriate to these members, whereas the actions at B, C, F, G, H are already known. The additions, being merely arithmetical, are quickly performed; they give the effects of one standard "operation", namely,

[1] *Loc. cit.*

the imposition of a displacement u_A at A. In the problem now under consideration these effects will comprise:

(i) component forces $X_A, X_B, X_C, X_F, X_G, X_H$,
(ii) ,, ,, $Y_A, Y_B, Y_C, Y_F, Y_G, Y_H$,
(iii) terminal moments $N_A, N_B, N_C, N_F, N_G, N_H$.

Values corresponding with a unit displacement ($u_A = 1$) are tabulated.

It is now necessary to explain how, with the aid of a complete Table of standard operations, any specified problem can be solved. This introduces the idea of " relaxation " which has given its name to the method.

According to the principle of superposition, the manner in which the applied loads are brought to their final values will make no difference to the stresses resulting from a given load-system. It is imagined (1) that motion of the joints is initially prevented by adjustable " constraints " (for example, jacks), which accordingly take the whole of the external loads, and (2) that subsequently the constraints are " relaxed ", so as to permit controlled displacements of the different joints in turn. In this way load will be transferred (in part) from the constraints to the framework: it is contemplated that the process of relaxation is performed systematically, so that the " residual forces " (that is, the forces which remain on the constraints) tend always to smaller values as it continues. Evidently, at any stage in the process, the loads which act upon the framework are the initial (that is, the specified) loads, less the residual forces: when the residuals are small enough to be deemed insignificant, the strained configuration is close enough to that which is required.

Since only one constraint is relaxed at a time, the effects of such relaxation can be found from the Table of standard operations, and by suitably choosing the type and magnitude of the permitted displacement the relaxed constraint can be relieved of any desired amount of load, new forces being added at the same time to the adjacent constraints (which are kept fixed). The load removed from the relaxed constraint is transferred to the framework: the loads which are transferred to adjacent constraints will have to be taken into account during subsequent relaxations, but eventually, as the result of these transfers, forces of opposite sign tend to meet and cancel out. They are then said to have been " liquidated ".

An examination of the " moment-distribution method " of Professor Hardy Cross will show that the relaxation method has many points of similarity. Indeed, on a certain simplifying assumption (that the members, although flexible, are inextensible), the two methods are essentially the same, and hence the first may be

regarded as a special case of the second. The new method, however, besides being more general, differs from that of Professor Hardy Cross in regard to the details of procedure.

From what has been said on p. 200 it will be evident that attention is confined, during the relaxation process, to the forces which act upon the constraints; the object is to bring these (sensibly) to zero, and with that object account is kept, firstly, of the residual forces, and secondly, of the displacements which it is found necessary to impose. The result (as the Author's example shows) is a complete knowledge of the final configuration, because when the displacements are given the actions are known, and hence the stresses in every member are known. The "moment-distribution method" (as it is usually propounded) is concerned only with "liquidating" unbalanced moments at the joints, and the notion of "constraints" is not (explicitly) introduced, attention being confined to the proportions in which a "distributed" moment is divided up between the adjacent members. Since (according to its simplifying assumption) the forces which result from relaxation can be neglected, the process of "liquidation" is more than usually rapid; at the close of the calculations, however, knowledge of the distortion is still lacking, unless additional labour is expended.

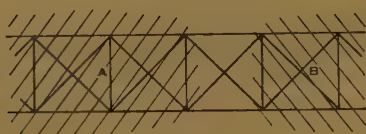
It remains to explain the term "block-relaxation" which appears in the Author's Paper. The procedure described on p. 200 will, if continued long enough, always result in a solution, but in relation to elongated frameworks (such as a bridge truss or an aeroplane fuselage) it will give a very slow approach to the required configuration—namely, that in which all "residuals" are sensibly zero. Inspection of *Figs. 1* reveals the reason: the resistance offered to a given displacement of A is much greater in the contemplated "operation" (because the surrounding joints are assumed to be fixed) than in the actual framework (where the surrounding joints are able to move in sympathy); therefore a very small displacement will suffice temporarily to "liquidate" the force at A, in comparison with the displacement which A will undergo by the time that the framework as a whole is in equilibrium.

The remedy can be explained by reference to *Fig. 2* (p. 202). If the effect of moving any one joint is known, it is possible (by the principle of superposition) to deduce the effect of any specified combination of joint-displacements, and in this way new "operations" can be added to the existing Table, to be used exactly like the more simple operations of "joint-displacement" in the process of "relaxation". For example, during an operation in which the parts A and B (indicated by shading in *Fig. 2*) are moved as rigid bodies, either by translation or by rotation, the members which lie wholly within

either shaded portion will keep their lengths unaltered, and so will be left unstressed; in general, however, all the members which connect A and B will be strained, because their ends will undergo a relative displacement. A displacement of this kind is termed a "block-displacement". Since the movement of each block is prescribed, the displacements of the ends of the members affected are known, and it is only necessary to superpose the effects of these "joint-displacements" (which, taken severally, are already known) in order to ascertain their resultant effect.

Various types of "block-displacement" were considered in the Papers [1] read before the Royal Society, and it was shown that by their

Fig. 2.



use the approach to a solution can be greatly accelerated. The Author has extended this idea by devising other combinations of "joint-displacements" which (for the sake of clarity) he has distinguished by the term "group-displacements".

INTRODUCTION.

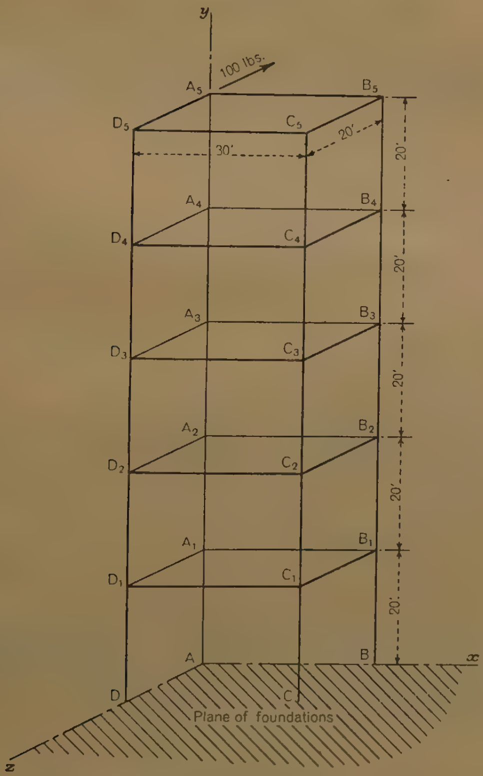
This Paper deals with a problem similar in kind (although not in complexity) to that which is presented in practice by steel-framed buildings. It was chosen in order to examine the applicability to such structures of the "method of systematic relaxation of constraints", recently developed by Professor R. V. Southwell.[2] The basic idea of the relaxation method is to assume that all joints in the structure are prevented by imaginary constraints both from changing position and from rotating. The given load-system is then applied, and is initially resisted wholly by the constraints; subsequently constraints are systematically "relaxed". The effect of any single relaxation (which is to transfer actions to adjacent constraints) can be calculated from simple formulas: account is kept of the relaxations (that is, displacements), and of the forces and moments left at every stage on the constraints. Systematic relaxation is continued until the residual or "unliquidated" actions on the constraints have become small enough to be neglected: then, since

[1] *Loc. cit.*
[2] Footnote 3, p. 196.

the total displacements are known, the final (distorted) form of the structure, and the stresses in the members which comprise it, are immediately calculable.

The particular example to be treated here is shown in *Fig. 3*: the structure, being stiff-jointed, has 114 degrees of redundancy.[1]

Fig. 3.



SKETCH OF FRAMEWORK AND LOADING.

Conventional methods, applied to such a frame, would entail extremely arduous if not prohibitive labour, whereas by the "relaxation method", and especially by using to the full the device

[1] The order of redundancy N = (number of initially unknown actions) — (number of relating equations which are obtainable by statics).

The unknown actions are six for each of the forty members, a total of two hundred and forty. The statical equations are six conditions of equilibrium for the frame as a whole, plus six conditions of equilibrium for each of the twenty unconstrained joints—a total of one hundred and twenty-six in all.

known as " block-relaxation ", a closely-approximate solution has been obtained in a reasonably short time. Accordingly it would seem that the practical designer (faced with far more elaborate structures, but for practical reasons satisfied by much less close approximation) may find this new method of considerable value.

Particulars of Structure.

The material is assumed to be steel with a Young's modulus $(E) = 30 \times 10^6$ lbs. per square inch and a Poisson's ratio $\sigma = 0{\cdot}3$.

Columns.

Length between floors = 20 feet.
Area = 40 square inches ; $EA = 12 \times 10^8$ lbs.
Flexural rigidity B (for all axes) $= 5 \times 10^8$ lbs.-feet^2.
Torsional rigidity $C = 4 \times 10^8$ lbs.-feet^2.

Girders AB *and* CD (*all floors*).

Length = 30 feet.
Area = 15 square inches ; $EA = 4{\cdot}5 \times 10^8$ lbs.
Flexural rigidity about vertical axis $B_1 = 4{\cdot}1\dot{6} \times 10^6$ lbs.-feet^2.
Flexural rigidity about horizontal axis $B_2 = 125 \times 10^6$ lbs.-feet^2.
Torsional rigidity $C = 51{\cdot}\dot{6} \times 10^6$ lbs.-feet^2.

Girders AD *and* BC (*all floors*).

Length = 20 feet.
Area = 15 square inches ; $EA = 4{\cdot}5 \times 10^8$ lbs.
Flexural rigidity about vertical axis $B_1 = 4{\cdot}1\dot{6} \times 10^6$ lbs.-feet^2.
Flexural rigidity about horizontal axis $B_2 = 125 \times 10^6$ lbs.-feet^2.
Torsional rigidity $C = 51{\cdot}\dot{6} \times 10^6$ lbs.-feet^2.

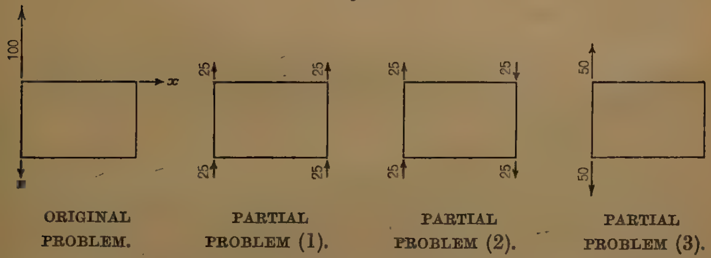
The Problem.

The structure and loading (*Fig. 3*) are simple, but the problem is essentially three-dimensional, and it reproduces an essential feature of steel-framed buildings in that rigidity of the joints is of prime importance. The horizontal loading shown as acting at one top corner will induce twisting, bending and shear of the whole structure, so that its effects are far more complex than those, for example, of vertical floor-loads, even when unsymmetrical. 100 lbs. is taken merely as a convenient unit: the effect of any other loading can be obtained by simple multiplication.

The first step towards a solution is the calculation of influence-coefficients for the constituent members. These are easily obtained, from Professor Southwell's general formulas (13)* and (14)*, in terms of the elastic and dimensional data : it is found that unit displacements or rotations imposed at the end of a member involve forces X_F, Y_F, Z_F and moments $(M_x)_F$, $(M_y)_F$, $(M_z)_F$ on the constraints at the end which is fixed, together with forces and moments X_M, . . . etc. on the constraints at the end which is moved, where X_F, . . etc. have the values given in Table I.†

Using Table I, it is an easy matter to calculate similar coefficients for any type of block-displacement or rotation (*cf.* §§ 18–25*, Part I*), or for displacements imposed, simultaneously, in any other com-

Figs. 4.



PLAN VIEW OF TOP FLOOR, EXPLAINING THE ANALYSIS OF THE COMPLETE PROBLEM INTO THREE PARTIAL PROBLEMS: (1) UNIFORM SHEAR, (2) UNIFORM TWISTING ACTION, (3) LOCAL TENSION IN MEMBER A_5D_5 (*Fig. 3*).

bination which may be found convenient. The latter (to distinguish them from block-relaxations proper) will here be termed " group-relaxations ". The problem now under discussion was solved almost entirely by permitting block- and group-relaxations of various types, influence-coefficients being calculated for suitable types as the work proceeded.

In applying the relaxation method it is important to proceed systematically and to make as much use as possible of considerations of symmetry. Accordingly the problem was divided into three " partial problems " (*Figs. 4*), each characterized by a symmetrical load-system : the separate solutions can be combined, finally, in accordance with the principle of superposition.

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* Asterisks refer to the Royal Society Papers cited in footnote 3, p. 196.

† More significant figures are given than would be justified in practice by the accuracy of the elastic data, because this Paper is concerned with the accuracy of a method of calculation.

TABLE I.—INFLUENCE-COEFFICIENTS.

Units: 1 foot, 1 radian, 1 lb. weight, 1 lb.-foot.

Columns.

Note.—In the ambiguities, the upper sign is to be taken when the upper joint of the column is moved, and the lower sign when the lower joint is moved.

	$-X_M=X_F$	$-Y_M=Y_F$	$-Z_M=Z_F$	$(M_x)_F$	$(M_y)_F$	$(M_z)_F$	$(M_x)_M$	$(M_y)_M$	$(M_z)_M$
$u=1$	$7{\cdot}5\times10^5$	0	0	0	0	$\mp75\times10^5$	0	0	$\mp75\times10^5$
$v=1$	0	600×10^5	0	0	0	0	0	0	0
$w=1$	0	0	$7{\cdot}5\times10^5$	$\pm75\times10^5$	0	0	$\pm75\times10^5$	0	0
$p=1$	0	0	$\mp75\times10^5$	-500×10^5	0	0	$-1{,}000\times10^5$	0	0
$q=1$	0	0	0	0	$+200\times10^5$	0	0	-200×10^5	0
$r=1$	$\pm75\times10^5$	0	0	0	0	-500×10^5	0	0	$-1{,}000\times10^5$

Girders of Types AB *and* CD.

Note.—In the ambiguities, the upper sign is to be taken when the joints denoted by A's or D's are moved, and the lower sign when the joints denoted by B's or C's are moved.

	$-X_M=X_F$	$-Y_M=Y_F$	$-Z_M=Z_F$	$(M_x)_F$	$(M_y)_F$	$(M_z)_F$	$(M_x)_M$	$(M_y)_M$	$(M_z)_M$
$u=1$	150×10^5	0	0	0	0	0	0	0	0
$v=1$	0	$0{\cdot}\dot{5}\times10^5$	0	0	0	$\mp8{\cdot}\dot{3}\times10^5$	0	0	$\mp8{\cdot}\dot{3}\times10^5$
$w=1$	0	0	$0{\cdot}0\dot{1}8\dot{5}\times10^5$	0	$\pm0{\cdot}2\dot{7}\times10^5$	0	0	$\pm0{\cdot}2\dot{7}\times10^5$	0
$p=1$	0	0	0	$17{\cdot}\dot{2}\times10^5$	0	0	$-17{\cdot}\dot{2}\times10^5$	0	0
$q=1$	0	0	$\mp0{\cdot}2\dot{7}\times10^5$	0	$-2{\cdot}\dot{7}\times10^5$	0	0	$-5{\cdot}\dot{5}\times10^5$	0
$r=1$	0	$\pm8{\cdot}\dot{3}\times10^5$	0	0	0	$-83{\cdot}\dot{3}\times10^5$	0	0	$-166{\cdot}\dot{6}\times10^5$

Girders of types AD *and* BC.

Note.—In the ambiguities, the upper sign is to be taken when the joints denoted by A's or B's are moved, and the lower sign when the joints denoted by C's or D's are moved.

	$-X_M=X_F$	$-Y_M=Y_F$	$-Z_M=Z_F$	$(M_x)_F$	$(M_y)_F$	$(M_z)_F$	$(M_x)_M$	$(M_y)_M$	$(M_z)_M$
$u=1$	$0{\cdot}0625\times10^5$	0	0	0	$\mp0{\cdot}625\times10^5$	0	0	$\mp0{\cdot}625\times10^5$	0
$v=1$	0	$1{\cdot}875\times10^5$	0	$\pm18{\cdot}75\times10^5$	0	0	$\pm18{\cdot}75\times10^5$	0	0
$w=1$	0	0	225×10^5	0	0	0	0	0	0
$p=1$	0	$\mp18{\cdot}75\times10^5$	0	-125×10^5	0	0	-250×10^5	0	0
$q=1$	$\pm0{\cdot}625\times10^5$	0	0	0	$-4{\cdot}1\dot{6}\times10^5$	0	0	$-8{\cdot}\dot{3}\times10^5$	0
$r=1$	0	0	0	0	0	$25{\cdot}8\dot{3}\times10^5$	0	0	$-25{\cdot}8\dot{3}\times10^5$

Partial Problem (1) (*Horizontal Shear*).

Fig. 5 shows the external forces in this problem, and *Figs. 6, 7, 8, 9* and *10* show the residual forces and moments which are left on

Fig. 5.

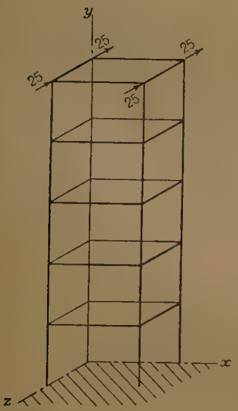


Fig. 6.

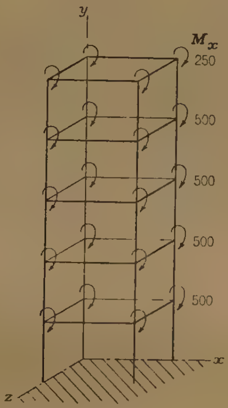
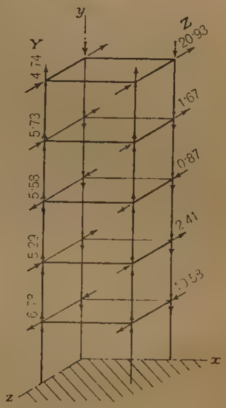


Fig. 7.



the constraints at different stages in the process of "liquidation". In these diagrams arrows are used to indicate residual forces and moments: on account of the symmetry of the problem it has not

Fig. 8.

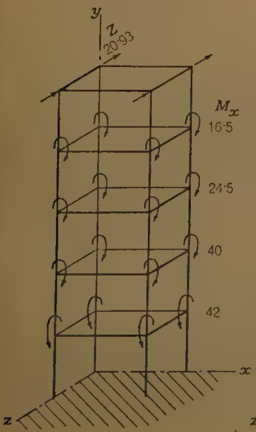


Fig. 9.

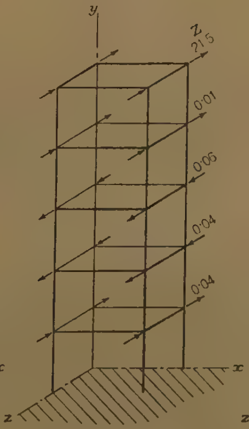
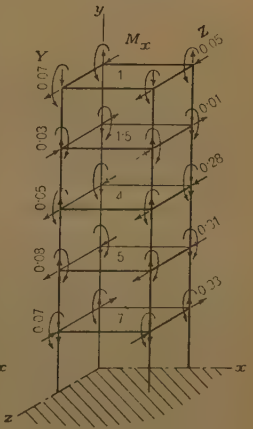


Fig. 10.



been deemed necessary to insert figures against every arrow. The strains involved are horizontal shear of floors in the $-w$-direction (that is, in a direction opposite to that of the z-axis), bending of the structure about the x-axis in the $-p$-direction (that is, in a direction

opposite to that which is taken as positive in measuring p), and distortions consequent on a release of the M_x-moments set up at the joints. By a judicious series of relaxations of these three types, the residual forces and moments on the constraints can be quickly reduced to negligible amounts.

First, the influence-coefficients for block-displacements in the $-w$-direction are worked out. It is merely necessary to tabulate the forces and moments induced at the ends of "cut" members when one block is permitted a shearing relaxation of unit magnitude. The block which is bounded by the floors $A_1B_1C_1D_1$ and $A_5B_5C_5D_5$ carries a total shearing force measured by -100 (lbs. weight units). It is allowed to move in the $-w$-direction just far enough to "liquidate" this action at the cost of bending and shear forces imposed

Fig. 11.

on the columns AA_1, BB_1, CC_1, DD_1; the block (2–5) is then moved similarly, and so on up to the top. *Fig. 6* shows the position at the end of this stage: all Z-forces have been "liquidated", but large M_x-moments have been induced at the joints.

In the next stage attention is concentrated on the M_x-moments. Treating the four joints on each floor as a group, this is rotated together with other groups in the $-p$-direction, until all the M_x moments have been "liquidated". The moments at the four base joints do not require to be liquidated, since these joints remain fixed. Here again the appropriate influence-coefficients have to be calculated from Table I. The coefficients for the top floor will differ from those below.

As shown in *Fig. 7*, when all the M_x-moments have been completely liquidated in this way, large Z-forces reappear, also a force in the Y-direction at every joint. Block-relaxations of the type indicated in *Fig. 11* serve to "liquidate" these Y-forces, and the necessary

rotations are so small that practically no other moments or forces are thereby imposed upon the constraints.

At this stage in the relaxation process the shear on the top floor is still more than 80 per cent. of its initial value. Mere repetitions of the three processes described above would thus give a very slow approach to complete "liquidation" of the Z-forces and M_x-moments, and accordingly the next step taken by the Author was to "liquidate" almost completely the Z-forces on floors 1, 2, 3 and 4, together with the M_x-moments and Y-forces on every floor except the base, without (for the moment) troubling about the Z-forces at the corners of the top floor. *Fig. 8* shows the comparatively small M_x-moments which remain after this partial block-displacement.

After several series of operations on the lines just described, the residual forces had the values shown in *Fig. 9*; on all floors except the top they may be regarded as negligible. At this stage is introduced a method which was explained in § 30* of Professor Southwell's Paper[1]:—The total effect of the displacements applied so far has been to reduce the shear forces on the top floor from 25 to 21·5: therefore multiplication of all displacements by the factor $\dfrac{25}{25 - 21{\cdot}5}$ (that is, by 7·14286) will result in an almost complete "liquidation" of the initial load-system. *Fig. 10* shows the final values of the forces remaining on the constraints: they are little more than 1 per cent., anywhere, of the forces which acted initially in this "partial problem".

Partial Problem (2) (*Torsion*).

The initial load-system is shown in *Fig. 12* (p. 210). Due to this torsional action several types of strain occur, each calling for treatment in a particular way. These are (i) twisting of the structure as a whole, (ii) distortion of floor rectangles into lozenge shapes, (iii) stretching of one pair of diagonally-opposite columns while the other pair are compressed, (iv) rotation of every joint (excepting those at the base) about all three axes.

The largest of these strains is the "lozenge effect" on the unbraced floors, producing a "scissor action" whereby the AD wall moves in the $-w$-direction and the BC wall in the $+w$-direction: this strain is dealt with first. It is convenient, for the purpose, to calculate influence-coefficients for a special type of "group-relaxation" which is represented[2] in *Fig. 13*. A "cut" is made through four

*, [1] Footnote 3, p. 196.

[2] On account of the exaggerated displacements, this diagram conveys a false impression that the upper floors have been contracted. It must be studied on the understanding that all displacements are horizontal.

columns, and all complete bays below the cut are imagined to be held fixed by appropriate constraints; above the "cut" complete bays are assumed to be distorted equally by rigid-body movements (in opposite directions) of a pair of opposite faces, as shown. The resistance to the displacement comes partly from those columns which are cut, but partly also from two girders which are thereby bent in every floor (*Fig. 13*); accordingly the influence-coefficients vary for each "cut", since at each successive floor upwards there are two less AB and CD members connecting the "blocks" moved.

Fig. 12.

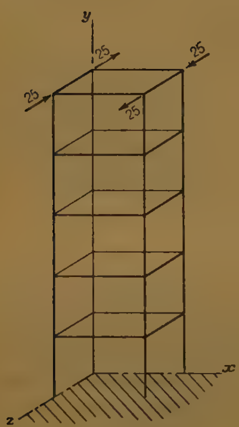
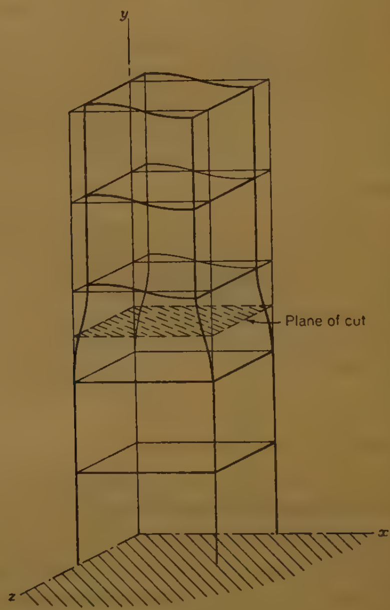


Fig. 13.



This fact accounts for the full advantage not being gained from "block-relaxation", but the effect is too small to reduce seriously the speed of "liquidation".

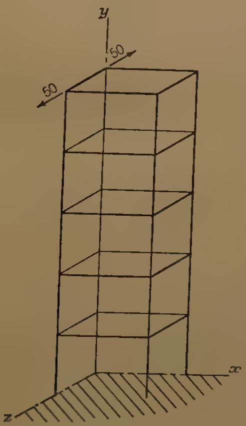
The greatest effect of this "scissor action" (which serves to "liquidate" the Z-forces) is to impose relatively large M_x-moments on all joints, which moments, when relaxed, restore the opposing Z-forces to about 80 per cent. of their original values. Accordingly, to accelerate the process of "liquidation", it was advisable at the start to overstep the mark by four—that is, to "liquidate" four times the Z-forces—and then to reduce the M_x-moments to zero. After the double operation the residual effects were quite small.

As the result of the processes described above, M_y-moments and Y-forces were induced at the joints, and on twisting all joints about the Y-axis to eliminate the M_y-moments small X- and Z-forces were again imposed on the joints. The Y-forces were easily "liquidated", and the X-forces were reduced simultaneously with the M_z-moments in the manner previously adopted for "liquidating" Z-forces and M_x-moments.

Partial Problem (3) (*Tension applied to member* A_5D_5 *in top floor*).

Fig. 14 shows the initial forces in this problem, which involves very little computation. Simultaneous relaxation of the two loaded

Fig. 14.



joints was found to "liquidate" the forces at the cost of very slight distortion of the rest of the frame, and there remain only two opposing Z-forces of magnitude 0·8 lb., with six very small M_x-moments. All of these may be regarded as negligible.

Conclusions.

The resultant displacements (which define the distortion of the frame when the three "partial" solutions have been combined) are given in Table II; they correspond with a unit load of 100 lbs. applied at A_5 (*Fig. 3*, p. 203). The nature of the distortion is indicated in *Fig. 15* (p. 213). The displacements and rotations are magnified 20×10^3 times in relation to the dimensions of the undistorted frame.

Table II.—Resultant Displacements and Rotations Produced by the Loading Shown in *Fig. 3.*

Displacements: feet.		
of type u (along x-axis)		
A_1, B_1	$+ 0·9637$	$\times 10^{-5}$
C_1, D_1	$- 0·9637$	,,
A_2, B_2	$+ 2·6087$	,,
C_2, D_2	$- 2·6087$	,,
A_3, B_3	$+ 3·9458$	,,
C_3, D_3	$- 3·9458$	,,
A_4, B_4	$+ 4·8671$	,,
C_4, D_4	$- 4·8671$	,,
A_5, B_5	$+ 5·3379$	,,
C_5, D_5	$- 5·3379$	,,
of type v (along y-axis)		
B_1, C_1	$\mp 0·1091$	$\times 10^{-5}$
A_1, D_1	$\mp 0·5691$	,,
B_2, C_2	$\mp 0·1967$	,,
A_2, D_2	$\mp 1·0437$	,,
B_3, C_3	$\mp 0·2579$	,,
A_3, D_3	$\mp 1·3955$	,,
B_4, C_4	$\mp 0·2932$	,,
A_4, D_4	$\mp 1·6162$	,,
B_5, C_5	$\mp 0·3078$	,,
A_5, D_5	$\mp 1·7106$	,,
of type w (along z-axis)		
A_1, D_1	$- 21·8182$	$\times 10^{-5}$
B_1, C_1	$- 3·9266$	,,
A_2, D_2	$- 64·9262$	,,
B_2, C_2	$- 12·0130$	,,
A_3, D_3	$- 114·7164$	,,
B_3, C_3	$- 20·9642$	,,
A_4	$- 165·4941$	,,
B_4, C_4	$- 29·0841$	,,
D_4,	$- 165·4905$	,,
A_5,	$- 210·8086$	,,
B_5, C_5	$- 35·4243$	,,
D_5,	$- 210·5900$	,,

Rotations: radians.		
of type p (about x-axis)		
A_1, D_1	$- 1·6061$	$\times 10^{-5}$
B_1, C_1	$- 0·3095$	,,
A_2, D_2	$- 2·1183$	,,
B_2, C_2	$- 0·4021$	,,
A_3,	$- 2·2740$	,,
B_3, C_3	$- 0·4039$	,,
D_3,	$- 2·2750$	,,
A_4,	$- 2·2097$	,,
B_4, C_4	$- 0·3375$	,,
D_4,	$- 2·2021$	,,
A_5,	$- 1·6969$	,,
B_5, C_5	$- 0·2531$	,,
D_5,	$- 1·6829$	,,
of type q (about y-axis)		
A_1, B_1, C_1, D_1	$- 0·2780$	$\times 10^{-5}$
A_2, B_2, C_2, D_2	$- 0·5560$	,,
A_3, B_3, C_3, D_3	$- 0·8091$	,,
A_4, B_4, C_4, D_4	$- 1·0123$	,,
A_5, B_5, C_5, D_5	$- 1·1298$	,,
of type r (about z-axis)		
A_1, B_1	$- 0·0689$	$\times 10^{-5}$
C_1, D_1	$+ 0·0689$	,,
A_2, B_2	$- 0·0693$	,,
C_2, D_2	$+ 0·0693$	,,
A_3, B_3	$- 0·0491$	,,
C_3, D_3	$+ 0·0491$	,,
A_4, B_4	$- 0·0291$	,,
C_4, D_4	$+ 0·0291$	,,
A_5, B_5	$- 0·0088$	,,
C_5, D_5	$+ 0·0088$	,,

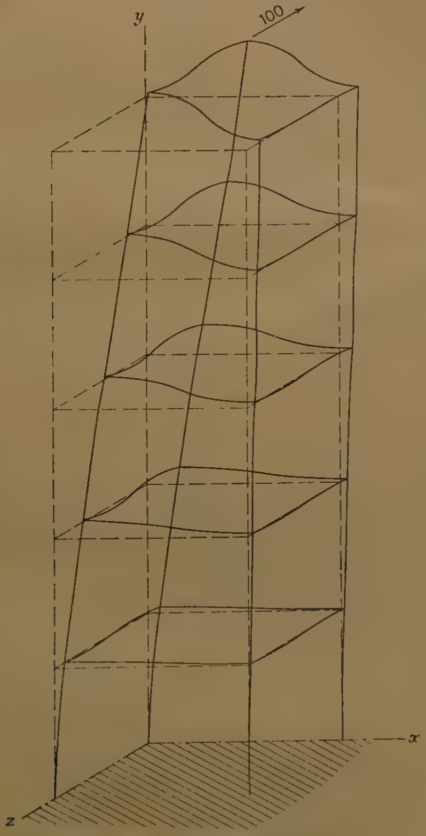
As a check on the accuracy of the numerical computations (not reproduced in this Paper) the forces and moments at each end of every member have been calculated from Table II by means of the influence-coefficients given in Table I. They are given in Table III—again, as figures corresponding with a unit load of 100 lbs. applied at A_5 (*Fig. 3*, p. 203).

Finally, the actions given in Table III (pp. 214–5) have been compounded to find the residual forces and moments which are left on the constraints when the displacements of Table II have been imposed. The results are given in Table IV (p. 216), and it will be seen that the residual forces are everywhere very small. In assessing the importance

of the residual moments it is to be remembered that a shearing action of only 1 lb., applied to one end of the shortest member, could entail a bending moment of 20 lbs.-feet at the other. Thus the accuracy throughout is greater than would be required in practical design.

For the numerical computations a Brunsviga calculating machine was employed, and account of the "unliquidated" forces and

Fig. 15.



SKETCH OF DISTORTED FRAMEWORK.

moments was kept on sheets containing blank line-diagrams which had been specially duplicated. This procedure had considerable advantages in the present connexion, since a very large number of columns would have been required if a Table had been drawn up. Moreover, a clearer vision of the residual forces and moments was obtained.

TABLE III.—RESULTANT ACTIONS IMPOSED ON

Units: 1 lb. weight;

Member	X	Y	Z	M_x	M_y	M_z
Top of column.						
A A_1	+ 2·1	− 341·5	− 43·2	+ 30	− 56	+ 3
A_1A_2	+ 2·0	− 284·8	− 44·0	+ 312	− 56	+ 20
A_2A_3	+ 1·1	− 211·1	− 44·0	+ 401	− 51	+ 17
A_3A_4	+ 1·1	− 132·4	− 44·6	+ 462	− 41	+ 15
A_4A_5	+ 0·7	− 56·6	− 46·9	+ 597	− 24	+ 12
B B_1	+ 2·1	− 65·5	− 6·2	− 15	− 56	+ 3
B_1B_2	+ 2·0	− 52·6	− 7·3	+ 50	− 56	+ 20
B_2B_3	+ 1·1	− 36·7	− 6·7	+ 66	− 51	+ 17
B_3B_4	+ 1·1	− 21·2	− 5·3	+ 70	− 41	+ 15
B_4B_5	+ 0·7	− 8·8	− 3·3	+ 54	− 24	+ 12
C C_1	− 2·1	+ 65·5	− 6·2	− 15	− 56	− 3
C_1C_2	− 2·0	+ 52·6	− 7·3	+ 50	− 56	− 20
C_2C_3	− 1·1	+ 36·7	− 6·7	+ 66	− 51	− 17
C_3C_4	− 1·1	+ 21·2	− 5·3	+ 70	− 41	− 15
C_4C_5	− 0·7	+ 8·8	− 3·3	± 54	− 24	− 12
D D_1	− 2·1	+ 341·5	− 43·2	+ 30	− 56	− 3
D_1D_2	− 2·0	+ 284·8	− 44·0	+ 312	− 56	− 20
D_2D_3	− 1·1	+ 211·1	− 43·9	+ 400	− 51	− 17
D_3D_4	− 1·1	+ 132·4	− 45·0	+ 458	− 41	− 15
D_4D_5	− 0·7	+ 56·6	− 46·9	+ 599	− 24	− 12
" A " end.						
A_1B_1	0	− 1·4	− 0·2	− 22	+ 3	− 21
A_2B_2	0	− 1·6	− 0·7	− 30	+ 10	− 24
A_3B_3	0	− 1·5	− 1·3	− 32	+ 19	− 22
A_4B_4	0	− 1·2	− 2·0	− 32	+ 29	− 18
A_5B_5	0	− 0·9	− 2·6	− 25	+ 39	− 14
" D " end.						
D_1C_1	0	+ 1·4	− 0·2	− 22	+ 3	+ 21
D_2C_2	0	+ 1·6	− 0·7	− 30	+ 10	+ 24
D_3C_3	0	+ 1·5	− 1·3	− 32	+ 19	+ 22
D_4C_4	0	+ 1·2	− 2·0	− 32	+ 29	+ 18
D_5C_5	0	+ 0·9	− 2·6	− 25	+ 39	0
" A " end.						
A_1D_1	− 0·2	+ 58·1	0	− 581	− 2	− 4
A_2D_2	− 0·4	+ 75·5	0	− 755	− 4	− 4
A_3D_3	− 0·5	+ 80·1	0	− 801	− 5	− 3
A_4D_4	− 0·7	+ 76·7	− 0·8	− 767	− 7	− 2
A_5D_5	− 0·8	+ 56·9	− 49·2	− 570	− 7	− 0
" B " end.						
B_1C_1	− 0·2	+ 11·2	0	− 112	− 2	− 4
B_2C_2	− 0·4	+ 14·3	0	− 143	− 4	− 4
B_3C_3	− 0·5	+ 14·2	0	− 142	− 5	− 3
B_4C_4	− 0·7	+ 11·6	0	− 116	− 7	− 2
B_5C_5	− 0·8	+ 8·3	0	− 83	− 7	− 0

MEMBERS BY THE LOADING SHOWN IN *Fig. 3.*

1 lb.-foot.

X	Y	Z	M_x	M_y	M_z
Bottom of column.					
− 2·1	+ 341·5	+ 43·2	+ 833	+ 56	+ 38
− 2·0	+ 284·8	+ 44·0	+ 568	+ 56	+ 20
− 1·1	+ 211·1	+ 44·0	+ 479	+ 51	+ 6
− 1·1	+ 132·4	+ 44·6	+ 429	+ 41	+ 5
− 0·7	+ 56·6	+ 46·9	+ 340	+ 24	+ 2
− 2·1	+ 65·5	+ 6·2	+ 140	+ 56	+ 38
− 2·0	+ 52·6	+ 7·3	+ 96	+ 56	+ 20
− 1·1	+ 36·7	+ 6·7	+ 67	+ 51	+ 6
− 1·1	+ 21·2	+ 5·3	+ 36	+ 41	+ 5
− 0·7	+ 8·8	+ 3·3	+ 11	+ 24	+ 2
+ 2·1	− 65·5	+ 6·2	+ 140	+ 56	− 38
+ 2·0	− 52·6	+ 7·3	+ 96	+ 56	− 20
+ 1·1	− 36·7	+ 6·7	+ 67	+ 51	− 6
+ 1·1	− 21·2	+ 5·3	+ 36	+ 41	− 5
+ 0·7	− 8·8	+ 3·3	+ 11	+ 24	− 2
+ 2·1	− 341·5	+ 43·2	+ 833	+ 56	− 38
+ 2·0	− 284·8	+ 44·0	+ 568	+ 56	− 20
+ 1·1	− 211·1	+ 43·9	+ 478	+ 51	− 6
+ 1·1	− 132·4	+ 45·0	+ 432	+ 41	− 5
+ 0·7	− 56·6	+ 46·9	+ 339	+ 24	− 2
"B" end.					
0	+ 1·4	+ 0·2	+ 22	+ 3	− 21
0	+ 1·6	+ 0·7	+ 30	+ 10	− 24
0	+ 1·5	+ 1·3	+ 32	+ 19	− 22
0	+ 1·2	+ 2·0	+ 32	+ 29	− 18
0	+ 0·9	+ 2·6	+ 25	+ 39	− 14
"C" end.					
0	− 1·4	+ 0·2	+ 22	+ 3	+ 21
0	− 1·6	+ 0·7	+ 30	+ 10	+ 24
0	− 1·5	+ 1·3	+ 32	+ 19	+ 22
0	− 1·2	+ 2·0	+ 32	+ 29	+ 18
0	− 0·9	+ 2·6	+ 25	+ 39	0
"D" end.					
+ 0·2	− 58·1	0	− 581	− 2	+ 4
+ 0·4	− 75·5	0	− 755	− 4	+ 4
+ 0·5	− 80·1	0	− 801	− 5	+ 3
+ 0·7	− 76·7	+ 0·8	− 766	− 7	+ 2
+ 0·8	− 56·9	+ 49·2	− 569	− 7	0
"C" end.					
+ 0·2	− 11·2	0	− 112	− 2	+ 4
+ 0·4	− 14·3	0	− 143	− 4	+ 4
+ 0·5	− 14·2	0	− 142	− 5	+ 3
+ 0·7	− 11·6	0	− 116	− 7	+ 2
+ 0·8	− 8·3	0	− 83	− 7	0

TABLE IV.—RESIDUAL FORCES AND MOMENTS LEFT ON THE CONSTRAINTS.

Units.—1 lb. weight; 1 lb.-foot.

	X	Y	Z	M_x	M_y	M_z
A_1	+ 0·13	+ 0·02	− 0·63	+ 5·16	− 0·41	+ 1·41
B_1	+ 0·13	+ 0·30	− 1·21	+ 8·73	− 0·41	+ 1·41
C_1	− 0·13	− 0·30	− 1·21	+ 8·73	− 0·41	− 1·41
D_1	− 0·13	+ 0·11	− 0·63	+ 5·16	− 0·41	− 1·41
A_2	− 0·45	− 0·21	+ 0·66	− 5·71	− 1·40	+ 1·90
B_2	− 0·45	− 0·13	− 0·08	− 3·32	− 1·40	+ 1·05
C_2	+ 0·45	+ 0·13	− 0·08	− 3·32	− 0·40	− 1·89
D_2	+ 0·45	+ 0·21	+ 0·74	− 5·21	− 1·40	− 1·89
A_3	+ 0·43	+ 0·05	+ 0·73	+ 1·94	− 4·15	+ 2·31
B_3	+ 0·43	− 0·01	+ 0·09	+ 6·88	− 4·15	+ 2·31
C_3	− 0·43	+ 0·09	+ 0·09	+ 6·86	− 4·15	− 2·31
D_3	− 0·43	− 0·05	+ 0·19	+ 0·75	− 4·15	− 2·31
A_4	+ 0·30	+ 0·34	+ 0·48	− 2·73	− 5·74	+ 2·54
B_4	+ 0·30	− 0·36	+ 0·08	+ 2·32	− 5·74	+ 2·54
C_4	− 0·30	+ 0·36	+ 0·08	+ 2·46	− 5·74	− 2·54
D_4	− 0·30	− 0·34	− 0·70	+ 0·88	− 5·74	+ 0·46
A_5	+ 0·07	+ 0·62	− 1·35	− 1·53	− 8·35	+ 2·39
B_5	+ 0·07	− 0·50	+ 0·54	+ 4·86	− 8·35	+ 2·41
C_5	− 0·07	+ 0·50	+ 0·64	+ 5·10	− 9·30	+ 11·14
D_5	− 0·07	− 0·62	+ 0·30	− 5·19	− 8·30	+ 11·86

From experience gained on this particular problem, the Author is convinced that more complex load-systems, applied to a similar framework, would not present greater difficulties. Vertical loading, especially if symmetrical, is very quickly dealt with, and the case of an isolated moment imposed at some particular joint can be subdivided into three partial problems in the manner here employed to deal with an isolated force.

In conclusion, the Author desires to acknowledge his indebtedness to Professor R. V. Southwell, M.A., F.R.S., for his interest and advice in the preparation of this Paper, and also to Mr. E. Warlow-Davies, for many suggestions and for assistance in the numerical computations.

The Paper is accompanied by nine sheets of diagrams, from which the Figures in the text have been prepared.

ENGINEERING RESEARCH.

THE INSTITUTION RESEARCH COMMITTEE.

Joint Sub-Committee on Special Cements.

An abstract has already been given[1] of the interim report of the work of the British Sub-Committee on Special Cements of the British Committee on Large Dams of the World Power Conference. (The constitution of the British Sub-Committee on Special Cements as a Joint Sub-Committee of The Institution Research Committee and the British Committee on Large Dams was announced in the February, 1936, Journal.)[2] Mr. F. M. Lea, D.Sc., F.I.C., of the Building Research Station who attended the Second International Congress on Large Dams in September as the representative of the British Sub-Committee has summarized the present position with regard to special cements in the following Report.[3]

Special Cements for Mass-Concrete Structures and their Specification.

The subject of special cements has aroused considerable interest in many countries during recent years. The term "special cements" is used to cover cements in which some modification of the properties of Portland cement are deliberately introduced to provide the user with a material more suited to certain requirements than the normal or rapid-hardening products of the present day. Special cements may be Portland cements falling within the composition limits of the British Standard Specification for Portland cement, but in some cases they fall into a class which is often termed "blended cements." Such cements are composed of a mixture of Portland cement and another material; for example, pozzuolanic cements contain a mixture of Portland cement and a pozzuolanic material. Portland blast-furnace cement also belongs to the class of blended cements

[1] Journal Inst. C.E., vol. 2 (1935–36), p. 363. (March, 1936.)
[2] Journal Inst. C.E., vol. 2 (1935–36), p. 175. (February, 1936.)
[3] Crown copyright reserved.

since it is composed of a mixture of Portland cement and granulated blast-furnace slag.

The most widespread demand for special cements has come from engineers engaged in the construction of large mass-concrete works. At the First International Congress on Large Dams held at Stockholm in 1933 attention was directed to the need for cements of special properties for use in concrete dams. Stress was laid in particular on the desirability of reducing shrinkage-cracking. It was recognized that a major factor involved in shrinkage-cracking was the high temperatures attained in such large concrete masses, owing to the heat evolved during the hydration of cement, and to the thermal contraction which occurs on subsequent cooling. The desirability of obtaining cements with a lower heat-evolution was thus brought to the fore.

In the present Report it is proposed to review the progress which has been made in the production of low-heat cements and to discuss the specification and properties of such cements. Particular attention will be paid to the forms of specification now used in the U.S.A. since they involve a radical departure from usual methods.

After the meeting in Stockholm of the International Congress on Large Dams, an International Sub-Committee on Special Cements was set up under its ægis and corresponding national committees were established in many countries. In Great Britain, for example, a Sub-Committee on Special Cements was set up in 1934 by the British National Committee on Large Dams, and later this committee was reconstituted as a joint committee of The Institution and the British Committee on Large Dams. The working programme of the International Sub-Committee included the enunciation and specification of the special properties desired in cements for large dams and the provision of suitable routine test methods by which cements could be examined. It was not a function of the committee to describe how such a cement should be manufactured, but to indicate to the manufacturers the properties desired and the tests to which it should conform. As a first stage in its work the International Sub-Committee directed its attention to the heat of hydration of cements, the drying-shrinkage, permeability and workability of concretes, and the action on cement of water percolating through concrete.

The International Sub-Committee in the early part of 1936 drew up an Interim Report which was presented at the Second International Congress on Large Dams held at Washington, D.C., U.S.A., in September, 1936, concurrently with the meeting of the World Power Conference. The sub-committee's Interim Report, in accordance with their terms of reference, made no attempt to lay down how cements of the type desired should be manufactured, but was confined

to recommendations on test methods and reports of experimental work which had been carried out under the auspices of various co-operating national committees. Tentative recommendations were made for methods of testing the heat-evolution and solubility of cements. For the former a simple adiabatic method developed by the Building Research Station for the British Joint Committee on Special Cements was proposed, and for the latter a method presented by the Swedish National Committee. Full descriptions of these test methods were included. These recommendations were subsequently endorsed at the Washington Congress by a resolution of the Executive Committee of the International Commission on Large Dams. The International Sub-Committee had not reached any final conclusion on test methods for workability or shrinkage, and as regards permeability tests its recommendations were limited to a brief statement of general principles. These and other matters remain for the future consideration of the sub-committee.

It may be well to note here that the sub-committee emphasized strongly that, whilst the use of special cements may be expected to be of valuable assistance in overcoming certain troubles in mass-concrete construction, their use will in no way obviate the necessity of taking every precaution to obtain sound concrete of a high quality. This is, and always will be, of fundamental importance for all concrete structures, and particularly for large dams.

Although the International Sub-Committee included amongst its members a representative of the U.S.A. who had submitted many extensive and valuable reports, it had not, for reasons of distance, been possible for the U.S.A. to be represented at the meetings held in various European cities. The sub-committee stated in their Interim Report that they felt they had not been able to take full advantage of American experience and they had dealt mainly with European practice. It was therefore particularly fortunate that their Interim Report could be presented to a Congress held in Washington, and an opportunity afforded for the sub-committee to obtain the views and criticisms of American authorities, and to study at first hand developments in the U.S.A.

During recent years considerable experience has been gained in the U.S.A. in the use of special cements for dams. Notable examples are to be found in Boulder dam on the Colorado river and Grand Coulee dam on the Columbia river together with many smaller structures coming under the authority of the U.S. Bureau of Reclamation; Norris dam in the Tennessee basin under the Tennessee Valley Authority; Bonneville dam on the Columbia river and Tygart river dam in the upper Ohio river basin under the U.S. Army Engineers; and Morris dam built by the Water Department of

Pasadena. Of these dams, Boulder, Morris, and Norris are completed, whilst construction is in progress on all the others.

The General Reporter on "Special Cements" to the Washington Congress was Mr. J. L. Savage, Chief Designing Engineer to the U.S. Bureau of Reclamation, and his report summarized the various Papers submitted on this subject. A separate addendum to his report dealt specifically with the Interim Report of the International Sub-Committee on Special Cements. General agreement was expressed with the conclusions of the International Sub-Committee as to methods of testing heat of hydration and solubility of cements and on the general principles enunciated for the testing of permeability. The reporter then went on to say: "As a complement to the plan for developing standard test methods for the five selected properties (heat-evolution, solubility, permeability, shrinkage, and workability) it is assumed that the sub-committee contemplates that cements tested by such methods will likewise be tested for revealing their chemical and physical properties. This is considered a matter of the greatest importance, particularly with regard to cement fineness and chemical composition which play so prominent a role in the physical behaviour of concrete and consequently in the interpretation of the various test data. So dominant, in fact, is the influence of fineness on some of the properties of cement in concrete, at least within a certain range of specific surface, that provision for its reliable determination might easily be considered of greater concern in the study of special cements than any of the test methods dealt with in the sub-committee's Interim Report."

The inference in this statement that, once the chemical composition and fineness of cements (of the Portland-cement type) are specified, the properties of the cements directly follow, forms the basis of the more recent U.S. specifications for special cements.

It has been considered worth while to outline the work and attitude of the International Sub-Committee for it serves to bring into marked contrast the different ideas of cement-specification methods which are at present held by the European and the American authorities. The European approach to the problem, as evidenced by the Interim Report of the International Sub-Committee, has been to endeavour to formulate tests for the properties desired and to leave it to the manufacturer to produce as best suits him a cement conforming to those tests. In the American form of specification as now used for special cements, a form which is also extending into other fields such as cements for concrete roads, the composition and fineness of the cement are prescribed to the manufacturer, and the engineer formulating the specification assumes the responsibility that the material will have the physical properties desired. This method of

specification, first introduced by the U.S. Bureau of Reclamation, has as its basis a very large amount of fundamental research work on cements and of experience in construction. A very valuable summary of this research work and of practical experience in construction was presented by Mr. J. L. Savage to the Washington Congress under the title "Special Cements for Mass Concrete." Before proceeding to discuss the recent American specifications in detail it should be mentioned that not all authoritative opinion in the U.S.A. is in agreement with the theoretical basis of these specifications, whilst it is also held by some that they may prove unduly restrictive of future developments.

A brief digression into the chemistry of Portland cement is necessary here for the full appreciation of the U.S. specifications. Portland cement is mainly composed of lime, silica, alumina, iron oxide, and magnesia. Modern research has established that the compounds formed from these oxides in the burning of cement clinker are tricalcium silicate ($3CaO . SiO_2$), dicalcium silicate ($2CaO . SiO_2$), tricalcium aluminate ($3CaO . Al_2O_3$), tetracalcium aluminoferrite ($4CaO . Al_2O_3 . Fe_2O_3$), and uncombined magnesia. From the ordinary oxide analysis of a cement, together with a determination of the amount of lime remaining uncombined, an arithmetical calculation can be made and the composition of the cement expressed in terms of these various compounds. The compound composition is only another method, though for many purposes a very convenient method, of representing the cement-analysis, and the accuracy of the calculated contents of the various compounds is dependent on certain assumptions which do not appear in the arithmetical procedure. It is known that other compounds beyond those mentioned may be present to some extent and that the calculation as described above is not entirely correct, but the extent of the errors is at present uncertain. Modern research has also shown that tricalcium silicate is mainly responsible for the early strength of cements and dicalcium silicate for the strength developed from 14 or 28 days onwards; the iron-containing compound contributes little strength at any age, and the tricalcium aluminate, though contributing somewhat to early strength, causes a reduced strength at long ages. Tricalcium aluminate has the greatest heat of hydration, tricalcium silicate next, and dicalcium silicate and the iron compound least. It will be evident that, as the compounds mainly responsible for the early strength-development in Portland cement have also a high heat-evolution on hydration, some sacrifice of strength at early ages (although not at long ages) must be made if a low heat-evolution is desired. The effect of these different compounds on shrinkage and durability is more uncertain, apart from

the special case of resistance to attack by sulphate-bearing waters in which the tricalcium aluminate is the most vulnerable. The basis of the U.S. specifications is a limitation of the compound content of cements in such a way as, when combined with a certain specified minimum fineness, to obtain the particular properties desired. The test of fineness is not a sieve test, but a much more refined measurement of the surface area per unit weight of the cement, for which suitable methods have been developed. A strength test is at present retained, but the specified strength is low.

That form of specification has clearly many attractions for it depends mainly on chemical tests and fineness tests which are both accurate and rapid, though probably most European authorities are in agreement with some dissenting American opinion that it represents a step which is not yet sufficiently supported by knowledge.

The low-heat cement used for Morris dam in California (originally called Pine Canyon dam) was the first to be supplied under a specification which placed a limit on the heat-evolution. This dam was built during 1932–34 and at the time the specification was drafted, the cement investigations carried out for Boulder dam were in progress and information on the properties of the low-heat cements was available. The Morris dam cement-specification was based mainly on physical properties and the only chemical limitation, apart from the limits for magnesia, sulphur trioxide and loss on ignition which are found in all cement specifications, was one on the content of tricalcium aluminate. The first specification for low-heat cement for Boulder dam was issued in 1933 and revised specifications followed later, the last specification being issued in 1934. Some difficulty was experienced by cement manufacturers in supplying cement to conform with the heat-evolution requirements of the original 1933 specification, and in the 1934 specification the maximum permissible heat-evolution was raised. In the Boulder dam specifications for low-heat cements limitations were placed on the contents of tricalcium silicate, dicalcium silicate, tetracalcium aluminoferrite, and the ratio of ferric oxide to alumina. Details of these specifications are given in Table I.

The original Morris dam specification permitted a maximum residue of 15 per cent. on a 200-mesh sieve, but when construction started it was found with the cement from one manufacturer that excessive separation of water from the concrete occurred in placing. The fineness limit was therefore altered to a maximum of 8 per cent. residue on the 200-mesh sieve [1] and thereafter no trouble was found.

[1] The U.S. 200-mesh sieve has an opening of 0·0074 centimetre compared with one of 0·0089 centimetre in the B.S. 170 mesh. For a given fineness the residues are therefore higher on the U.S. 200 sieve than on the B.S. 170 sieve.

Table I.—American Specifications for Low-Heat and Modified Portland Cements.*

		Low-heat Portland cements.			Modified Portland cements.		
		Morris dam 1932–34.	Boulder dam June 1933.	Boulder dam March 1934.	Norris dam 1934.	Grand Coulee dam 1935.	Tygart river dam 1935.
Chemical composition.							
Tricalcium silicate: per cent.	Not less than	—	—	—	35	35	35
	Not greater than	—	40	40	55	55	50
Dicalcium silicate: per cent.	Not greater than	—	60	65	—	—	—
Tricalcium aluminate: per cent.	Not greater than	6	6	7	8	7	8
	Not less than	1	—	—	—	—	—
Tetracalcium aluminoferrite: per cent.	Not greater than	—	20	20	—	—	—
Ratio of Fe_2O_3 to Al_2O_3:	Not greater than	—	1·5	1·5	1·5	—	—
Uncombined lime (CaO): per cent.		—	—	—	—	1·25	—
Fineness.							
Residue on 200-mesh sieve: per cent.	Not greater than	8	—	—	—	—	—
	Not less than	2	—	—	—	—	—
Specific surface: square centimetre per gram.	Not less than	—	1,800	1,700	1,600 (later 1,800)	1,800	1,600
	Not greater than	—	2,300	2,300	2,200	—	2,200
Heat of hydration: calories per gram.							
7 days	Not greater than	65	60	65	—	—	—
28 days	” ”	80	70	75	—	—	—
Compressive strength: lbs. per square inch.							
1 : 3 standard sand mortar cylinders 2 inches by 4 inches. Water about 10 per cent. by weight of dry materials.							
7 days		800	1,000	1,000	—	—	—
28 days		2,000	2,400	2,000	—	—	—
Minimum per cent. increase: 7 to 28 days		35	35	35	—	—	—
1: 2·77 graded sand plastic mortar 2-inch cubes. Water about 14 per cent. by weight of dry materials.							
3 days		—	—	—	750	—	750
7 days		—	—	—	1,500	1,500	1,500
28 days		—	—	—	2,500†	2,500	2,500
Minimum per cent. increase: 7 to 28 days		—	—	—	28-day strength greater than 7-day.	25	—

* These specifications also include setting-time, soundness tests and limits for magnesia, sulphur trioxide and ignition loss.

† Not required as acceptance test.

It may be noted here that the concrete in Morris dam was placed with the aid of internal vibrators. This was also the case for all the later dams, with the exception of Boulder dam of which only parts were vibrated. American authorities attach much importance to high fineness in cements, as it has been found that the separation of water from concrete during vibration is thereby reduced. In the specifications for cements for the dams constructed subsequent to Morris dam, the fineness is controlled by a requirement of a minimum surface area per gram of cement. The actual figures inserted in the specifications for surface area cannot be directly compared with the sieve-residue values used in this country, but from measurements made at the Building Research Station of the surface area of British cements it appears that the fineness of the cements used in the American dams is comparable with that of the rapid-hardening Portland cements marketed here. It should be noted that sieve tests are of very little value for controlling the fineness of cements, and that, when considerable importance is attached to this property, the use of more refined methods of measurement, such as the American methods for surface area, becomes essential.

At Morris dam where the climate was hot in summer and mild in winter the rate of hardening of the low-heat cement was satisfactory and the shuttering could be removed within the intended time, about 3 days. It was originally intended to use the low-heat cement throughout at Boulder dam, but the rate of hardening of this cement in the winter months was found to be rather too slow for the rapid rates of construction employed ; a blend of 60 per cent. low-heat and 40 per cent. normal Portland cements was therefore used during cold weather. It is stated, however, that rather slight changes in construction procedure would have eliminated the necessity for this expediency.

In the dams on which construction has been started subsequent to Boulder dam, such as Norris dam in the Tennessee valley, Grand Coulee dam on the Columbia river, and Tygart river dam in the upper Ohio river basin, a type of cement known as " modified " or " moderate heat " Portland cement has been used. This cement, which has been adopted as a compromise, evolves on the average about 10 per cent. less heat than normal Portland cement, as compared with a figure of about 27 per cent. less for low-heat cement, but it has a rate of strength development about the same as that of normal Portland cement. It differs mainly from normal Portland cement in having a low content of tricalcium aluminate. There are certain differences in the specifications, as shown in Table I, for the modified cement for the three dams mentioned and attention may be directed to that for Grand Coulee dam as it represents the most

highly-developed form of the new American type of specification. Maximum and minimum limits are placed on the tricalcium silicate content and a maximum limit on the content of tricalcium aluminate. A maximum limit is also set to the content of uncombined lime. This limitation will rule out a cement which has a high lime-content, but is underburnt and, owing to combination of the lime not being complete, still contains a tricalcium silicate content below the maximum specified. No direct test is required for heat-evolution as it is considered that the composition requirements are adequate to control this. A high fineness, similar to that of rapid-hardening Portland cements in this country, is again required. The specified strength is at a low level though somewhat above that in the Boulder dam low-heat cement. The strength test in both the Boulder and Grand Coulee cement-specifications is a compressive test on a mortar, but the form and composition of the test piece differ. After allowing for the difference introduced in this way the minimum strength required in the Coulee cement is about one-third higher at 7 days than that in the Boulder cement, and slightly higher at 28 days. The strength test appears only to be retained in the Grand Coulee cement-specification as a precautionary measure and reliance is placed primarily on the composition and fineness requirements to ensure a cement of the required properties.

The average heat-evolution of American cements of various types is given by Mr. J. L. Savage as follows :

TABLE II.

Cement.	Heat of hydration: calories per gram.		
	3 days.	7 days.	28 days.
Rapid-hardening Portland	102	108	114
Normal Portland	79	86	91
Modified Portland	63	74	82
Low-heat Portland	44	52	65
Boulder dam low-heat cement-specification	—	65	75

These values are all based on the American "heat of solution" method. Relatively few directly-comparable values on British cements are available as practically all tests in this country have been made by the adiabatic calorimetric method. Values on three British normal Portland cements by the American "heat of solution" method have, however, given values of 72–96 calories per gram at 7 days and 94–114 at 28 days.

As mentioned before, the strength requirements in the American

low-heat and modified cement specifications are low and the actual strength of the cements supplied has been considerably above the limits specified. The average values for the low-heat cements from five different manufacturers supplied for Boulder dam are shown in Table III.

TABLE III.—MORTAR TESTS ON LOW-HEAT CEMENTS SUPPLIED FOR BOULDER DAM.

Cement.	Compressive strength: lbs. per square inch. (1 : 3 standard sand mortar with 10 per cent. water.) 4-inch by 2-inch cylinder.	
	7 days.	28 days.
A	1,750	3,420
B	1,660	3,630
C	1,530	3,300
D	2,320	4,570
E	1,680	5,300
Specification requirement	1,000	2,000

The data on the modified cements supplied for Grand Coulee dam given in Table IV also show actual strengths much above the specification requirement.

TABLE IV.—MORTAR TESTS ON MODIFIED CEMENTS SUPPLIED FOR GRAND COULEE DAM.

Cement.	Compressive strength: lbs. per square inch. (1 : 2·77 graded sand mortar with about 14 per cent. water.) 2-inch cubes.	
	7 days.	28 days.
1	2,137	4,401
2	2,940	5,250
3	3,435	5,060
4	3,200	5,330
5	3,160	6,200
Specification requirement	1,500	2,500

From various data which are available, it appears that the compressive strength of concrete made from the Boulder dam low-heat cements is rather below one-half of that obtained with average normal Portland cement at 7 days and about three-quarters at 28 days. The modified cements give a slightly lower concrete strength than average normal Portland cement at 7 days and an equal strength at 28 days. Both the low-heat and modified cements

show rather higher strengths at long ages, 6 months and upwards, than normal Portland cement.

Particular interest attaches to the cement being used for the construction of Bonneville dam on the Columbia river in the U.S.A. This cement is a pozzuolanic cement composed of 75-per-cent. cement clinker of the modified Portland-cement type and 25 per cent. of a pozzuolana prepared by burning, at about 1,650° F., a dredged mud. The clinker and pozzuolana are ground together to a fineness which is even greater than that required for the low-heat and modified Portland cements discussed earlier. The heat-evolution of this cement is intermediate between that of low-heat and modified Portland cement. Though no advantage is gained in compressive strength, the tensile strength of concrete made with this pozzuolanic cement has been found somewhat superior to that of modified Portland-cement concrete. It is also stated that the concrete shows improved workability and less tendency to water-separation when placed by internal vibration methods. Pozzuolanic cements in general have a higher temperature-coefficient of strength-development than Portland cements and are apt to become very slow in hardening at low temperatures. It has been found, however, at Bonneville dam that in cold weather (35–40° F.) the shuttering can still be stripped after 3 days provided that this is done carefully to avoid damage to the concrete. An addition of 10 per cent. more cement to the concrete is made in cold weather as a safeguard. The specification for this cement requires separate tests on the pozzuolana and the Portland-cement clinker and also tests on the finished blended cement.

Although low-heat cements have now been manufactured in a number of countries in addition to the U.S.A. the only other country in which appreciable practical experience is available is Sweden. The development of low-heat cement in Sweden was contemporary with that in the U.S.A. Three forms of low-heat cement have been made, one a low-heat Portland cement first marketed in 1932 and the other two low-heat pozzuolanic cements which were first produced commercially in 1934. The two pozzuolanic cements were made by grinding a pozzuolana with normal and low-heat Portland cement respectively, and they have different rates of heat-evolution and hardening. Details of these cements and experience in their use were included in reports from Sweden to the Washington Congress. The heat-evolution of these cements as determined by the American heat of solution method is shown in Table V (p. 228).

The low-heat Portland cement is intermediate between the American low-heat and modified Portland cements in its heat-evolution up to 7 days and similar to the American modified cement in the

Table V.—Comparative Heat-Evolution of Swedish Cements.

Cement.	Heat evolution: calories per gram.		
	3 days.	7 days.	28 days.
Normal Portland	80	94	104
Low-heat Portland	57	69	86
Pozzuolanic containing normal Portland cement	61	75	85
Pozzuolanic containing low-heat Portland cement	57	66	72

28-day value. The strength properties also appear to follow a similar course. The Swedish low-heat Portland cement differs, however, from the American cements in that it is more coarsely, instead of more finely, ground than the average normal Portland cement. This is done in order to obtain a rather prolonged time of initial set, thus assisting in the placing of large masses of concrete. It involves, however, the disadvantage that the workability of the concrete is reduced and in practice it has been found desirable to add a small amount of diatomaceous earth (3 per cent. by weight of the cement) to the concrete mix to offset this. The different views held in the U.S.A. and Sweden in regard to the fineness desirable probably arise, in part at least, from differences in the methods of construction and the extent to which large-scale mechanical aids are used to facilitate rapid placing of the concrete.

The Swedish pozzuolanic cements are ground more finely than the low-heat Portland cement. Details of practical experience are only available for the cement blended with a normal Portland cement. This is stated to yield concrete with a workability similar to that obtained from the normal Portland cement. The strength-characteristics are similar to those of the low-heat Portland cement, the 7-day compressive strength being about 70 per cent. of that of normal Portland cement and the 28-day strength equal or higher. The method of specification adopted in Sweden is to specify the heat-evolution and strength required and not to use detailed requirements as to the chemical composition and compound content of the cement as in the American specifications.

No low-heat cement has been produced in Great Britain on a commercial scale up to the present, but manufacturers are now actively interested in the production of such special cements. The Joint Sub-Committee on Special Cements of The Institution and the British National Committee on Large Dams is at the same time endeavouring, with the aid of the information now at its disposal, to

formulate the tests to which such cements should be required to conform.

RESEARCH WORK IN ENGINEERING AT WOOLWICH POLYTECHNIC, JANUARY, 1937.

The following researches are being carried out at Woolwich Polytechnic under the Departments of Civil and Mechanical Engineering and of Electrical Engineering.

Civil and Mechanical Engineering Department.

The variation of the coefficient of expansion of concrete with differing proportions of aggregate is being studied by comparing the differential expansion of concrete and steel. The distribution of stress and the direction and magnitude of the principal stresses in a deep riveted plate-girder with the usual complications of web stiffeners and additional flange plates at the centre has been determined by applying by hydraulic jack a central load between two similar beams placed back to back and tied together at their ends. A large amount of research has been carried out on wire ropes. At the moment a research is in progress to ascertain the requisite conditions so that the loops which form in a wire rope when released from tension will pull-out without kinking on a load being re-applied. This is of importance in connexion with mooring hawsers. The torsion and bending of wire ropes are also being studied. The fatigue strength in torsion of steel specimens of non-circular cross-section is being investigated. The specimen is subjected to repetition of predetermined angular strain and the stress measured optically. Tests are being carried out on various steels with various heat-treatments.

Electrical Engineering Department.

A research is being carried out into the variation, with change of current-density, of the potential-difference at electrodes of different materials in various electrolytes. This research is of particular importance in connexion with electro-plating.

Many researches are being carried out on the subject of wireless communications. The stabilization of valve circuits at supersonic frequencies has been effected by the use of a rod of invar steel vibrating at its natural longitudinal frequency in a magnetic circuit. The choice of material is found to affect greatly the range over which

stabilization is effective. In another research an attempt is being made to develop an amplifier which shall be distortionless at all frequencies, a matter of great importance in wireless reception. The development of measuring instruments suitable for operation at very high frequencies is attended by difficulties and it has not so far been possible to obtain accurate measurements for wave-lengths of less than about 5 metres. An attempt is being made by the production of special valves in which capacities are reduced to a minimum to extend the range of measurement down to a wave-length of 1 metre. A watt-meter is being developed capable of measuring down to micro-watts. This is based upon the employment of valves permitting the passage of a current proportional to the square of the voltage. It is possible to arrange electrical circuits so that the voltage-difference between two such valves is always proportional to the product of current and voltage. A start is being made on a research into television.

The above researches are being carried out under the direction of Edward Mallett, D.Sc. (Eng.), Principal, and Head of the Electrical Engineering Department, W. A. Scoble, M.B.E., D.Sc. (Eng.), Head of the Civil and Mechanical Engineering Department, and T. B. Vinycomb, M.C., M.A., Head of the Department of Physics.

REPORT OF THE BUILDING RESEARCH BOARD FOR THE YEAR 1935.

In the Report for the year 1935 prominence is given to certain major researches which have originated in an inquiry into methods of construction for the building of flats for working-classes. These concern sound-insulation, bug infestation, and fire resistance, and research in connexion with these problems is in progress.

With regard to general research and co-operative investigations work on building stones is proceeding. Research on asphalt and bitumens with particular reference to roofing material has continued, and a method has been devised whereby specimens are subjected to artificial weathering conditions. Special low-heat cements are being studied as a co-operative research with the British Sub-Committee on Special Cements of the World Power Conference.[1] The suitability of air-cooled and "foamed" blast-furnace slag as concrete aggregates has been investigated. Research

[1] Now constituted as a Joint Sub-Committee of The Institution Research Committee and the British Sub-Committee on Special Cements.—See footnote 2, p. 217.

has been continued on cast concrete products with particular reference to crazing, on renderings, asbestos-cement roofing-materials, sand-lime bricks, and limes and plasters. Another line of research concerns clay building-materials, work having been done on the firing of clays, florescence of bricks, and their weathering properties. The drawing-up of recommendations for laying jointless (magnesium oxychloride) floors is in hand. Research on the stability of paint on plaster and cement backgrounds has continued.

A large amount of research has been carried out on structures and strength of materials. The redistribution of moments during loading in continuous beams and frames of reinforced concrete have been investigated on large-scale specimens. A study of cracking due to atmospheric exposure and due to restraint of shrinkage has been made. The effective modulus of elasticity in eccentrically loaded reinforced-concrete columns has also been determined. The investigation of the action of sea-water on reinforced concrete which is being carried out in co-operation with the Sea Action Committee of The Institution of Civil Engineers has continued. A description of work done in connexion with investigations on the driving of reinforced-concrete piles is given.[1] Tests have been continued on the strain of a reinforced-concrete road slab under a concentrated load. A vibrating machine has been devised for the compacting of mortar test-cubes.[2] Work on the grading of aggregates and the workability of concrete has continued.

Among other researches may be mentioned an investigation into the strength of brickwork built in lime mortar gauged with cement, and tests on high-tensile steel beams and struts. The erection of the fire-testing station at Elstree has presented the Building Research Station with several technical problems during the year. A study of the stresses in floor beams due to vibration in buildings has been made. In connexion with the research on wind-pressures the experiments on the Severn bridge have now been concluded and work is continuing at the National Physical Laboratory on the shielding effect of neighbouring buildings. On the subject of soil-physics work is being carried out on the compressibility, rate of compression, and shear strength of soils.[3]

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[1] See also W. H. Glanville, Geoffrey Grime, and W. W. Davies, "The Behaviour of Reinforced-Concrete Piles during Driving." Journal Inst. C.E., vol. 1 (1935–36), p. 150. (December, 1935.) Further research is being carried out in co-operation with the Joint Sub-Committee on Pile-Driving. Journal Inst. C.E., vol. 2 (1935–36), p. 177. (February, 1936.)

[2] Reference may be made to the work of the Joint Sub-Committee on Vibrated Concrete. Journal Inst. C.E., vol. 1 (1935–36), p. 42. (November, 1935.)

[3] See work of the Sub-Committee on Earth-Pressures. Journal Inst. C.E., vol. 2 (1935–36), p. 361. (March, 1936.)

A further class of research concerns the efficiency of buildings from the standpoint of the user, and tests have been made on emission of heat from panel radiators, the thermal conductance of a brick wall, the heat-transmission through roofs, and the exclusion of solar heat by various kinds of paint. The illumination of buildings by daylight and their acoustic properties have been studied. In addition, a large number of special investigations have been carried out.

NOTES ON RESEARCH PUBLICATIONS.

Measurement.

A method of determining the elastic constants of soils by means of vibration methods is described in *Bulletin of the Earthquake Research Institute, Tokyo Imperial University*, **14,** *632.* A new method of measuring surface quality by the capacity of a condenser is given in *Masch.*, **15,** *669* (*Eng. Abs.* **72,** 3). Flow through measuring nozzles and orifices is discussed in *Forschungsheft, No. 381* of the *Verein deutscher Ingenieure* (*Eng. Abs.* **72,** 161). The measurement of static and total pressure in pipes is discussed in *Am. Soc. Mech. Engineers, Power Test Codes, 1936, Part 2.* An absolute determination of the ampere as carried out at the National Physical Laboratory is described in *Phil. Trans. Roy. Soc., Series A.*, **236,** *133.* The following researches deal with electrical measurement: Temperature compensation of millivoltmeters, *U.S. Nat. Bur. Stand. J. Research*, **17,** *497*; Voltage-measurements at extremely high frequencies by the Diode voltmeter, *Hochfrequenz.*, **48,** *117* (*Eng. Abs.* **72,** 8); and in the same journal, *p. 158*, Measurement of peak voltages at all frequencies; A capacitance-attenuator and its application to the measurement of very small capacitances, *J. Sci. Instruments*, **13,** *407.*

Engineering Materials: Properties and Testing.

Bricks, Cement, and Concrete.

Investigations into the influence of repeated freezing and thawing on the relation of water absorbed to pore volume of bricks are

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described in *U.S. Nat. Bur. Stand. Technical News Bulletin, 1936, (234), 86.* The applications of aluminous cement and its influence on concrete construction are discussed in *Chemistry and Industry*, **55,** *1037.* A report on cement setting temperatures, "*Temperatura di presa dei cementi*," by L. Santarella (1936), has been published by U. Hoepli at Milan. An electrical conductometric analysis of Portland cement pastes and mortars as a simple means of investigating various properties such as the ratio of gel to granular substance is described in *J. Am. Conc. Inst.*, **8,** *131*, and on *p. 107* of the same journal the effect of calcium and sodium chlorides on concrete when used for ice-removal is discussed, whilst, on *p. 123*, a report of a Committee of the Institute on the effect of plastic flow and volume changes on design is given. Researches on resistance to deterioration include an article on the resistance of cement to the corrosive action of sodium sulphate solutions, a research carried out in view of the corrosiveness of the soils in the Colorado river, *J. Am. Conc. Inst.*, **8,** *83*; and a research on the sulphate resistance of Portland cement described in *U.S. Nat. Bur. Stand. Technical News Bulletin, 1936 (234), 87.* The determination of the mix proportions and cement-content of cement mortar and concrete is discussed in *Zement*, **25,** *411.* An article on the deformation of concrete under load appears in *J. Inst. Mun. and County Engineers*, **63,** *982.* Tests on the shrinkage of concrete, carried out at the Ohio State University, are given in *Eng. Expt. Stn. News*, **8,** *6.* The flow of water under constant pressure through concrete is discussed in *Comptes Rendus*, **203,** *1351* (*Eng. Abs.* **72,** 10).

Metals.

A theory of creep under the action of combined stresses is given in *Ingeniørs-Vetenskaps-Akademien, Handlingar, No. 141* (*Eng. Abs.* **72,** 19). The correlation between metallography and mechanical testing is discussed in *Univ. Illinois Eng. Expt. Stn. Bull.*, **34,** *No. 31.* Impact strength and notch sensibility of cast iron are discussed in *Giesserei*, **23,** *674* (*Eng. Abs.* **72,** 18). The influence of grain-size upon the properties of steel and its control are dealt with in *Stahl und Eis.*, **56,** *1412* (*Eng. Abs.* **72,** 32). A Paper on the effect of specimen form on the resistance of metals to combined alternating stresses is contained in *Proc. Inst. Mech. Engineers*, **132,** *549.* Tests on the ageing of mild-steel wire are included in the Fourth Report of the Corrosion Committee of the Iron and Steel Institute, *p. 209.* The corrosion of metals by water and carbon dioxide under pressure is discussed in *Ind. & Eng. Chem.*, **28,** *1078*, and the mechanism of diffusion through protective oxides and the rate of pressure on the

oxidation of nickel are dealt with in *Comptes Rendus*, **203**, *154* (*Eng. Abs.* **72**, 37). Electrolytic measurement of the corrosiveness of soils is described in *U.S. Nat. Bur. Stand. J. Research*, **17**, *363* (*Eng. Abs.* **72**, *13*).

Engineering Materials: Production, Manufacture, and Preservation.

The effect of the rate of air circulation on the rate of drying of timber is considered in *J. Council for Scientific and Industrial Research, Australia*, **9**, *171*. The compositions of protective plaster impermeable to gasoline and paraffin are detailed in *Chem. Abs.*, **30**, *6914*. Researches dealing with preservation include: an article on the surface differences of protective coatings, *Paint*, **6**, *265*, and a description of tests of organic inhibitors of corrosion, *Ind. & Eng. Chem.*, **28**, *1048*.

Structures.

Mass Structures.

A review of the subject of soil mechanics and concrete pile driving is given in *Structural Engineer*, **15**, *26*. The pressure under weirs with depressed floors and without piles is considered in *Punjab Irrigation Research Inst.*, **5**, (*6*) (*Eng. Abs.* **72**, 163). The results of tests of the penetration of moisture into masonry walls are given in *U.S. Nat. Bur. Stand. Technical News Bulletin*, *1936* (*234*), *85*. The following papers have been published in connexion with the Second Congress on Large Dams: Question VI. Geotechnical studies of foundation materials: D.*63*, Geotechnical study of the foundation soils for dams; D.*7*, Application of electrical prospecting to the study of dam sites; D.*8*, Geotechnical studies of foundation materials; D.*15*, The geology of reservoir-dam sites; D.*21*, Examples of dam construction contributing to geotechnical study of foundation; D.*35*, *Untersuchung unter Berücksichtigung der geotechnischen Beschaffenheiten des Baugrundes bei Talsperrenprojektierung*; D.*36*, *Rapport sur les essais géotechniques des terrains de fondation*; D.*37*, *Untersuchung eines Taluntergrundes auf seine Eignung als Stauwerkträger*; D.*46*, Determination of depth to bedrock; D.*54*, A method for taking earth samples of the most undisturbed natural consistency; D.*57*, Engineering geology of dam sites. Question VII. Calculation of the stability of earth dams: D.*64*, Stability of earth dams; D.*58*, A symposium. Proposed methods of calculating the stability

of earth dams ; Application of the planimeter to the Swedish method of analysing the stability of earth slopes ; Rational design of earth dams ; D.*59*, Stability of embankment foundations ; D.*40*, Stability conditions of earth bodies and analysis of slopes and foundations of earth dams ; D.*16*, *Standfestigkeitsberechnungen von Erddammen ;* D.*20*, Problems concerning stability calculation of earth dam on movement and action of infiltrating water through it ; D.*22*, Seismic stability of the earth dam ; D.*30 Calculs de stabilité des barrages en terre ;* D.*38*, *Standfestigkeitsberechnung von Staudammen ;* D.*39*, Approximate determination of stability of earthen dams ; D.*41*, Rolling the soil in dams ; D.*47*, *Détermination des caractéristiques des matériaux employés dans la confection des digues en terre. Application au calcul de la stabilité des digues ;* D.*48*, Calculation of the stability of earth dams ; D.*50*, Calculation of the stability of earth dams D.*56*, Calculation of the stability of earth dams.

Framed Structures.

General formulas for the calculation of the deflexion of straight beams are given in *Gén. Civ.*, **109,** *220* (*Eng. Abs.* **72,** 45). The strength of eccentrically-loaded steel struts is dealt with in *Bauing.*, **17,** *366* (*Eng. Abs.* **72,** 49). The influence of long-standing loads on the deformation and compressive strength of concrete and r.-c. columns is discussed in *Deutsch. Materialprüf*, **83,** *13* (*Eng. Abs.* **72,** 61). A short method for computing moments in continuous frames is given in *J. Am. Conc. Inst.*, **8,** *147*, and a solution of rigid frames of constant section by the theorem of joint translation is contained in *Univ. Washington Eng. Expt. Stn. Bull. 89.* The action of wind on structures is dealt with in *Ricerche di Ingegneria*, **4,** *123* (*Eng. Abs.* **72,** 50). Trial loading tests with a specimen frame in steel are detailed in *International Assoc. for Bridge and Structural Engineering, Second Congress, 1936, Preliminary Publication, 859.* The effect of shrinkage on the structural design of buildings is discussed in *Concrete and Constr. Eng.*, **31,** *585.* The stresses in a concrete subaqueous tunnel tube with flexible horizontal ties are analysed in *Am. Soc. Civ. Eng. Proc.*, **62,** *1519.* An investigation of the heat losses through tiled roofs is summarized in *J. Inst. Heat. Vent. Eng.*, **4,** *313.*

Constructional Operations and Methods.

The uses of steel scaffolding are dealt with in *The Structural Engineer*, **14,** *490.* A specification for concrete pile-driving has been issued by the Institution of Structural Engineers.

Transformation, Transmission and Distribution of Energy.

Research in connexion with combustion includes a study of the combustion process in the Diesel engine in *Auto. Zeit.*, **39,** *501* (*Eng. Abs.* **72,** 92), and a description of experiments with tubes on corrosion from the combustion products of gas, work carried out jointly by the Institution of Gas Engineers and Leeds University, *Gas Journal*, **216,** *434*. Heat-transfer and pressure-drop of liquids in tubes is discussed in *Ind. & Eng. Chem.*, **28,** *1429* (*Eng. Abs.* **72,** 79). The following electrical researches have been noted: The influence of voltage-variations on impulse motors, *Elek. und Masch.*, **54,** *581* (*Eng. Abs.* **72,** 98); The present limitations and future possibilities of voltage-amplification by thermionic valves, *J. Scientific Instruments*, **13,** *381*; The development and application of grid-controlled rectifiers, *Assoc. Suisse Elec.*, **27,** *685* (*Eng. Abs.* **72,** 104), and in the same Journal, *p. 698*, Progress and experience in rectifier construction (*Eng. Abs.* **72,** 103). Researches in connexion with the transmission of energy are as follows: An investigation of relative stresses in solid spur gears by the photoelastic method *Univ. Illinois, Eng. Expt. Stn. Bull.*, No. 288, and in connexion with electrical transmission: the development of stranded cables for transmission lines, *Elek. Zeit.*, **57,** *1388* (*Eng. Abs.* **72,** 107); in the same Journal, *p. 1391*, insulators for overhead transmission-lines (*Eng. Abs.* **72,** 113); Calculations of electrical surge-generator circuits, *U.S. Nat. Bur. Stand. J. Research*, **17,** *585*; the present status of the electrical earthing problem as related to the formation of the American Research Committee on Grounding, *J. Am. Water-Works Assoc.*, **28,** *1735*.

Mechanical Processes, Appliances, and Apparatus.

The following researches on welding have been noted: A list of fundamental research problems in welding and reviews of recent welding literature are given in *Am. Weld. Soc. J.*, **15,** (*11*), *1*: metallurgical study of welding, *Weld. Ind.*, **4,** *427*; the influence of slag inclusions on the quality of electric resistance butt welds, *Weld. Ind.*, **4,**, *451*; change in structure of mild steel due to welding, *Werft*, **17,** *401* (*Eng. Abs.* **72,** 126); and new developments in spot and seam welding, *Elec. Eng.*, **55,** *1371*.

Specialized Engineering Practice.

Transport.

The applications of asphalt mixtures and bituminous emulsions laid cold for road construction are described in *J. Inst. Mun. and*

County Engineers, **63**, *853*. Various Papers presented at the twenty-third annual conference on highway engineering, 1936, are published in *Univ. Illinois Eng. Expt. Stn., Circular, No. 27*. A method of non-destructive testing for faults in railway axles is given in *Organ für die Fortschritte des Eisenbahnwesens*, **91**, *504*. Wave-action against vertical-faced breakwaters is dealt with in *Ann. Lav. Pubb.*, **74**, *764*. Fire resistance tests of materials to be used in the construction of ships are enumerated in *Recherches et Inventions*, **17**, *69*.

The Report on the progress of Civil Aviation, 1935, has been issued by the Air Ministry. The following researches in connexion with air transport appear in *Luftfahrt*, Vol. **13** : *p. 371*, variation of velocity and dynamic pressure in a vertical nose-dive (*Eng. Abs.* **72**, 159) ; *p. 374*, the plane problem in wing vibration (*Eng. Abs.* **72**, 157) ; *p. 388*, determination of internal damping of vibrating wings (*Eng. Abs.* **72**, 158) ; *p. 391*, plane plate girders with non-parallel spars (*Eng. Abs.* **72**, 48) ; *p. 405*, theory of small-spin aerofoils ; *p. 410*, non-stationary lift on wings ; *p. 425*, motion of a plate entering a jet boundary ; *p. 430*, mass balancing of control surfaces ; *p. 433*, origin of airscrew noise. The following Aeronautical Research Committee Reports and Memoranda have been noted ; *No. 1691*, comparison of drag of trousered and retractable under-carriages ; *No. 1709*, experiments on a Heinkel He.70 aeroplane in the compressed-air tunnel ; *No. 1711*, a successive approximation process for solving simultaneous linear equations ; *No. 1712*, full-scale and model porpoising tests of the Singapore IIc ; *No. 1713*, full-scale tests of slots and flaps on a Heinkel He.64, with special reference to landing ; *No. 1716*, oscillations of elastic blades and wings in an airstream ; *No. 1717*, tests of aerofoils R.A.F. 69 and R.A.F. 89, with and without split flaps, in the compressed-air tunnel. Some experiences and lessons learned in spinning aeroplanes are described in *J. Roy. Aer. Soc.*, **41**, *1*.

Water-Supply and Sewage-Disposal.

The construction and testing of hydraulic models in connexion with the Muskingum watershed project are considered in *Am. Soc. Civ. Eng. Proc.*, **62**, *1501*. The results of investigations on the biologic digestion of garbage with sewage sludge are given in *Univ. Illinois Eng. Expt. Stn., Bull. No. 287*.

Heating, Ventilation, and Acoustics.

A general review of developments in heating research is contained in *J. Inst. Heat. Vent. Eng.*, **4**, *198*, and on *p. 227* a method of

determining the equivalent temperature of a warm room by means of a new form of eupatheoscope is explained. The results of tests on the thermal conductance of windows carried out at the Building Research Station are given in *J. Inst. Heat. Vent. Eng.*, **4,** *369.* The resistance of elbows in ventilating ducts and means whereby it may be reduced are described in *J. Inst. Heat. Vent. Eng.*, **4,** *269.* Hygienic and noise-reduction requirements in rooms are somewhat antagonistic, and in *Archt. and Build.*, **148,** *63,* methods of hygienic noise-absorption are described.

Miscellaneous.

A *Summary of the progress of the Geological Survey of Great Britain for the year 1935, Part I,* has been published; mention is made of work in connexion with the Inland Water Survey, soil science, building stone research, fuel research, and water pollution research. A calculation of the virtual mass and damping of a body oscillating vertically in water is set out in *Werft,* **17,** *385* (*Eng. Abs.* **72,** 150). A mathematical study of streamline flow through channels with rectangular and inclined obstructions is made in *Memoirs of the Faculty of Engineering, Hokkaido Imperial University,* **4,** *No. 1, 13.* In *Proc. Cambridge Phil. Soc.*, **32,** *598,* is a note on the slow two-dimensional motion of a viscous liquid past a sharp edge projecting into and normal to the undisturbed direction of the stream.

NOTE.

OBITUARY.

SIR BRODIE HALDANE HENDERSON, K.C.M.G., C.B., was born on the 6th March, 1869, and died at Upp Hall, Braughing, Herts, on the 28th September, 1936. He was educated in Germany, at Owen's College, Manchester, and at King's College, London. When 16 years old he became a pupil of Messrs. Beyer, Peacock and Company, and later continued his pupilage under Mr. James Livesey, M. Inst. C.E., during part of which time he acted as assistant engineer on the construction through mountainous country of the Bobadilla-Algerciras (Gibraltar) Railway. He was subsequently employed in the Civil Engineer's Department of the Lancashire and Yorkshire Railway.

On the 6th October, 1891, he entered into partnership with Mr. James Livesey and Sir (then Mr.) H. Livesey, M. Inst. C.E., under the style of Livesey, Son and Henderson, and at the time of his death was senior partner of this firm, which is now known as Livesey and Henderson. Sir Brodie, through his firm, was connected for many years with railway, dock, and harbour developments throughout Latin America as well as in Spain, China and Africa, and more recently his firm were Consulting Engineers with Messrs. Rendel, Palmer, and Tritton for the construction of the Lower Zambesi bridge, the longest bridge in the world at the time of opening and now the second longest bridge. His firm were the engineers for the construction of the Transandine tunnel driven through the Andes Mountains, and for the new docks at Buenos Aires, the construction of which was commenced in 1911 and completed in 1925.

During the War he received a commission in the Royal Engineers and rose to be Deputy Director-General of Transportation. He retired from the Army with the rank of Brigadier-General, after being mentioned four times in dispatches and being decorated with the orders of Commander of the Crown of Belgium, Officer of the Legion of Honour and the Croix de Guerre, and became a Companion of the Order of St. Michael and St. George in 1918. In 1919 he became a Companion of the Bath, and was also promoted in 1919 to be a Knight Commander of the Order of St. Michael and St. George in recognition of his services.

Sir Brodie served on the Commission of the Peace, was High

Sheriff of Hertfordshire in 1925, and was also a Deputy Lieutenant of that county.

Sir Brodie was elected an Associate Member of The Institution in 1894 and was transferred to full membership in 1899. He was elected as Member of Council in 1915 and as Vice-President in 1925, and held the office of President in 1928–29.

Sir Brodie was for many years one of the Governors of the Imperial College of Science and Technology, a member of the Delegacy of the City and Guilds College, and Hon. Consulting Engineer to the Imperial War Graves Commission.

He married in 1901 Ella, daughter of James Jones of Lechlade, who survives him and by whom he had three sons and one daughter; one of his sons, Mr. Neil B. Henderson, is a partner in the firm of Livesey and Henderson.

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